Le16

Tuesday, 11 April 2023 21:45

Es 4 $s(t)$

- REALE e DISPARI
- PERIODICO $T_p = 2 \rightarrow \omega_0 = \frac{2\pi}{T_p} = \pi$
- COEF. FOURIER $S_k = 0$ per $|k| > 1$
- $P_s = 1$

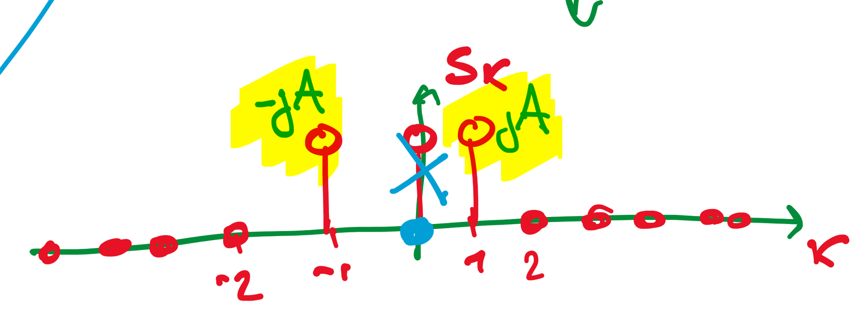
S_k HERMITIANI e DISPARI

$s(t) = ?$

PARTE REALE PARI $= 0$

PARTE IMM. DISPARI

i coeff. S_k sono IMMAGINARI e DISPARI



$S_0 = -S_{-0} = -S_0$

$2S_0 = 0$

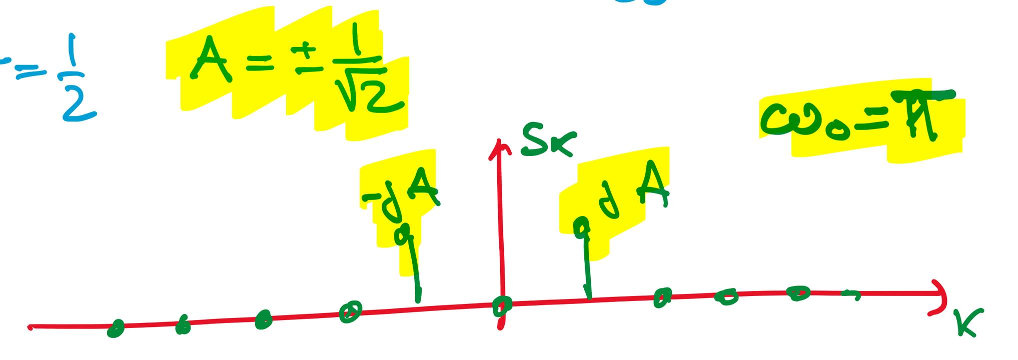
$S_0 = 0$

$P_s = 1 = \sum_k |S_k|^2$

$= A^2 + A^2 = 2A^2$

$A^2 = \frac{1}{2}$ $\quad A = \pm \frac{1}{\sqrt{2}}$

$\omega_0 = \pi$



$s(t) = \sum_k S_k e^{jk\omega_0 t}$

$= jA\, e^{j\pi t} - jA\, e^{-j\pi t}$

$= jA \left(e^{j\pi t} - e^{-j\pi t}\right)$

$\cancel{\cos\pi t} + j\sin\pi t - \left(\cancel{\cos(\pi t)} - j\sin(\pi t)\right)$

