Neutrinos Oscillations

- Neutrinos flavours
- Mass and weak interactions eigenstates
- Two flavours neutrinos oscillations
- Three flavours oscillations
- Experiments of neutrinos oscillations
- Reactor experiments
- Long-baseline experiments at the accelerators
- Solar neutrinos
- Super-Kamiokande and SNO experiments
- Present situation

Neutrinos flavours

- Neutrinos cannot be observed directly (they are neutral), they are seen through their weak interactions.
- The neutrinos flavours are verified from the flavours of the charged leptons produced in the CC interactions.
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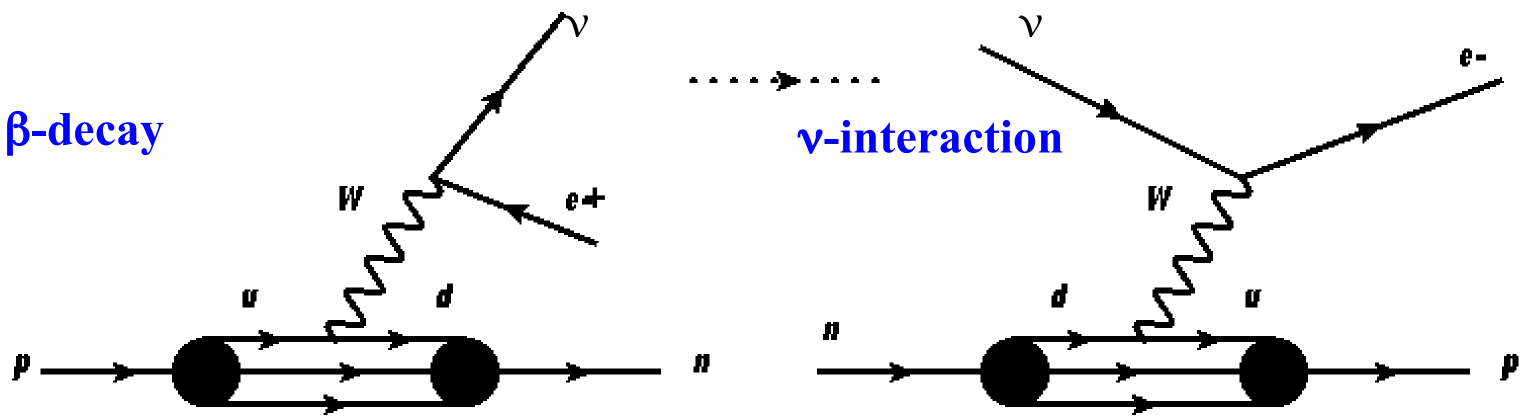
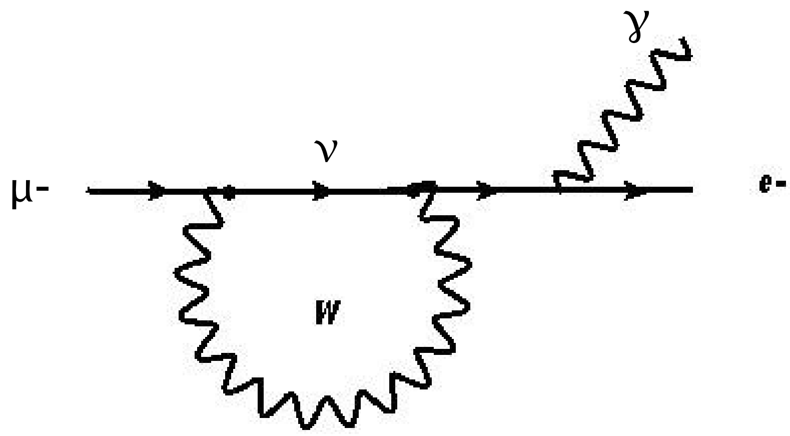


- In a natural manner the idea of conservation of the electronic lepton number was born.
- The same observation can be done for the muon neutrinos.

Neutrinos flavours

- Further evidence of the difference between the muon and electron neutrinos: non observation of the decay: $\mu \rightarrow e^{-}\gamma$
  From the experimental point of view one knows that the branching ratio is very small: $< 10^{-11}$ even if it can procede in the following manner:



Wμν vertex different from the Weν vertex

Neutrinos mixing

The neutrinos are the only known elementary particles which do not conserve the flavour with which they were born. The change of neutrinos flavours happen in **two different manners**:

- **oscillation in the vacuum** (similar but not identical to those of K^0, the latters are composite objects)
  - in the **kinetic energy** of the Hamiltonian
  - observed in
    - ν_μ indirectly produced by the cosmic rays collisions in the atmosphere;
    - neutrinos produced by the accelerators;
    - neutrinos produced by nuclear reactors.
- **transformation in the matter** (Mikehev-Smirnov-Wolfenstein effect)
  - **dynamical phenomena**, due to the interaction of the ν_e with the electrons
  - observed as dominant process in the ν_e coming from the Sun for energies > 2 MeV

Because both processes happen it is necessary that, differently from the Standard Model:

- the masses of the various neutrinos have to be not all equal, then different from zero
- the lepton flavours do not conserve themselves

Neutrinos mixing

The neutrinos produced and observed (through the weak interactions), ν_e, ν_μ, ν_τ are not the eigenstates with definite mass ν_1, ν_2, ν_3 (**masses =** $\boldsymbol{m_1}$**,** $\boldsymbol{m_2}$ **e** $\boldsymbol{m_3}$) but linear combination of them.

The very small differences between the squared masses of the neutrinos imply that the characteristic times both of the vacuum oscillations and of the matter transformations are very long →
the " oscillations" distances with the available energies are of the order of thousands kilometres →
they cannot be observed in a standard experiment at an accelerator.

Mass eigenstates and Weak eigenstates

- The **mass eigenstates** are the stationary states of the free particle Hamiltonian:

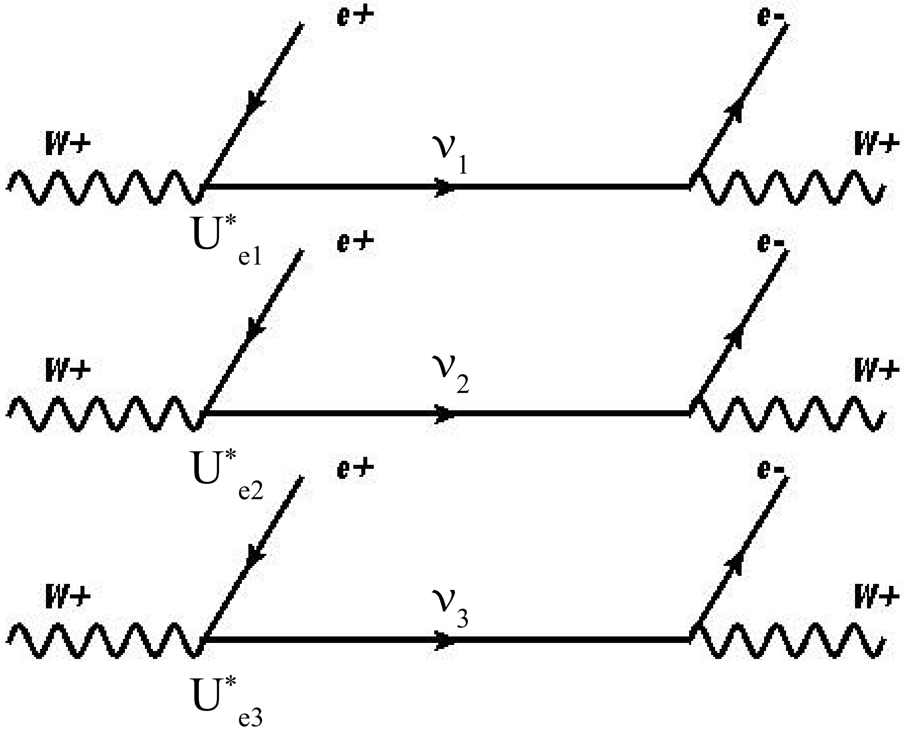
$$H\psi = i\frac{\partial\psi}{\partial t} = E\psi$$

- The mass eigenstates have the following time evolution:

$$\psi(\boldsymbol{x},t)=\phi(\boldsymbol{x})e^{-iEt}$$

- The mass eigenstates (the fundamental particles) are indicated with: ν_1, ν_2, ν_3
- They do not coincide with the weak interaction eigenstates: ν_e, ν_μ, ν_τ produced with the same flavour of the charged lepton in the weak interactions.

Mass eigenstates and Weak eigenstates



- At the vertex one of the mass eigenstates is produced
- It is not possible to know which mass eigenstates has been produced: coherent linear superposition of ν_1, ν_2, ν_3

Mass eigenstates and Weak eigenstates

- Relationship between the mass eigenstates and the weak eigenstates through the unitary matrix U:

Flavour states

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Stationary states

- The electron neutrino, which is the quantum state produced together with a positron in a CC interaction, is a linear combination of the mass eigenstates defined by the CC couplings of ν_1, ν_2, ν_3 at the vertex $W \rightarrow e^+ \nu$

$$|\psi> \; = \; U^*_{e1}|\nu_1> + U^*_{e2}|\nu_2> + U^*_{e3}|\nu_3>$$

- The electron neutrino later propagates as a coherent linear superposition of ν_1, ν_2, ν_3 and later interacts and the wave function collapses into a weak eigenstate, with the corresponding charged lepton of defined flavour.
- If the masses of ν_1, ν_2, ν_3 are not equal, phase differences are produced between the different components of the wave function, the phenomenon of the **neutrinos oscillations** happens.
- A ν produced with a charged lepton of a certain type flavour can later interact to produce a charged lepton of different type of flavour.

