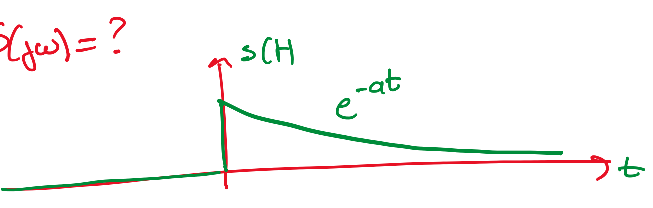
Le17

Wednesday, 12 April 2023 22:30

Es 1a

$s(t) = 1(t)\, e^{-at}$ $\quad a>0$

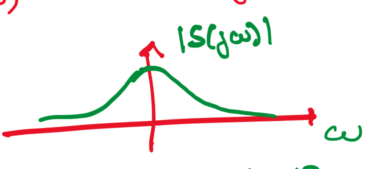
$S(j\omega) = ?$



$$S(j\omega) = \int_{-\infty\ (0)}^{+\infty} 1(t)\, \underbrace{e^{-at}\, e^{-j\omega t}}_{e^{-(a+j\omega)t}}\, dt$$

$$= \left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{+\infty}$$

$$= \frac{0 - 1}{-(a+j\omega)} = \frac{1}{a+j\omega}$$

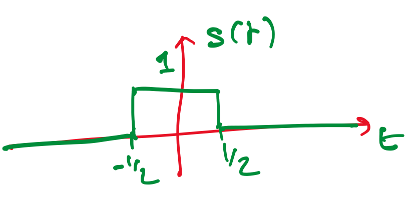


PROVATE IN MATLAB

Es 1c

$s(t) = rect(t)$

$S(j\omega) = ?$

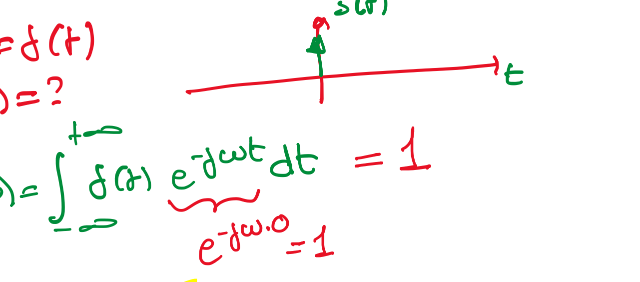


$$S(j\omega) = \int_{-\frac{1}{2}}^{1/2} rect(t)\, e^{-j\omega t}\, dt$$

$$= \left. \frac{e^{-j\omega t}}{-j\omega} \right]_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{-e^{-j\omega/2} + e^{j\omega/2}}{+j\omega}$$

$$= \frac{e^{j\omega/2} - e^{-j\omega/2}}{2j\,\frac{\omega}{2}} = \frac{\sin(\pi\omega/2\pi)}{\pi\omega/2\pi}$$

$$= \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$$



$$rect(t) \xrightarrow{\mathcal{F}} \mathrm{sinc}(\omega/2\pi)$$

Es 1d

$s(t) = \delta(t)$

$S(j\omega) = ?$

$$S(j\omega) = \int_{-\infty}^{+\infty} \delta(t)\, \underbrace{e^{-j\omega t}}_{e^{-j\omega \cdot 0} = 1}\, dt = 1$$

$$\delta(t) \xrightarrow{\mathcal{F}} 1$$

Es 1d bis

$s(t) = 1$

$S(j\omega) = ?$

~~$S(j\omega) = \int_{-\infty}^{+\infty} 1 \cdot e^{-j\omega t}\, dt = \left. \frac{e^{-j\omega t}}{-j\omega} \right]_{-\infty}^{+\infty}$~~

$X(j\omega) = \delta(\omega)$

$x(t) = ?$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \delta(\omega)\, \underbrace{e^{j\omega t}}_{e^{j0\cdot t} = 1}\, d\omega = \frac{1}{2\pi}$$

$$2\pi\ \frac{1}{2\pi} \xrightarrow{\mathcal{F}} \delta(\omega) \cdot 2\pi$$

$$1 \xrightarrow{\mathcal{F}} 2\pi\,\delta(\omega) \quad \leftarrow \delta(\omega/2\pi)$$

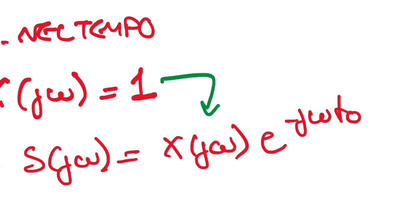
$$\delta(t) \xrightarrow{\mathcal{F}} 1$$

dualità segnale costante e delta

Es 1e

$s(t) = \delta(t - t_0)$

$S(j\omega) = ?$



(A) $$S(j\omega) = \int_{-\infty}^{+\infty} \delta(t - t_0)\, e^{-j\omega t}\, dt = e^{-j\omega t_0}$$