$= 2j \sin \pi t$

$s(t) = jA \cdot 2j \sin \pi t$

$= -2A \sin \pi t$

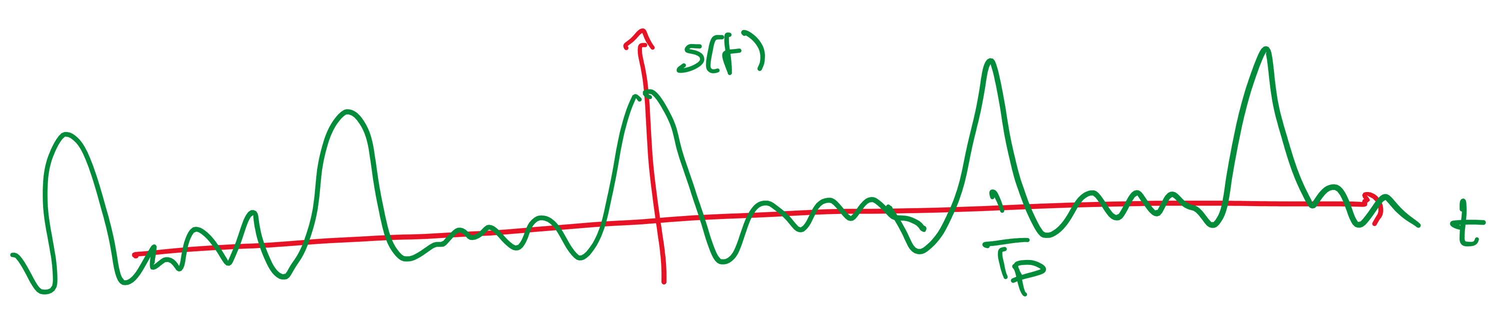
$= \pm\sqrt{2} \sin \pi t$

Es 5

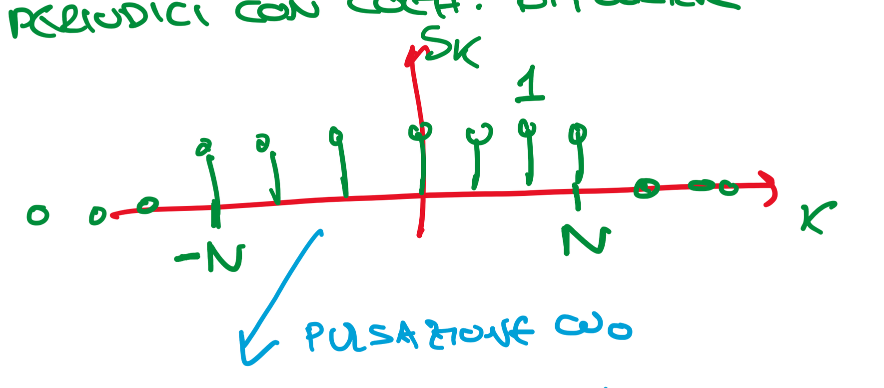
sinc periodico

$s(t) = \frac{3}{5} \cdot \frac{\sin(\pi t)}{\sin\left(\frac{\pi t}{5}\right)}$ $\quad$ PERIODICO $T_p = 10$

CALCOLARE $P_s = ?$



IL SEGNALE APPARTIENE ALLA CLASSE DI SEGNALI PERIODICI CON COEFF. DI FOURIER



PULSAZIONE ω_0

$s(t) = \sum_k S_k e^{jk\omega_0 t}$

$= \sum_{k=-N}^{N} e^{jk\omega_0 t}$

$= \sum_{m=0}^{2N} e^{j(m-N)\omega_0 t}$ $\quad k = m - N$

$= e^{-j\omega_0 N t} \sum_{m=0}^{2N} \underbrace{e^{jm\omega_0 t}}_{\alpha^m}$ $\quad \alpha = e^{j\omega_0 t}$

$= e^{-j\omega_0 N t} \frac{1-\alpha^{2N+1}}{1-\alpha}$

$= \frac{1 - e^{j\omega_0(2N+1)t}}{1 - e^{j\omega_0 t}} \cdot e^{-j\omega_0 N t}$

$= \frac{e^{-j\omega_0 N t} - e^{j\omega_0 (N+1)t}}{1 - e^{j\omega_0 t}} \cdot \frac{e^{-j\frac{\omega_0}{2}t}}{e^{-j\frac{\omega_0}{2}t}}$

$= \frac{-e^{-j\omega_0 (N+\frac{1}{2})t} + e^{j\omega_0 (N+\frac{1}{2})t}}{-e^{-j\omega_0 \frac{1}{2} t} + e^{j\omega_0 \frac{1}{2} t}}$