The CC leptonic vertex revisited

- The CC vertex is usually written in this manner:

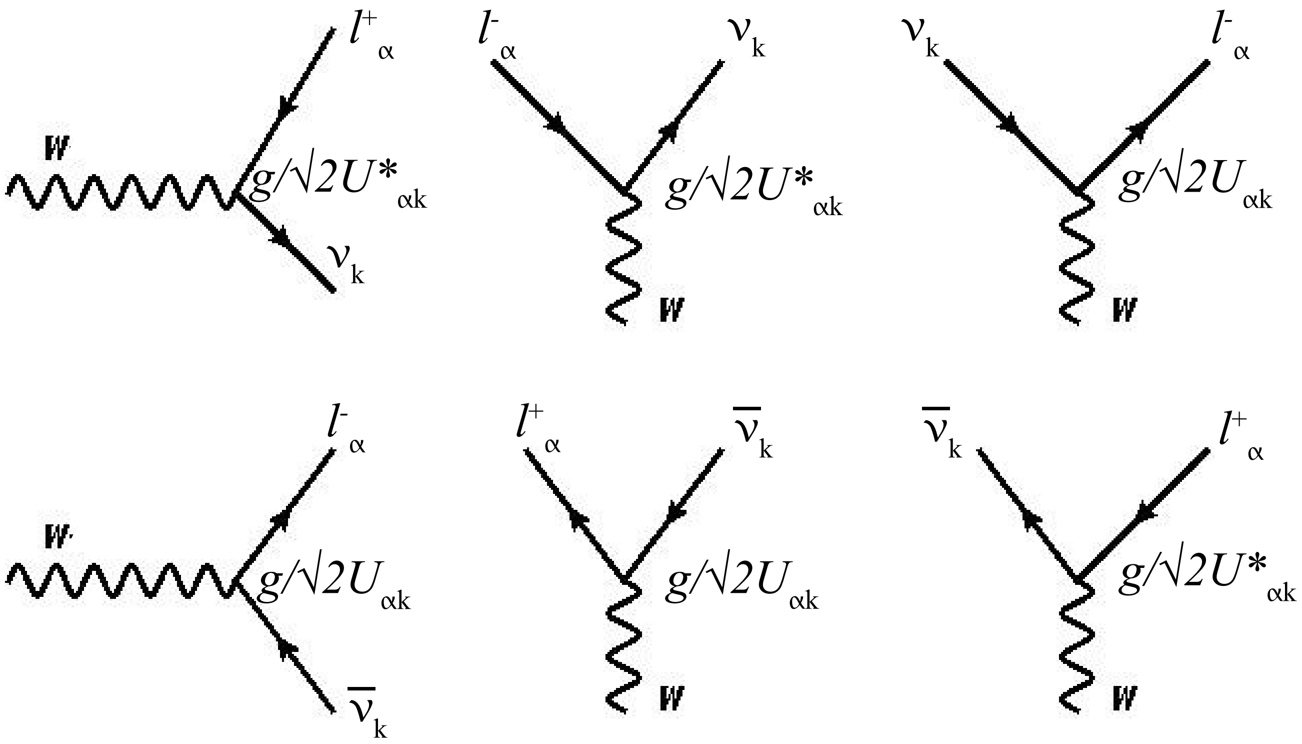
$$-i\frac{g}{\sqrt{2}}\bar{e}\gamma^{\mu}\frac{1}{2}\left(1-\gamma_5\right)\nu_e$$

- The CC vertex rewritten as a function of the mass eigenstates becomes:

$$-i\frac{g}{\sqrt{2}}\bar{l}_{\alpha}\gamma^{\mu}\frac{1}{2}\left(1-\gamma_5\right)U_{\alpha k}\nu_k$$

- Defining the neutrino state produced in a weak interaction through the U matrix, implies that, when the neutrino appears as adjoint spinor, the factor $U^*_{\alpha k}$ appears at the weak interaction vertex.

The CC leptonic vertex revisited



Two flavour neutrinos oscillation

- Supposing to have the two weak eigenstates: ν_e and ν_μ , they are linear superposition of the two mass eigenstates: ν_1 and ν_2
- ν_1 and ν_2 propagates in the vacuum as:

$$|\nu_1(t)> \;=\; |\nu_1> e^{-ip_1 x}$$
$$|\nu_2(t)> \;=\; |\nu_2> e^{-ip_2 x}$$

- the relationship between the mass eigenstates and those of the weak interactions is:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

- Supposing that at the time $t = 0$, a ν_e neutrino is produced, the wave function at the time $t = 0$ is:

$$|\psi(0)> \;=\; |\nu_e> \;\equiv \cos\theta\,|\nu_1> + \sin\theta\,|\nu_2>$$

Two flavour neutrinos oscillation

- The state evolves with time in the following manner:

$$|\psi(\boldsymbol{x},t)> \;=\; \cos\theta|\nu_1>e^{-ip_1x}+\sin\theta|\nu_2>e^{-ip_2x}$$

- If the neutrino interacts after a time T and at a distance L, the wave function is:

$$|\psi(L,T)> \;=\; \cos\theta|\nu_1>e^{-i\phi_1}+\sin\theta|\nu_2>e^{-i\phi_2}$$

- where the phases of the two mass eigenstates are:

$$\phi_i \;=\; p_i\cdot x \;=\; E_iT-p_iL$$

- Knowing that

$$\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

- one obtains:

$$\begin{aligned}|\psi(L,T)> &= \cos\theta\left(\cos\theta|\nu_e>-\sin\theta|\nu_\mu>\right)e^{-i\phi_1}+\sin\theta\left(\sin\theta|\nu_e>+\cos\theta|\nu_\mu>\right)e^{-i\phi_2}\\ &= \left(e^{-i\phi_1}\cos^2\theta+e^{-i\phi_2}\sin^2\theta\right)|\nu_e>-\left(e^{-i\phi_1}-e^{-i\phi_2}\right)\cos\theta\sin\theta|\nu_\mu>\\ &= e^{-i\phi_1}\left[\left(\cos^2\theta+e^{i\Delta\phi_{12}}\sin^2\theta\right)|\nu_e>-\left(1-e^{i\Delta\phi_{12}}\right)\cos\theta\sin\theta|\nu_\mu>\right]\end{aligned}$$

Two flavour neutrinos oscillation

- where:

$$\Delta\phi_{12} = \phi_1 - \phi_2 = (E_1 - E_2)T - (p_1 - p_2)L$$

- If $\Delta\phi_{12} = 0$ the neutrino remains in a pure electronic neutrino state, in a following CC interaction it will produce an electron.
- If $\Delta\phi_{12} \neq 0$ there will be a muonic neutrino component:

$$|\psi(L,T)\rangle = \langle\nu_e|\psi\rangle|\nu_e\rangle + \langle\nu_\mu|\psi\rangle|\nu_\mu\rangle = c_e|\nu_e\rangle + c_\mu|\nu_\mu\rangle$$

- The probability that the neutrino born as electronic neutrino interacts producing a muon is:

$$P(\nu_e \to \nu_\mu) = c_\mu c_\mu^* = \left(1 - e^{i\Delta\phi_{12}}\right)\left(1 - e^{-i\Delta\phi_{12}}\right)\cos^2\theta\sin^2\theta$$

$$= \frac{1}{4}\left(2 - 2\cos\Delta\phi_{12}\right)\sin^2(2\theta)$$

$$= \sin^2(2\theta)\sin^2\left(\frac{\Delta\phi_{12}}{2}\right)$$

- then the oscillation probability $\nu_e \to \nu_\mu$ depends on the mixing angle θ and on the phase difference between the two mass eigenstates, $\Delta\phi_{12}$

Two flavour neutrinos oscillation

- Supposing that the tri-momenta are equal $p_1 = p_2 = p$:

$$\Delta \phi_{12} = (E_1 - E_2) T = \left[p \left(1 + \frac{m_1^2}{p^2} \right)^{1/2} - p \left(1 + \frac{m_2^2}{p^2} \right)^{1/2} \right] T$$

- because $m \ll \mathrm{E}$ one has:

$$\left(1 + \frac{m^2}{p^2} \right)^{1/2} \approx 1 + \frac{m^2}{2\, p^2}$$

- where the phases of the two mass eigenstates are:

$$\Delta \phi_{12} \approx \frac{m_1^2 - m_2^2}{2\, p} L$$

assuming that $T \approx L$ (in natural units) and that the neutrinos are ultrarelativistic.

- The same phase difference is obtained eliminating the hypothesis about the tri-momenta and treating the neutrinos as wave packets propagating in a coherent manner.

