Answer the following questions, providing a motivation for each answer.
(In case of negative answer the "motivation" is a counterexample)

1. Is it true that a continuous function defined on the interval $]1, 3]$ has an absolute maximum and an absolute minimum?

2. Is it true that a continuous function defined on the interval $]1, 3]$ has a relative maximum and a relative minimum?

3. Consider a function $f : ]1, 1[ \rightarrow \mathbb{R}$, and assume that it is differentiable on $]1, 1[ \backslash \{0\}$ and that the limit $\lim_{x \to 0} f'(x)$ does NOT exist. Can we conclude that the function $f$ is not differentiable at $x = 0$?

4. Suppose that a function $f : [0, 1] \rightarrow \mathbb{R}$ is differentiable at all points of $[0, 1]$ and that $f$ is continuous at $x = 0$ and $x = 1$. Can we conclude that $f$ is integrable?

5. Let $\sum_{n=1}^{\infty} a_n$ be a convergent series which does not converge absolutely. Can we conclude that $\lim_{n \to \infty} (a_n)^4 = 0$?

6. Let $\sum_{n=1}^{\infty} a_n$ be a convergent series. Is it true that the series $\sum_{n=1}^{\infty} b_n$, with $b_n = -a_n \ \forall n \in \mathbb{N}$, is a convergent series as well?

7. Consider a complex polynomial $P(z) = b_5 z^5 + b_4 z^4 + b_3 z^3 + b_2 z^2 + b_1 z + b_0 \ (b_0, \ldots, b_5 \in \mathbb{C})$. Is it true that the equation $P(z) = 0$ has at least one real solution?

8. Same as in the previous question but with $b_0, \ldots, b_5 \in \mathbb{R}$

9. Consider a function $f : [0, +\infty) \rightarrow \mathbb{R}$ and assume that the series $\sum_{n=1}^{\infty} a_n$, where $a_n := f(n) \ \forall n \in \mathbb{N}$, converges. Can we conclude that $\lim_{x \to \infty} f(x) = 0$?