$s(t) = \frac{\cancel{2j} \sin\left(\omega_0 (N+\frac{1}{2}) t\right)}{\cancel{2j} \sin\left(\omega_0 \frac{1}{2} t\right)}$

$s(t) = \frac{3}{5} \cdot \frac{\sin(\pi t)}{\sin\left(\frac{\pi}{5} t\right)}$

$\cancel{\pi} = \omega_0 \left(N + \frac{1}{2}\right)$

$= \frac{2\cancel{\pi}}{5}\left(N+\frac{1}{2}\right)$

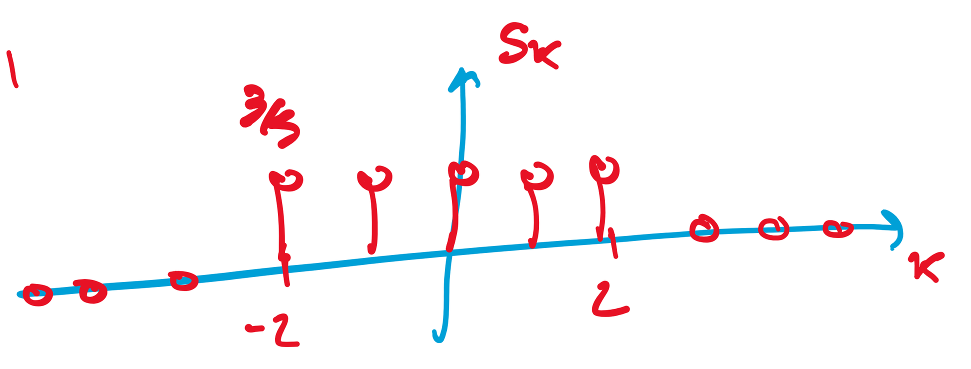
$\frac{\omega_0}{2} = \frac{\pi}{5}$

$\omega_0 = \frac{2\pi}{5}$

$T_p = 5$

$5 = 2N + 1$

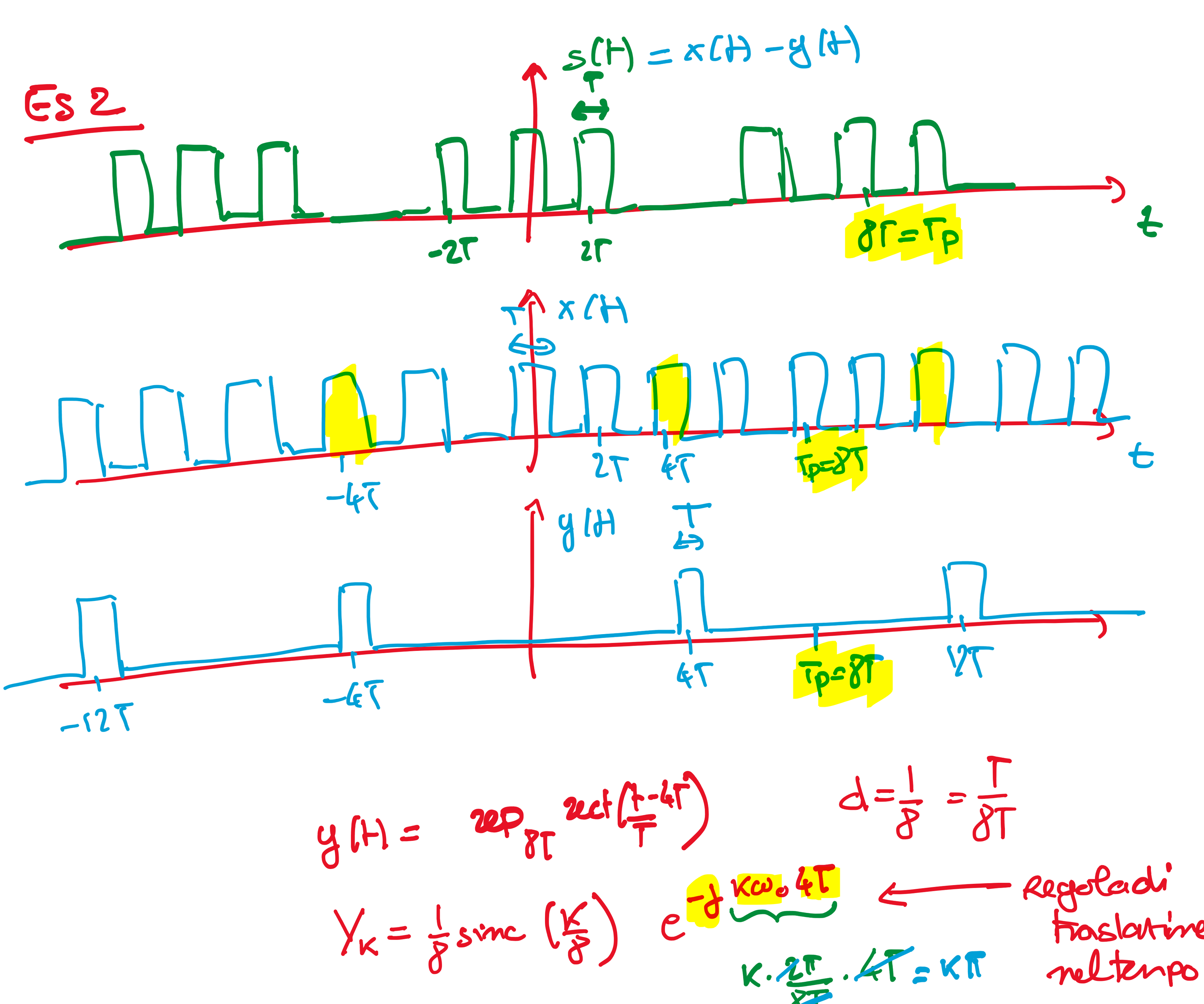
$N = 2$



$P_s = \sum_k |S_k|^2 = 5 \cdot \frac{9}{25} = \frac{9}{5}$

$m_s = S_0 = 3/5$

Es 2



$y(t) = \text{rep}_{8T}\, \text{rect}\left(\frac{t-4T}{T}\right)$ $\quad d = \frac{1}{8} = \frac{T}{8T}$

$Y_k = \frac{1}{8}\text{sinc}\left(\frac{k}{8}\right) e^{-j\underbrace{k\omega_0 4T}}$ $\leftarrow$ Regola di traslazione nel tempo