Two flavour neutrinos oscillation

- Combining the preceding formulas and assuming $p = E_\nu$:

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta)\sin^2\left(\frac{(m_1^2 - m_2^2)L}{4E_\nu}\right)$$

appearance probability

- rewriting using more convenient units:

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta)\sin^2\left(1.27\frac{\Delta m^2[eV^2]L[km]}{E_\nu[GeV]}\right)$$

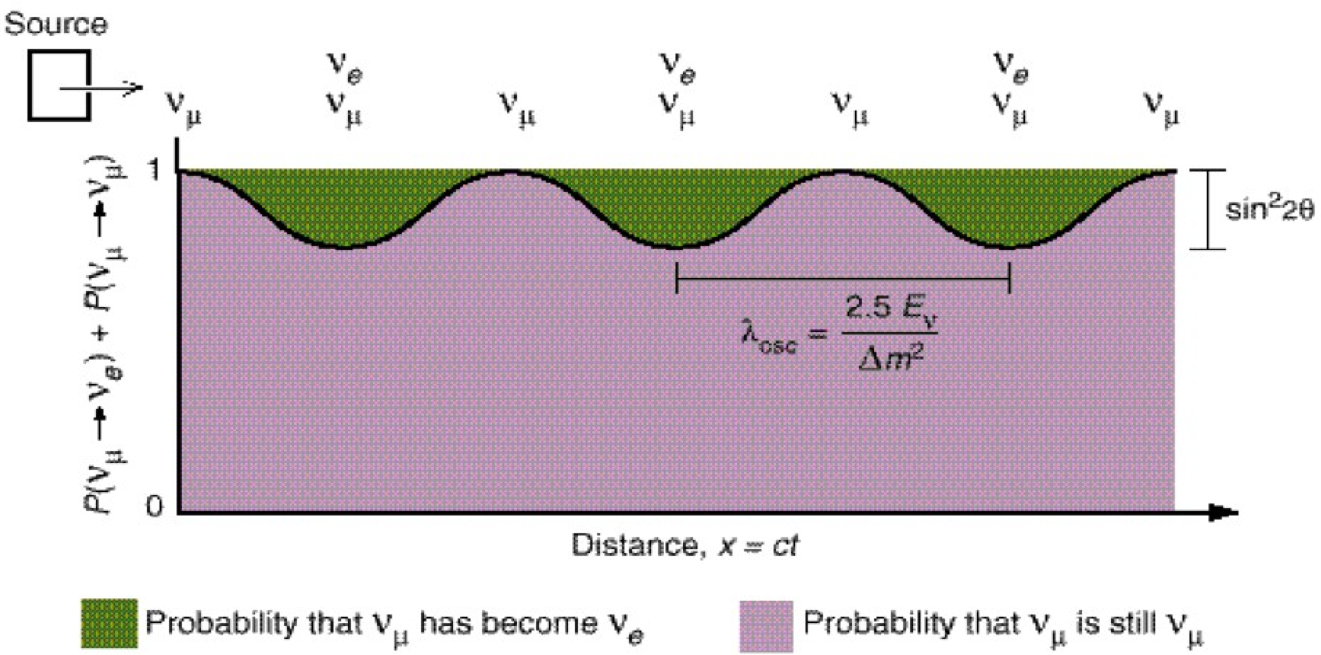
where L is in km, Δm^2 in eV^2 and the neutrinos energy in GeV.

- One has also :

$$P(\nu_e \rightarrow \nu_e) = 1 - P(\nu_e \rightarrow \nu_\mu) = 1 - \sin^2(2\theta)\sin^2\left(\frac{(m_1^2 - m_2^2)L}{4E_\nu}\right)$$

disappearance probability

Two flavour neutrinos oscillation



$$\lambda_{osc}[km] = \frac{\pi E_\nu [GeV]}{1.27\, \Delta m^2 [eV^2]}$$

If $\Delta m^2 = 0.002$ eV^2 and $E_\nu = 1$ GeV → $\lambda_{osc} = 1236$ km

For small values of Δm^2 the flavour oscillations develop only for long distances. This explains why the neutrino flavours is seen conserved in the first experiments with neutrinos.

Three flavour neutrinos oscillation

- With three flavours the relationship becomes (U = unitary matrix of Pontecorvo-Maki-Nakagawa-Sakata (PMNS)):

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

- The elements of the PMNS matrix are fundamental parameters of the leptonic flavour sector of the Standard Model.
- The mass eigenstates are expressed as:

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U^*_{e1} & U^*_{\mu 1} & U^*_{\tau 1} \\ U^*_{e2} & U^*_{\mu 2} & U^*_{\tau 2} \\ U^*_{e3} & U^*_{\mu 3} & U^*_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

- The unitarity condition, $UU^\dagger = 1$, implies that:

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} U^*_{e1} & U^*_{\mu 1} & U^*_{\tau 1} \\ U^*_{e2} & U^*_{\mu 2} & U^*_{\tau 3} \\ U^*_{e3} & U^*_{\mu 3} & U^*_{\tau 3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Three flavour neutrinos oscillation

- which provides 9 relationships between the elements of the PMNS matrix, for example:

$$U_{e1}U^*_{e1}+U_{e2}U^*_{e2}+U_{e3}U^*_{e3} = 1$$
$$U_{e1}U^*_{\mu 1}+U_{e2}U^*_{\mu 2}+U_{e3}U^*_{\mu 3} = 0 \quad (*)$$

- Supposing that at the time $t = 0$ an electronic neutrino is produced:

$$|\psi(0)> = |\nu_e> = U^*_{e1}|\nu_1> + U^*_{e2}|\nu_2> + U^*_{e3}|\nu_3>$$

- its time evolution is:

$$|\psi(\boldsymbol{x},t)> = U^*_{e1}|\nu_1>e^{-i\phi_1} + U^*_{e2}|\nu_2>e^{-i\phi_2} + U^*_{e3}|\nu_3>e^{-i\phi_3}$$

- the successive CC weak interactions can be described by:

$$\begin{aligned}|\psi(\boldsymbol{x},t)> = {} & U^*_{e1}(U_{e1}|\nu_e> + U_{\mu 1}|\nu_\mu> U_{\tau 1}|\nu_\tau>)e^{-i\phi_1} \\ & + U^*_{e2}(U_{e2}|\nu_e> + U_{\mu 2}|\nu_\mu> + U_{\tau 2}|\nu_\tau>)e^{-i\phi_2} \\ & + U^*_{e3}(U_{e3}|\nu_e> + U_{\mu 3}|\nu_\mu> + U_{\tau 3}|\nu_\tau>)e^{-i\phi_3}\end{aligned}$$

Three flavour neutrinos oscillation

- the equation can be rewritten as:

$$
\begin{aligned}
|\psi(\boldsymbol{x},t)> \;=\; & (U^*_{e1} U_{e1} e^{-i\phi_1} + U^*_{e2} U_{e2} e^{-i\phi_2} + U^*_{e3} U_{e3} e^{-i\phi_3})|\nu_e> \\
+\; & (U^*_{e1} U_{\mu 1} e^{-i\phi_1} + U^*_{e2} U_{\mu 2} e^{-i\phi_2} + U^*_{e3} U_{\mu 3} e^{-i\phi_3})|\nu_\mu> \\
+\; & (U^*_{e1} U_{\tau 1} e^{-i\phi_1} + U^*_{e2} U_{\tau 2} e^{-i\phi_2} + U^*_{e3} U_{\tau 3} e^{-i\phi_3})|\nu_\tau>
\end{aligned}
$$

- The oscillation probability $\nu_e \rightarrow \nu_\mu$ is:

$$
\begin{aligned}
P(\nu_e \rightarrow \nu_\mu) \;&=\; |<\nu_\mu|\psi(\boldsymbol{x},t)>|^2 \;=\; c_\mu c^*_\mu \\
&=\; |U^*_{e1} U_{\mu 1} e^{-i\phi_1} + U^*_{e2} U_{\mu 2} e^{-i\phi_2} + U^*_{e3} U_{\mu 3} e^{-i\phi_3}|^2
\end{aligned}
$$

- such equation can be simplified using the following identity between complex numbers:

$$
|z_1+z_2+z_3|^2 \;=\; |z_1|^2+|z_2|^2+|z_3|^2+2\,\Re\{z_1 z^*_2 + z_1 z^*_3 + z_2 z^*_3\}
$$

Three flavour neutrinos oscillation

- $$P(\nu_e \to \nu_\mu) = |U^*_{e1}U_{\mu 1}|^2 + |U^*_{e2}U_{\mu 2}|^2 + |U^*_{e3}U_{\mu 3}|^2 + 2\Re\{U^*_{e1}U_{\mu 1}U_{e2}U^*_{\mu 2}e^{-i(\phi_1-\phi_2)}\}$$
$$+2\Re\{U^*_{e1}U_{\mu 1}U_{e3}U^*_{\mu 3}e^{-i(\phi_1-\phi_3)}\} + 2\Re\{U^*_{e2}U_{\mu 2}U_{e3}U^*_{\mu 3}e^{-i(\phi_2-\phi_3)}\}$$

- using the identity between complex numbers and the unitarity condition (*) of the U matrix ($z_1 = U^*_{e1}U_{\mu 1}; z_2 = U^*_{e2}U_{\mu 2}; z_3 = U^*_{e3}U_{\mu 3}$):

$$U^*_{e1}U_{\mu 1} + U^*_{e2}U_{\mu 2} + U^*_{e3}U_{\mu 3} = 0 \quad \to \quad |U^*_{e1}U_{\mu 1} + U^*_{e2}U_{\mu 2} + U^*_{e3}U_{\mu 3}|^2 = 0$$
$$|U^*_{e1}U_{\mu 1}|^2 + |U^*_{e2}U_{\mu 2}|^2 + |U^*_{e3}U_{\mu 3}|^2 +$$
$$2\Re\{U^*_{e1}U_{\mu 1}U_{e2}U^*_{\mu 2} + U^*_{e1}U_{\mu 1}U_{e3}U^*_{\mu 3} + U^*_{e2}U_{\mu 2}U_{e3}U^*_{\mu 3}\} = 0$$