(B) REGOLA DI TRASL. NEL TEMPO

$$x(t) = \delta(t) \xrightarrow{\mathcal{F}} X(j\omega) = 1$$

$$s(t) = x(t - t_0) \xrightarrow{\mathcal{F}} S(j\omega) = X(j\omega)\, e^{-j\omega t_0}$$

$$S(j\omega) = e^{-j\omega t_0}$$

Es 1f

$s(t) = e^{j\omega_0 t}$

$S(j\omega) = ?$

$x(t) = 1$

$s(t) = x(t)\, e^{j\omega_0 t}$ segnale costante modulato

$\downarrow \mathcal{F}$

$X(j\omega) = 2\pi\,\delta(\omega)$

$S(j\omega) = X(j\omega - j\omega_0)$

$= 2\pi\,\delta(\omega - \omega_0)$

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} 2\pi\,\delta(\omega - \omega_0)\, e^{j\omega t}\, d\omega = e^{j\omega_0 t}$$

$$\delta(t - t_0) \xrightarrow{\mathcal{F}} e^{-j\omega t_0}$$

$$e^{j\omega_0 t} \xrightarrow{\mathcal{F}} 2\pi\,\delta(\omega - \omega_0)$$

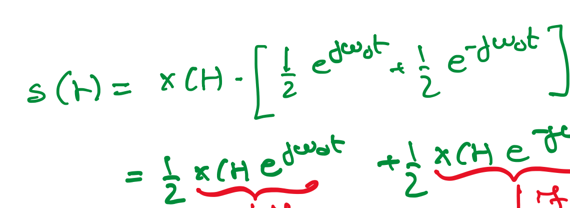
dualità esponenziale compl. a fase lineare e delta traslata

Es 1g

$$s(t) = \cos(\omega_0 t + \varphi_0) = \frac{e^{j\varphi_0}}{2} e^{j\omega_0 t} + \frac{e^{-j\varphi_0}}{2} e^{-j\omega_0 t}$$

$S(j\omega) = ?$

$$S(j\omega) = \frac{e^{j\varphi_0}}{2}\, 2\pi\,\delta(\omega - \omega_0) + \frac{e^{-j\varphi_0}}{2}\, 2\pi\,\delta(\omega + \omega_0)$$

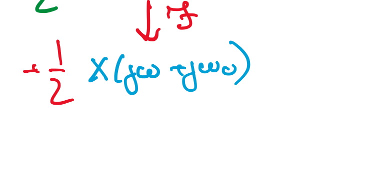


SIMMETRIA HERMITIANA

Es 1j

$s(t) = x(t)\cos(\omega_0 t)$

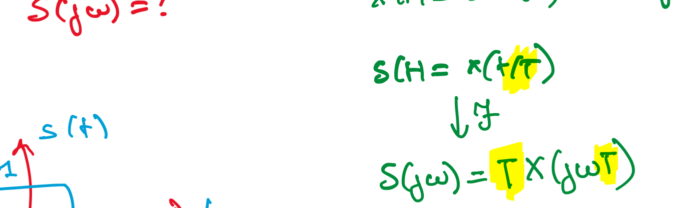
$S(j\omega) = ?$



$$s(t) = x(t) \cdot \left[\frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t}\right]$$

$$= \frac{1}{2} \underbrace{x(t)\, e^{j\omega_0 t}}_{\downarrow \mathcal{F}} + \frac{1}{2} \underbrace{x(t)\, e^{-j\omega_0 t}}_{\downarrow \mathcal{F}}$$

$$S(j\omega) = \frac{1}{2} X(j\omega - j\omega_0) + \frac{1}{2} X(j\omega + j\omega_0)$$



MODULAZIONE DSB = DOUBLE SIDE BAND

Es 1b

$s(t) = rect(t/T)$

$S(j\omega) = ?$

$$x(t) = rect(t) \xrightarrow{\mathcal{F}} X(j\omega) = \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$$

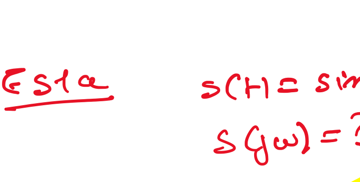
$s(t) = x(t/T)$

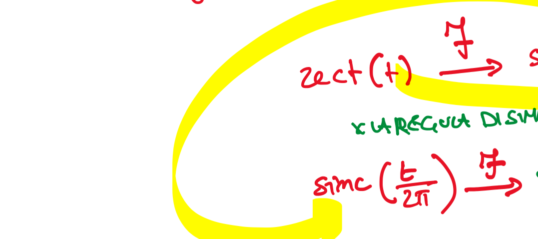
$\downarrow \mathcal{F}$

$$S(j\omega) = T\, X(j\omega T)$$

$$= T\, \mathrm{sinc}\left(\frac{\omega T}{2\pi}\right)$$

$$= T\, \mathrm{sinc}\left(\frac{\omega}{2\pi/T}\right)$$





Es 1a

$s(t) = \mathrm{sinc}(t)$

$S(j\omega) = ?$

$$rect(t) \xrightarrow{\mathcal{F}} \mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$$

x LA REGOLA DI SIMMETRIA

MOLTIPLICO PER π $\quad t \to -\omega$

$$\mathrm{sinc}\left(\frac{t}{2\pi}\right) \xrightarrow{\mathcal{F}} 2\pi\, rect(-\omega) = 2\pi\, rect(\omega)$$

$\omega \to t$

PROVATE X CASA AD ANTITRASFORMARE

$$x(t) = \mathrm{sinc}\left(\frac{t}{2\pi}\right) \xrightarrow{\mathcal{F}} X(j\omega) = 2\pi\, rect(\omega)$$

$$s(t) = x(t \cdot 2\pi) \xrightarrow{\mathcal{F}} S(j\omega) = \frac{1}{2\pi} X\left(j\omega \cdot \frac{1}{2\pi}\right)$$

SCALA $a = \frac{1}{2\pi}$

$$= \frac{1}{2\pi} \cdot 2\pi\, rect\left(\frac{\omega}{2\pi}\right)$$

$$\mathrm{sinc}(t) \xrightarrow{\mathcal{F}} rect\left(\omega/2\pi\right)$$

$$rect(t) \xrightarrow{\mathcal{F}} \mathrm{sinc}\left(\omega/2\pi\right)$$

DUALITA' SINC-RECT