$k \cdot \frac{2\pi}{\cancel{8T}} \cdot \cancel{4T} = k\pi$

$Y_k = \frac{1}{8}\text{sinc}\left(\frac{k}{8}\right) \cancel{e^{-jk\pi}}$ $(-1)^k$

$\tilde{X}_k = \begin{cases} \frac{1}{2}\text{sinc}\left(\frac{\cancel{m}}{\cancel{2}}\right)^{k/8} & k = 4m \\ 0 & \text{altrove} \end{cases}$

$S_k = \tilde{X}_k - Y_k = \begin{cases} \overset{3/8}{\cancel{\frac{1}{2}}}\text{sinc}\left(\frac{k}{8}\right) \quad \cancel{-\frac{1}{8}\text{sinc}\left(\frac{k}{8}\right)} \overset{1}{\cancel{(-1)^k}} & k = 4m \\ +\frac{1}{8}\text{sinc}\left(\frac{k}{8}\right)(-1)^{k+1} & \text{altrove} \end{cases}$

$S_k = \frac{1}{8}\text{sinc}\left(\frac{k}{8}\right) \cdot \begin{cases} 3 & k = 4m \\ (-1)^{k+1} & \text{altrove} \end{cases}$

$\overset{?}{=} \frac{1}{8}\text{sinc}\left(\frac{k}{8}\right) \left(1 + 2\cos(k\pi/2)\right)$

uguale