- one obtains:

$$P(\nu_e \to \nu_\mu) = 2\Re\{U^*_{e1}U_{\mu 1}U_{e2}U^*_{\mu 2}[e^{-i(\phi_1-\phi_2)}-1]\} +$$
$$2\Re\{U^*_{e1}U_{\mu 1}U_{e3}U^*_{\mu 3}[e^{-i(\phi_1-\phi_3)}-1]\} +$$
$$2\Re\{U^*_{e2}U_{\mu 2}U_{e3}U^*_{\mu 3}[e^{-i(\phi_2-\phi_3)}-1]\}$$

Three flavour neutrinos oscillation

- while for the survival probability of the electronic neutrino:

$$P(\nu_e \to \nu_e) = 1 + 2|U_{e1}|^2 |U_{e2}|^2 \Re\{[e^{-i(\phi_1-\phi_2)} - 1]\}$$
$$+ 2|U_{e1}|^2 |U_{e3}|^2 \Re\{[e^{-i(\phi_3-\phi_1)} - 1]\}$$
$$+ 2|U_{e2}|^2 |U_{e3}|^2 \Re\{[e^{-i(\phi_3-\phi_2)} - 1]\}$$

simplifying with:

$$\Re\{e^{i(\phi_j-\phi_i)} - 1\} = \cos(\phi_j - \phi_i) - 1 = -2\sin^2\left(\frac{\phi_j - \phi_i}{2}\right) = -2\sin^2\Delta_{ji}$$

where:

$$\Delta_{ji} = \frac{\phi_j - \phi_i}{2} = \frac{(m_j^2 - m_i^2)L}{4E_\nu}$$

- and then:

$$P(\nu_e \to \nu_e) = 1 - 4|U_{e1}|^2 |U_{e2}|^2 \sin^2\Delta_{21}$$
$$-4|U_{e1}|^2 |U_{e3}|^2 \sin^2\Delta_{31} - 4|U_{e2}|^2 |U_{e3}|^2 \sin^2\Delta_{32}$$

Three flavour neutrinos oscillation

- only two of the squared mass differences are independent, in fact:

$$\Delta_{31} = \Delta_{32} + \Delta_{21}$$

Neutrinos masses and mass hierarchy

- The neutrinos oscillations give only information about the squared mass differences of the neutrinos. **They do not give constraints on the neutrino absolute mass scale.**
- In this moment there are not direct measurements of the neutrinos masses, only upper limits.
- From the study of the **end-point of the tritium beta decay:**

$$m_\beta = \sqrt{\sum_{k=1}^{3} |U_{ek}|^2 m_k^2} < 0.8\, eV$$

- From the study of the **neutrinoless double beta decay**:

$$m_{\beta\beta} = \sum_{k=1}^{3} U_{ek}^2 m_k < 0.1\, eV$$

- Indirect, model dependent, measurements from the **cosmology**. The relic neutrinos coming from Bing Bang have low energy and a density: $O(100)$ cm^{-3}. Given this high density, the neutrinos masses have an impact on the evolution of the Universe. From recent data of the large scale of the Universe:

$$\sum_{k=1}^{3} m_k \leqslant 0.12\ eV$$

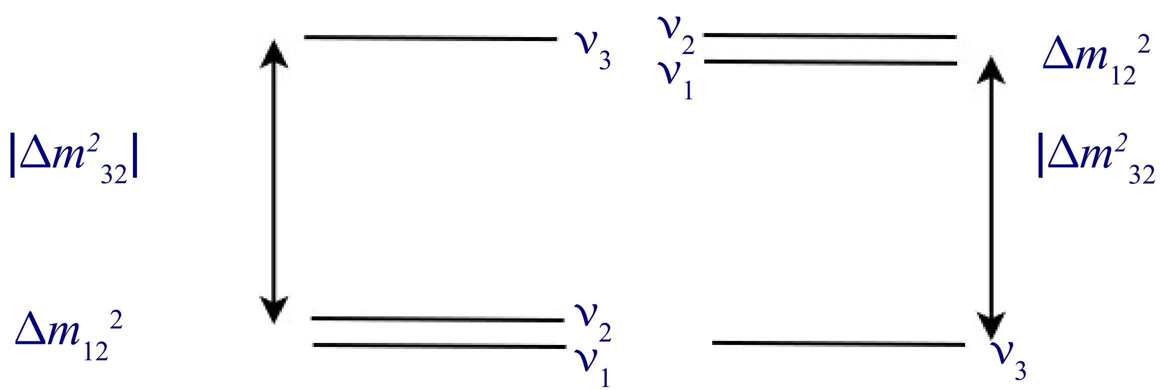
- Even if we do not know their masses, certainly they are much lower than the masses of leptons and quarks (factors bigger than 10^6 respect to the electron mass).
- Theoretical hypothesis to explain such difference: **see-saw mechanism**.

Neutrinos masses and mass hierarchy

- The results of the oscillation experiments give:

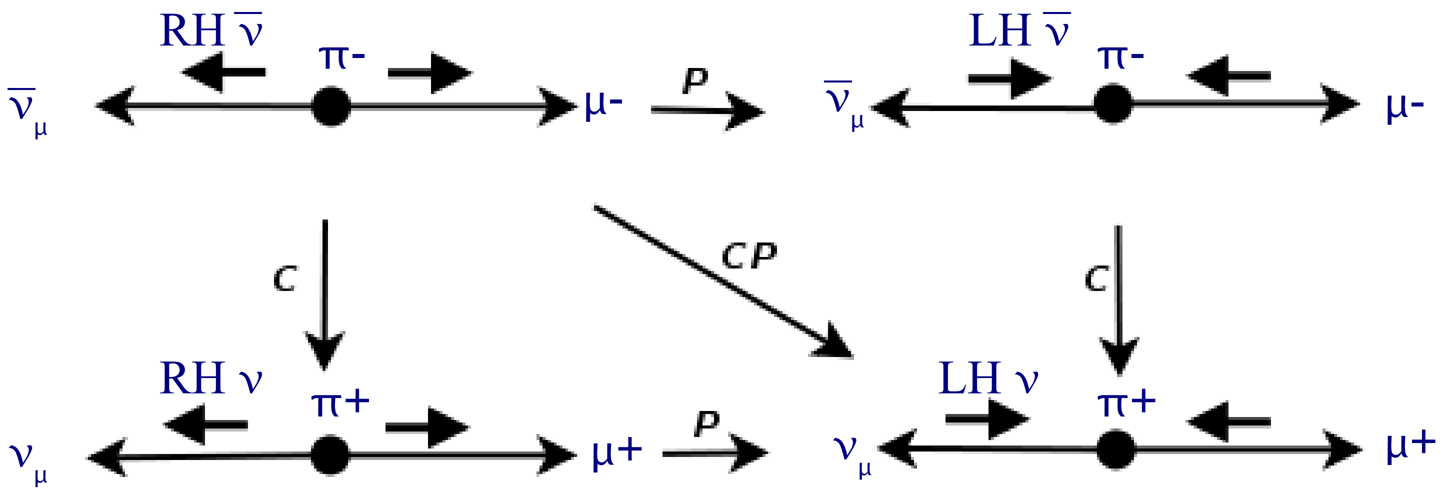
$$\Delta m_{21}^2 = m_2^2 - m_1^2 \approx 8\times 10^{-5}\, eV^2$$
$$|\Delta m_{32}^2| = |m_3^2 - m_2^2| \approx 2\times 10^{-3}\, eV^2$$

- It is possible to have two hierarchies for the neutrinos masses, independently from the absolute scale of the mass of the lightest neutrino:



- In the normal hierarchy: $m_3 > m_2$ and in the inverse one: $m_3 < m_2$.
- The present experiments start to be sensitivie to the two possibilities.
- Indipendently from the type of hierarchy, being $\Delta m_{12}^2 \ll \Delta m^2_{32}$ it is reasonable to do the approximation: $|\Delta m^2_{32}| \approx |\Delta m^2_{31}|$

CP violation in the neutrino interaction



- The weak interactions maximally violate P and C symmetries
- They seem to conserve the CP symmetry

Time reversal and CPT

- All the field theories locally Lorentz-invariant must also be invariant respect to CPT
- This means that particle and antiparticle must have identical masses, identical magnetic moments, …
- Better experimental limit:

$$\frac{|m(K^0)-m(\bar{K}^0)|}{m(K^0)} < 10^{-18}$$

- CPT is considered an exact symmetry of the Universe
- This implies that if CP is conserved also T is conserved
- But if CP is violated also T is violated and vice versa.

Violation of CP and T in the neutrino oscillation

- If T is valid then $P(\nu_e \rightarrow \nu_\mu) = P(\nu_\mu \rightarrow \nu_e)$

$$P(\nu_e \rightarrow \nu_\mu) = 2\Re\{U_{e1}^* U_{\mu 1} U_{e2} U_{\mu 2}^* [e^{-i(\phi_1 - \phi_2)} - 1]\} + \ldots$$

while

$$P(\nu_\mu \rightarrow \nu_e) = 2\Re\{U_{\mu 1}^* U_{e1} U_{\mu 2} U_{e2}^* [e^{-i(\phi_1 - \phi_2)} - 1]\} + \ldots$$

then except for the case in which all the elements U_{ei} and $U_{\mu j}$ are real, T is not valid in the neutrino oscillations, this implies the possibility that CP is violated

- The CP operation gives:

$$CP: \quad \nu_e \rightarrow \nu_\mu \qquad \rightarrow \qquad \bar{\nu}_e \rightarrow \bar{\nu}_\mu$$

The oscillation probability $P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$ can be obtained from $P(\nu_e \rightarrow \nu_\mu)$ noting that the element of the PMNS matrix appears as U or U^* if the neutrino is a spinor or an adjoint spinor in the vertex of the weak interaction. Consequently:

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) = 2\Re\{U_{e1} U_{\mu 1}^* U_{e2}^* U_{\mu 2} [e^{-i(\phi_1 - \phi_2)} - 1]\} + \ldots$$

another time except for the case in which all the elements U_{ei} e $U_{\mu j}$ are real, $P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) \neq P(\nu_e \rightarrow \nu_\mu)$ and CP will be violated in the neutrino oscillations.

