R^3 m = e_1 - 3e_3, V = -2e_2 + e_3, W = 2e_1 + e_2 + e_3, t = e_1 + e_3
L'insieme dei vettori base canonici R^3

\[ \left( \begin{array}{c} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{array} \right) \] 
\[ \left( \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right) 
\quad \left( \begin{array}{c} 0 \\ 1 \\ 1 \end{array} \right) \]

(2) Mostre dip. lineare m, v, w, t

\[ \begin{align*}
a - 16a + d &= 0 \\
-15a + d &= 0 \\
a &= 1 \\
b &= 4 \\
c &= -8 \\
d &= 15a
\end{align*} \]

\[ m - 4v - 8w + 15t = \left( \begin{array}{c} 0 \\ 0 \end{array} \right) \]

trovare basi per U, W e per U+W e per V+W

(6) \( U = \langle m, v \rangle, W = \langle w, t \rangle \)

\[ \vec{a} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \vec{b} \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \vec{a} = \vec{b} = 0 \Rightarrow m \in V \text{ lin. ind.} \]

base \( \{m, v\} \Rightarrow \dim_{R^3}(U) = 2 \)

Analogo base per \( W = \{w, t\} \),

\[ \dim_{R^3}(W) = 2 \]

\[ U \cap W \cap m - 4v - 8w + 15t = 0 = 0_{R^3} \]

\[ U \cap W \cap m \leq w \]

\[ \dim(U \cap W) \geq 1 \]

Può \( \dim(U \cap W) = 2 \)? Se lo fosse avrei \( U \cap W = U = W \)

\( m \in W \) \( \Leftrightarrow \exists n, m \text{ t.c. } n \in W + mt = m \)

\[ \begin{pmatrix} 2n + m \\ m \end{pmatrix} = \begin{pmatrix} n + m \\ n \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]

\[ 2n + m = n + (n + m) \]

\[ 1 = 0 - 3 = -3 \]

\[ m \not\in W \Rightarrow W \neq U \]

\[ \Rightarrow \dim(U \cap W) = 1 \]
\[ U + W = \mathbb{R}^3 \]

\[ \dim(U + W) = \dim U + \dim W - \dim(U \cap W) = 2 + 2 - 1 = 3 \]

Base for \( U + W = \{ e_1, e_2, e_3, e_4, e_5, e_6, e_7 \} \)

**INTERMEZZO**

Proof \( U \cap W = \{ 0 \} \neq \mathbb{R}^3 \)?

\[ \dim(U + W) = \dim U + \dim W - \dim(U \cap W) \]

Case 1: \( \dim(U \cap W) = 0 \) \( \iff U \cap W = \{ 0 \} \neq \mathbb{R}^3 \)

Case 2: \( \dim(U + W) = 2 + 2 - 0 = 4 \)

Let \( V' = 2e_1 - 3e_2 + e_3 = \begin{pmatrix} 2 & -3 & 1 \end{pmatrix} \)

\( V' \in U \cap W \iff \exists a \in \mathbb{R} \) s.t. \( a \begin{pmatrix} 1 \\ -7 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \)

\( V' \in U \cap W \iff \exists a \) s.t. \( a \begin{pmatrix} 1 \\ -7 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \)

\( V \in U \cap W \iff \exists a \) s.t. \( a \begin{pmatrix} 1 \\ -7 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \)

\( V \in U \cap W \iff \exists a \) s.t. \( a \begin{pmatrix} 1 \\ -7 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \)

Analogous to \( W = \{ (x_1, x_2, x_3) \mid x_1 = x_2 + x_3, \ x_2 = a, \ x_3 = a + b \} \),

\[ x_1 = x_2 + x_3 \]

\[ x_1 - x_2 - x_3 = 0 \]

\( U \cap W = \{ 6x_1 + x_2 + 2x_3 = 0, \ x_1 - x_2 - x_3 = 0 \} \)
\[ U = \begin{pmatrix} 1 & 0 \\ -2 & 0 \end{pmatrix}, \quad U_2 = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}, \quad U_3 = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} \quad \mathbb{C}^4 \]

\[
\begin{cases}
a + b = 0 \\
b + c = 0 \\
-2a - b + c = 0 \\
-c = 0
\end{cases} \Rightarrow a = b = c = 0
\]

\[ U = \langle U_1, U_2, U_3 \rangle \]

\[ X_1 = a + b \\
X_2 = b + c \\
X_3 = -2a - b + c \\
X_4 = -c
\]

\[ a = x_1 - x_2 - x_4 \\
b = x_2 + x_4 \\
c = -x_4
\]

\[ X_3 = -2x_1 + 2x_2 + 2x_4 - x_2 - x_4 - x_4 = -2x_1 + x_2 \]

\[ U : 2x_1 - x_2 + x_3 = 0 \]

\[ \text{Completere ad una base di } \mathbb{C}^4 \]

\[ \mathbb{C}^4 = \langle U_1, U_2, U_3, e_4 \rangle \]

\[ \text{Determinare } W_0 \text{ t.c. } \mathbb{C}^4 = U \oplus W_0 \]

\[ \dim(U) = 3 \]

\[ \dim(W_0) = 4 - 3 = 1 \]

\[ W_0 = \langle e_4 \rangle \]

\[ \text{Determinare tutti } W \text{ t.c. } \mathbb{C}^4 = U \oplus W \]

\[ W = \langle x_1 u_1 + x_2 u_2 + x_3 u_3 + x_0 e_1 \rangle \]

\[ x_0 = 1 \]

\[ W = \langle x_1 u_1 + x_2 u_2 + x_3 u_3 + e_4 \rangle \]

\[ W \not\subset U \Rightarrow x_0 \neq 0 \]

\[ \dim(\mathbb{C}^4) = 4 \\
\dim(U) = 3 \\
\dim(U \cap W_0) = 0 \\
\dim(W) = 1 \]