Violation of CP and T in the neutrino oscillation

- Finally considering CPT:

$$CPT: \quad \nu_e \rightarrow \nu_\mu \quad \rightarrow \quad \bar{\nu}_\mu \rightarrow \bar{\nu}_e$$

the effect of T is to change *e* with μ and the effect of CP those to change *U* with *U**, therefore:

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = 2\Re\{U_{\mu 1} U^*_{e1} U^*_{\mu 2} U_{e2} [e^{-i(\phi_1 - \phi_2)} - 1]\} + \ldots = P(\nu_e \rightarrow \nu_\mu)$$

The neutrino oscillations are CPT invariant.

- The imaginary components of the PMNS matrix give a possible source of CP violation in the SM.
- The relative size of the CP violation in the neutrino oscillations is given by:

$$P(\nu_e \rightarrow \nu_\mu) - P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) = 16\Im\{U^*_{e1} U_{\mu 1} U_{e2} U^*_{\mu 2}\} \sin\Delta_{12} \sin\Delta_{13} \sin\Delta_{23}$$

- With the present knowledges such difference is of few percent. The effects of the CP violation in the neutrino oscillations is small. The sensitivities of the experiments of the present generation maybe are not enough to establish it without reasonable doubts.

The PMNS matrix

- The unitarity matrix U is defined by 3 real parameters (3 angles) and a complex phase (δ) responsible of the CP violation in the leptonic sector of the SM.

atmospheric ν_μ — ν_e disappearance oscill. $\nu_\mu \Leftrightarrow \nu_e$ — ν from the Sun

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{+i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{+i\phi_2} & 0 \\ 0 & 0 & e^{+i\phi_3} \end{pmatrix} = U_D U_M$$

accelerator beams — beams of high intensity — ν from reactors — $0\nu2\beta$

9 real indipendent parameters

- 3 masses: m_1, m_2, m_3
- 3 "mixing angles" θ_{12}, θ_{13}, θ_{23} $\theta_{ij} \in [0,\pi/2]$
- 1 phase (δ ⇒ CP violation). CP conserved in 2 cases: $\delta=0$, $\delta=\pi$
- +2 phases (ϕ_2,ϕ_3), if neutrinos are of Majorana type ⇒ L violation - irrelevant for oscill.

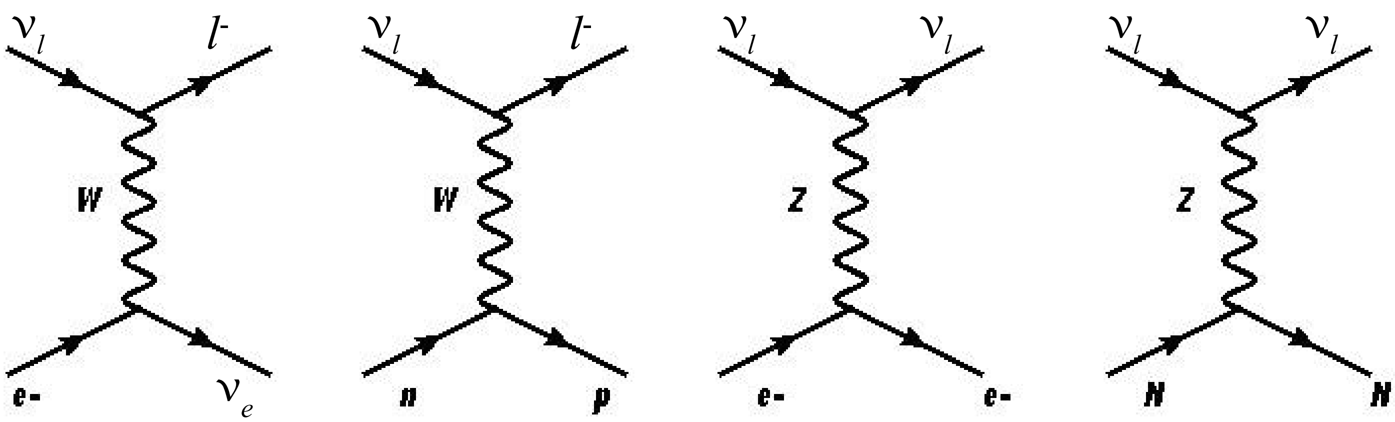
Experiments of neutrino oscillation

- **Many neutrino sources** to study the neutrino oscillations:
  - atmospheric neutrinos (coming from cosmic rays)
  - neutrinos from nuclear reactors
  - neutrinos from accelerators
  - neutrinos from suns
  - neutrinos from galactic and extra-galactic sources

- Two types of experiments:
  - experiments in **appearance mode**: one searches the appearance of the wrong flavour of the charged lepton in a beam of known flavour neutrinos (for example the appearance of e and/or τ in un beam initially made of ν_μ)
  - experiments in **disappearance mode**: disappearance of the correct flavour of the charged lepton (for example one observes less μ events respect to what it is expected from a beam initially made of only ν_μ)

Thresholds for the neutrino interactions

- Neutrinos in the matter are detected through their CC and NC interactions both with atomic electrons and with the nucleons.
- The interactions with the nuclei are dominant ($\sigma \propto s \approx 2mE_\nu$) unless they are not kinematically prohibited.



- An appearance signal is seen if the interaction is kinematically permitted. The CC interactions are permitted if the center of mass energy is sufficient to produce a charged lepton and an hadronic final state.
- The threshold is given by the process with the lowest W^2: $\nu_l n \rightarrow l^- p$
- In the laboratory system, where the neutron is a at rest, the squared center of mass energy is:

$$s = (p_\nu + p_n)^2 = (E_\nu + m_n)^2 - E_\nu^2 = 2 E_\nu m_n + m_n^2$$

Thresholds for the neutrino interactions

- The reaction $\nu_l n \rightarrow l^- p$ is kinematically possible if $s > (m_l + m_p)^2$

$$E_\nu > \frac{(m_p^2 - m_n^2) + m_l^2 + 2 m_p m_l}{2 m_n}$$

- From this expression, the threshold energies for the CC interactions of neutrinos with a nucleon are:

$$E_{\nu_e} > 0, \quad E_{\nu_\mu} > 110\, MeV, \quad E_{\nu_\tau} > 3.5\, GeV$$

for electronic neutrinos with energies of few MeV, it is necessary to consider the nuclear binding energy

- CC interactions with electrons $\nu_l e^- \rightarrow \nu_e l^-$ are kinematically allowed if $s > m^2_l$ In the laboratory system:

$$s = (p_\nu + p_e)^2 = (E_\nu + m_e)^2 - E_\nu^2 = 2 E_\nu m_e + m_e^2$$

and then:

$$E_\nu > \frac{m_l^2 - m_e^2}{2 m_e}$$

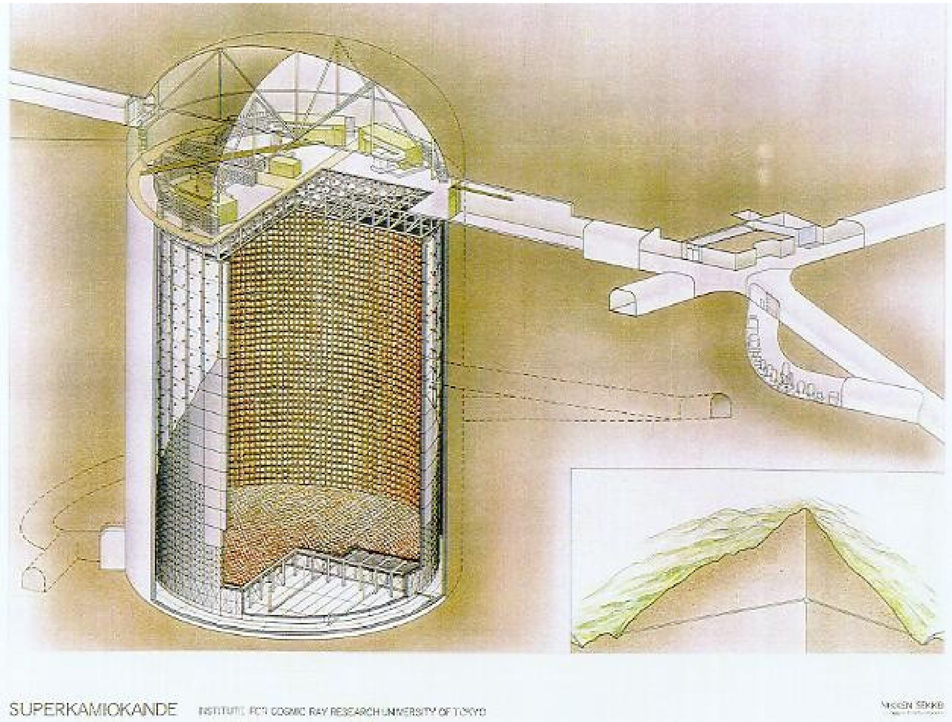
Thresholds for the neutrino interactions

- The thresholds in the laboratory system for CC scattering with electrons are:

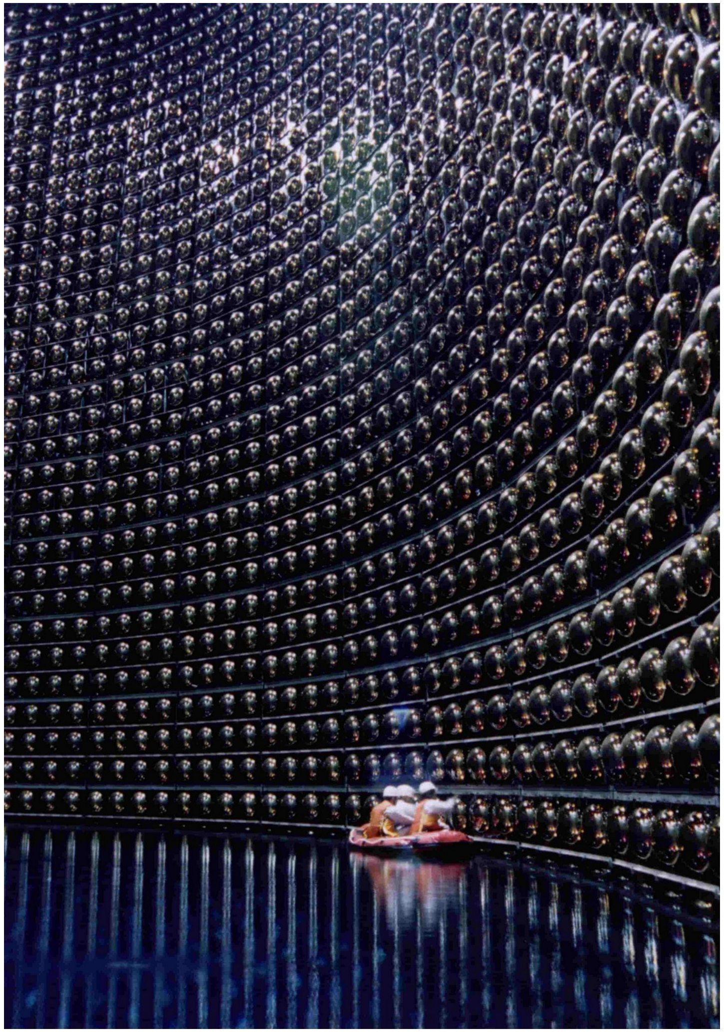
$$E_{\nu_e} > 0, \qquad E_{\nu_\mu} > 11\,GeV, \qquad E_{\nu_\tau} > 3090\,GeV$$

Therefore the interactions with the atomic electrons are important only for electronic neutrinos/antineutrinos.

Experiments with atmospheric neutrinos
Super-Kamiokande experiment



Controls during the filling



cylindric detector containing 50kton of ultrapure water
The Cherenkov radiation is used to detect the vertices of the event, estimate the energy, discriminate the type of particle (e-like, muon-like)

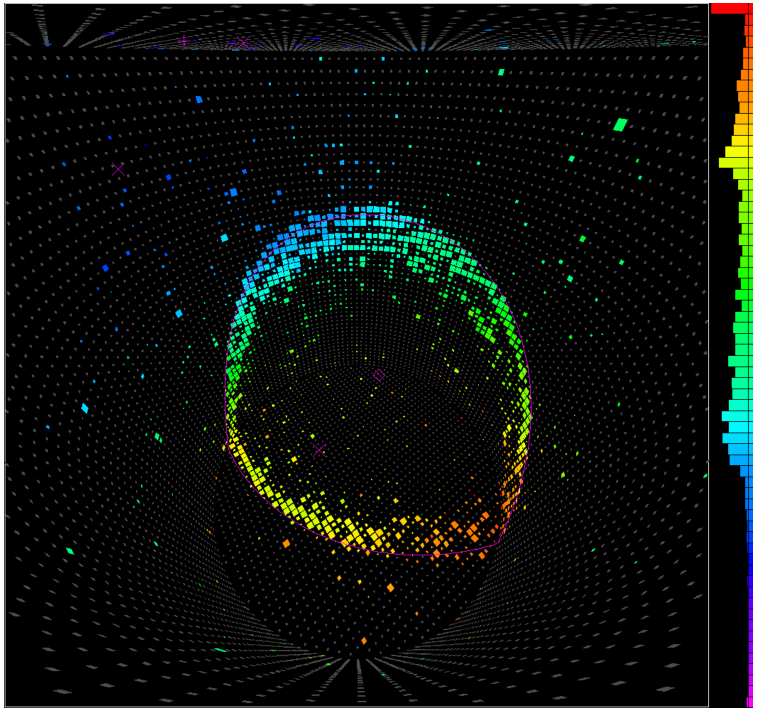
SuperK. The disappearance of the mu neutrinos

- The atmospheric neutrinos are detected through their CC interactions:

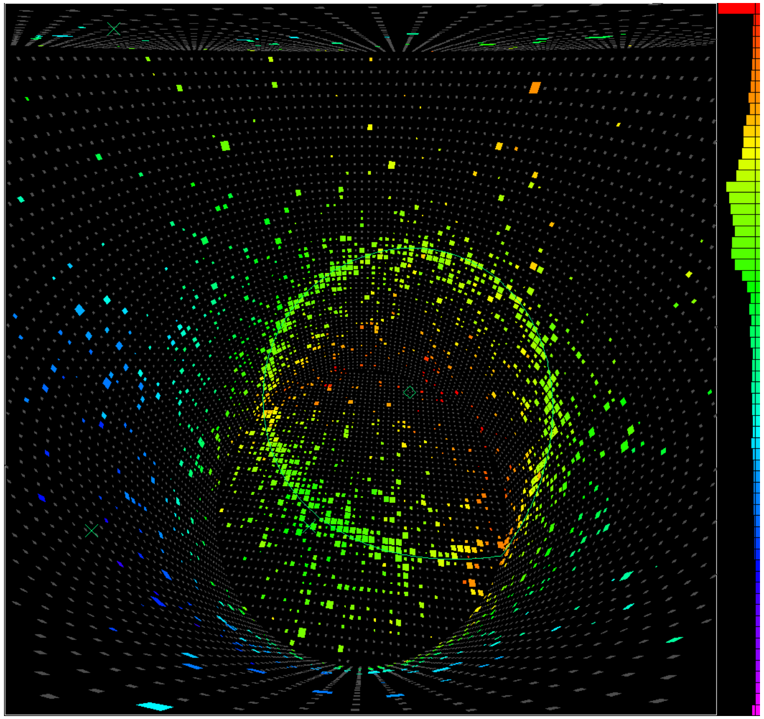
$$\nu_\mu + N \rightarrow \mu + N \qquad\qquad \nu_e + N \rightarrow e + N$$

- In both cases only one Cherenkov ring is detected (the lepton in the final state). In the case of a muon the ring has the edges well defined, in the case of an electron the edges are not well defined due to the bremsstrahlung process. The events with a single ring can be subdivided in events: ***e-like*** and **μ-*like.*** The charge remains unknown.

μ-*like*

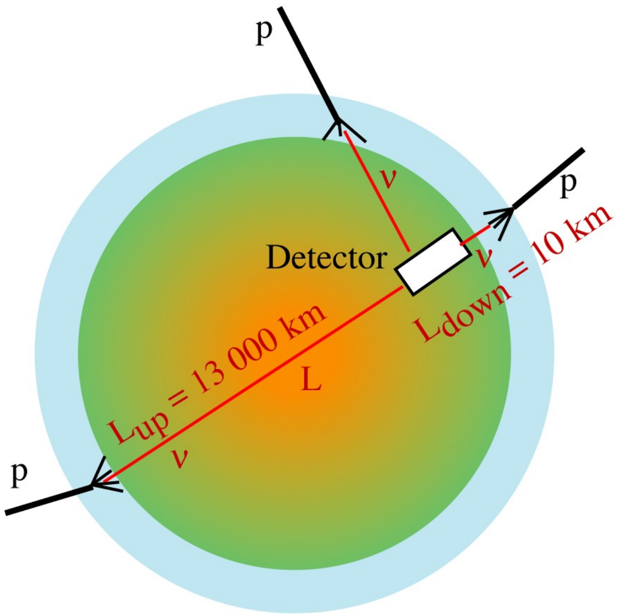


e-like

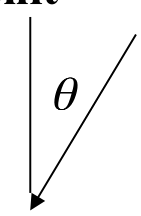


SuperK. The disappearance of the mu neutrinos

- The energies of the atmospheric neutrinos can vary from few hundreds MeV to several GeV.
- At these energies the differential cross section have a pronounced forward peak, due to this the direction of the final state lepton is always into the same direction of the neutrino.
- Knowing the direction of the neutrino one knows also the distance which it has gone through from the production point in the atmosphere.



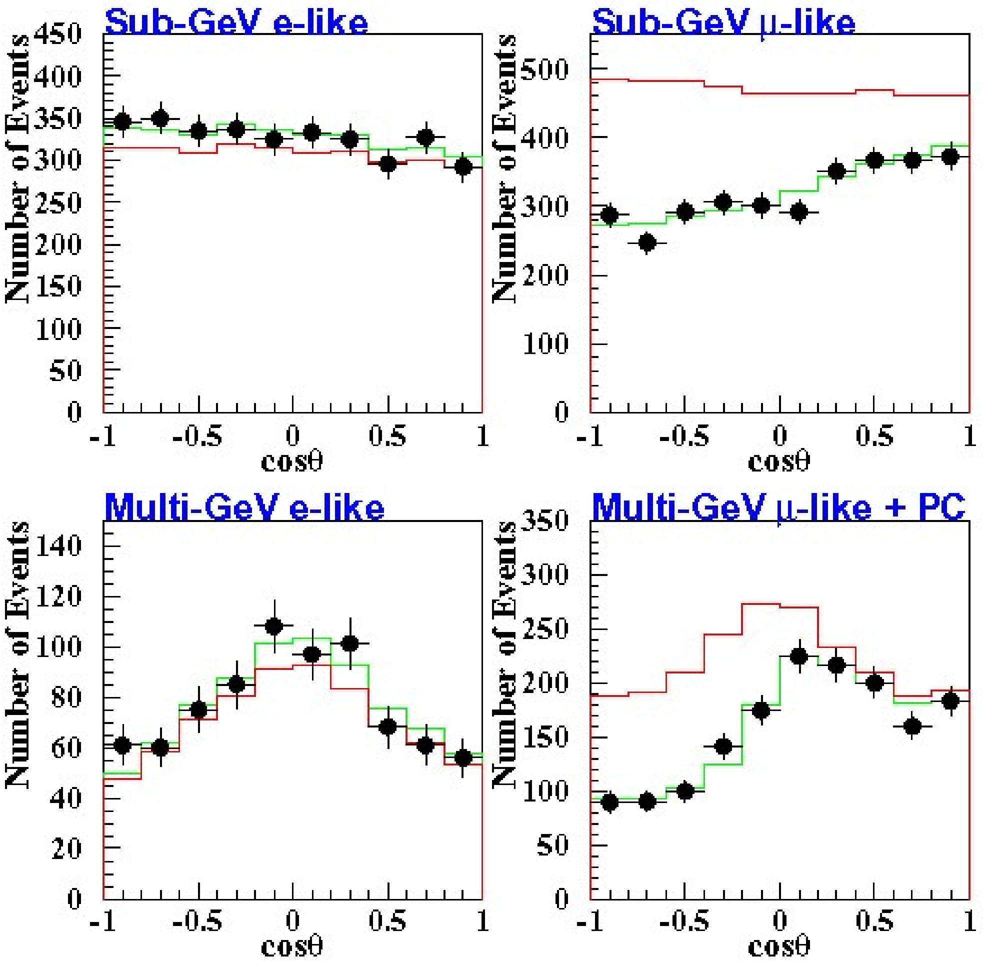
- Noting θ the angle between the direction of the neutrino with the zenith



the lenght of flight varies from ~10 km (for $\theta = 0$) to more than 12000 km for $\theta = \pi$

SuperK. The disappearance of the mu neutrinos

- The detector permits to have a rough estimation of the energy of the charged lepton, which is statistically correlated to the energy of the incoming neutrino.
- The ***e-like*** and ***μ-like*** events can be further subdivided into ***sub-GeV*** (energy below ~ 1 GeV) and ***multi-GeV*** events



The observations are not in agreement with the absence of oscillations

In agreement with disappearance oscillation of ν_μ with a period of $\Delta m^2 \approx 2400$ meV^2

SuperK. The disappearance of the mu neutrinos

- In general the oscillation probability between all the pairs of flavours can be written so:

$$P(\nu_x \rightarrow \nu_y, t) = A(\nu_x \rightarrow \nu_y)\sin^2[1.27\Delta m^2(L/E)]$$

- The costant A is the maximum of the oscillation probability between two flavours.
- The specific phenomenon discovered by SuperK is the disappearance of muonic neutrinos. For this phenomenon the maximum of the probability is:

$$A(\nu_\mu \rightarrow \nu_\mu) = \sin^2 2\theta_{23}\cos^2\theta_{13}(1-\sin^2\theta_{23}\cos^2\theta_{13}) \approx \frac{1}{2}$$

assuming $\cos^2\theta_{13} \approx 1$ and $\sin^2\theta_{23} \approx ½$.

- The disappearence of muonic neutrinos will be combined by the appearance of electron and tau neutrinos.
  The maximum of the corresponding probabilities are:

$$A(\nu_\mu \rightarrow \nu_e) = \sin^2\theta_{23}\sin^2 2\theta_{13} \approx 2\theta_{13}^2$$
$$A(\nu_\mu \rightarrow \nu_\tau) = \sin^2\theta_{23}\cos^4\theta_{13} \approx 1$$

SuperK. The disappearance of the mu neutrinos

- The data of the electronic neutrinos do not show oscillation, this implies that θ_{13} is small.
- Muonic neutrinos of high energy and with small zenith angle arrive to the detector (small distance of flight). Beyond a certain distance the flux of muonic neutrinos is about an half of the expected flux.
- The value of this distance determines Δm^2, while the value ½ of reduction implies that $\theta_{23} \approx \pi/4$. The neutrinos of low energy oscillate also for small distance.

Experiments at the reactors

- Nuclear reactors produce a large flux of electron antineutrinos through the beta decays of the isotopes: ^{235}U, ^{238}U, ^{239}Pu, ^{241}Pu
- Their mean energy is about 3 MeV, while the flux is precisely known from the power produced by the reactor.
- The antineutrinos are detected through the inverse beta process:

$$\bar{\nu}_e + p \rightarrow e^+ + n$$

- If an antineutrino oscillates into a neutrino of different flavour, it cannot be detected because under threshold. It is possible to observe the disappearance of the electronic antineutrinos.
- With the approximation $|\Delta m^2_{32}| \approx |\Delta m^2_{31}|$ the survival probability becomes:

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - 4|U_{e1}|^2 |U_{e2}|^2 \sin^2\Delta_{21} - 4|U_{e3}|^2 [|U_{e1}|^2 + |U_{e2}|^2] \sin^2\Delta_{32}$$

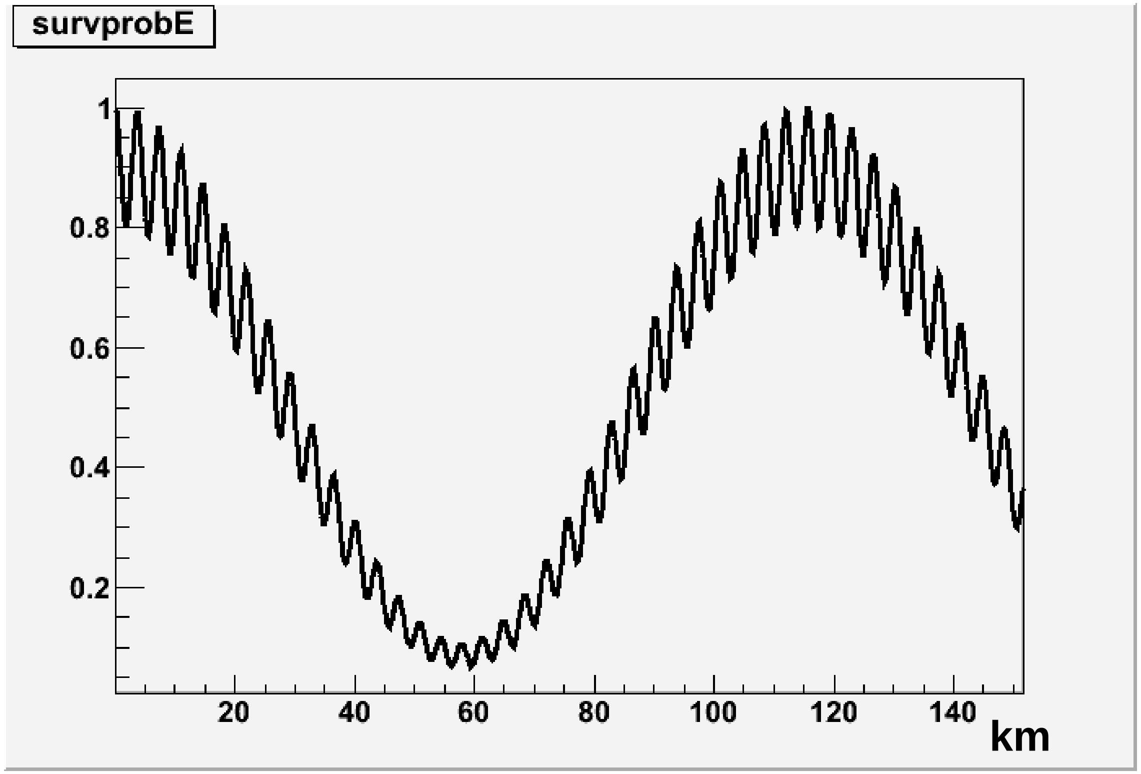
- using the unitarity relationship, it is possible to write:

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - 4|U_{e1}|^2 |U_{e2}|^2 \sin^2\Delta_{21} - 4|U_{e3}|^2 [1 - |U_{e3}|^2] \sin^2\Delta_{32}$$

Experiments at the reactors

- using the elements of the PMNS matrix one has:

$$P(\bar{\nu}_e \to \bar{\nu}_e) \approx 1 - 4(c_{12}c_{13})^2 (s_{12}c_{13})^2 \sin^2\Delta_{21} - 4 s_{13}^2 (1 - s_{13}^2) \sin^2\Delta_{32}$$
$$= 1 - \cos^4\theta_{13} \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m_{21}^2 L}{4 E_{\bar{\nu}}}\right) - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{32}^2 L}{4 E_{\bar{\nu}}}\right)$$



E_ν = 3.5 MeV
Δm^2_{21} = 7.5 10^{-5} eV^2
Δm^2_{32} = 2.3 10^{-3} eV^2
$\sin^2 \theta_{12}$ = 0.857
$\sin^2 \theta_{13}$ = 0.098

There are 2 different scales of length. The component with short wave length depends on Δm^2_{32} and oscillates with $\sin^2 \theta_{13}$ amplitude along that with long wave length (which dependes on Δm^2_{21}) Measurements at distances of $O(1)$ km sensitive to θ_{13}, measurements at distances of $O(100)$ km sensitive to Δm^2_{21} and θ_{12}

Short-baseline experiments with reactors

- At small distances from the reactors, the survival probabiliy of the electron antineutrinos is:

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\,\theta_{13} \sin^2\left(\frac{\Delta m_{32}^2 L}{4 E_{\bar{\nu}}}\right)$$

- **Daya Bay** (China) experiment

6 reactor cores; they produce 2.9 GW each
6 detectors, 2 at a mean distance of 470 m from the reactors; 1 at 576 m; 3 at 1.65 km

Short-baseline experiments with reactors

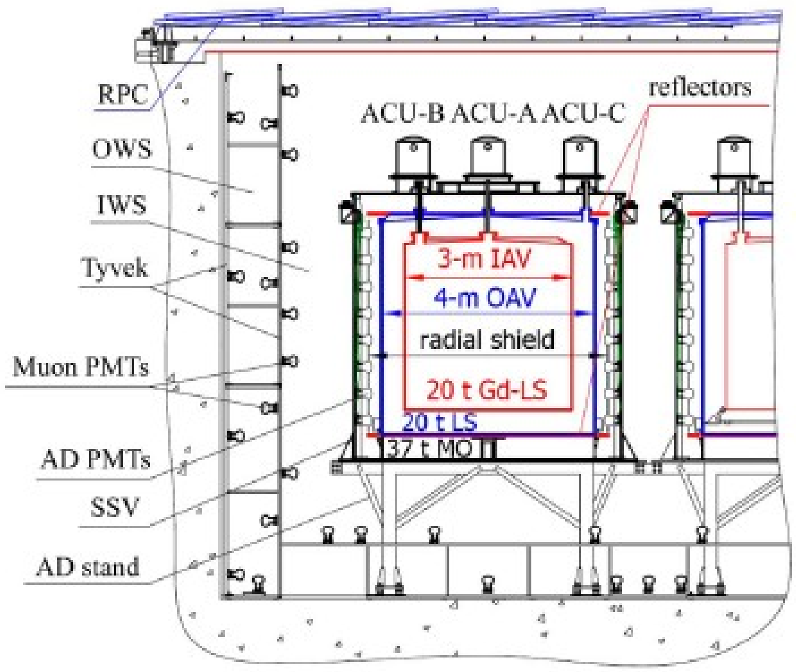


Fig. 2. Schematic diagram of the Daya Bay detectors.

Inner vessel with 20 t of liquid scintillator doped with gadolinium
The vessel is equipped with photomultipliers
The reaction for the detection is the inverse beta decay

Short-baseline experiments with reactors

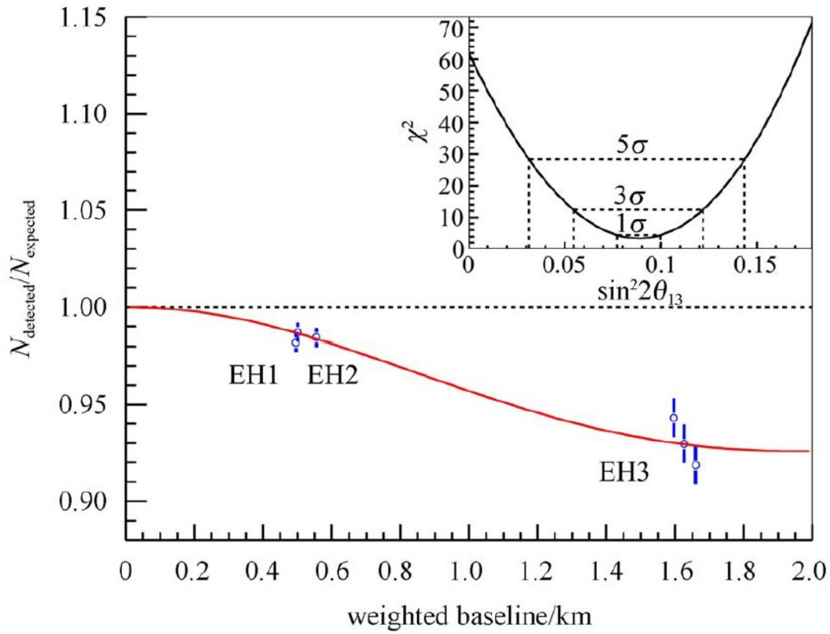


Fig. 23. Ratio of measured versus expected signals in each detector, assuming no oscillation. The error bar is the uncorrelated uncertainty of each AD, including statistical, detector-related, and background-related uncertainties. The expected signal has been corrected with the best-fit normalization parameter. Reactor and survey data were used to compute the flux-weighted average baselines. The oscillation survival probability at the best-fit value is given by the smooth curve. The AD4 and AD6 data points were displaced by -30 and +30 m for visual clarity. The χ^2 value versus $\sin^2 2\theta_{13}$ is shown in the inset.

$$\sin^2 2\theta_{13} = 0.089 \pm 0.010(stat) \pm 0.005(syst)$$

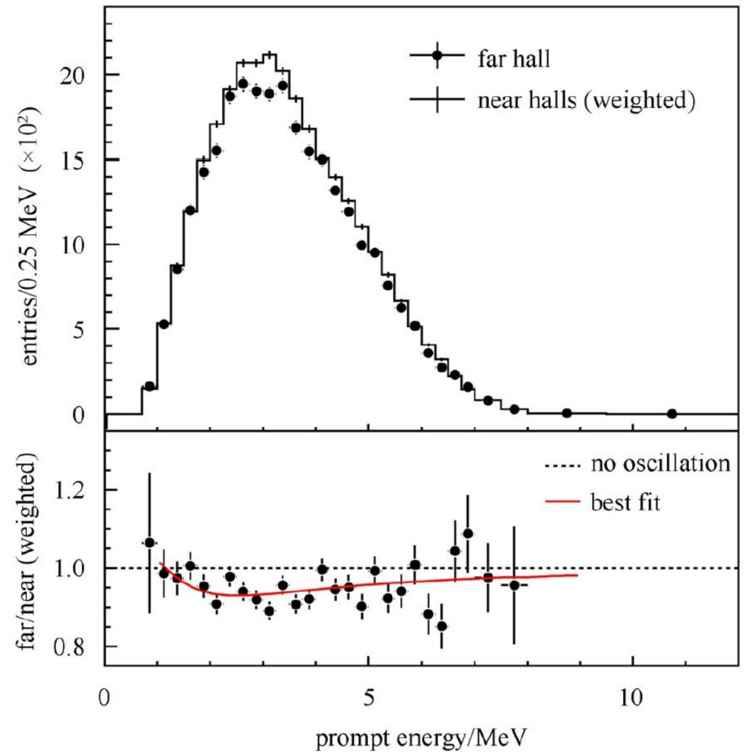
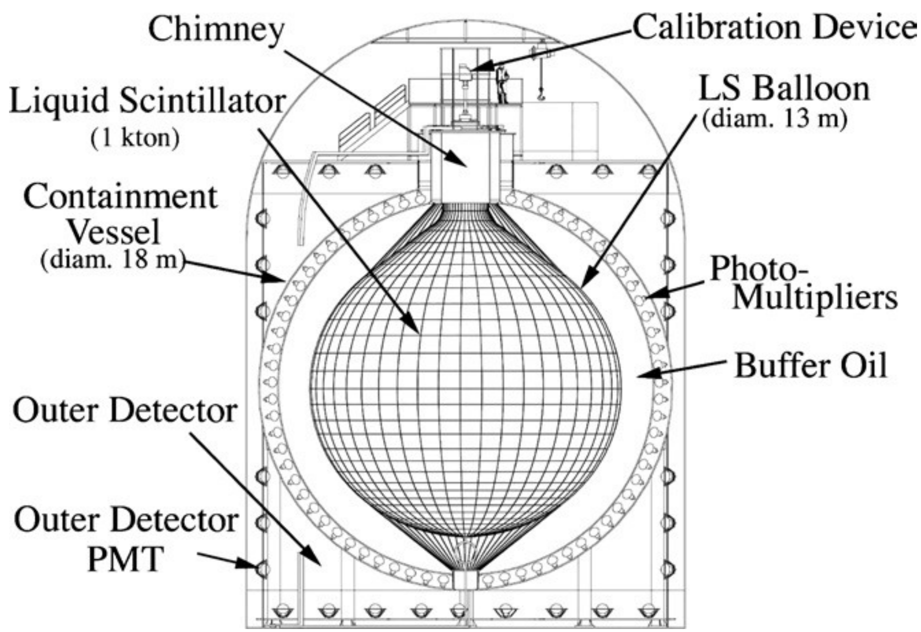


Fig. 24. Top: Measured prompt energy spectrum of the far hall (sum of three ADs) compared with the no-oscillation prediction based on the measurements of the two near halls. Spectra were background subtracted. Uncertainties are statistical only. Bottom: The ratio of measured and predicted no-oscillation spectra. The solid curve is the expected ratio with oscillations, calculated as a function of neutrino energy assuming $\sin^2 2\theta_{13}=0.089$ obtained from the rate-based analysis. The dashed line is the no-oscillation prediction.

Long-baseline experiments with reactors



KamLAND (Japan) experiment

placed at 130-240 km from the reactors (total power: 70 GW)
Vessel filled with liquid scintillator (1kton) surrounded by 1800 photomultipliers.
The reaction for the detection is the inverse beta decay

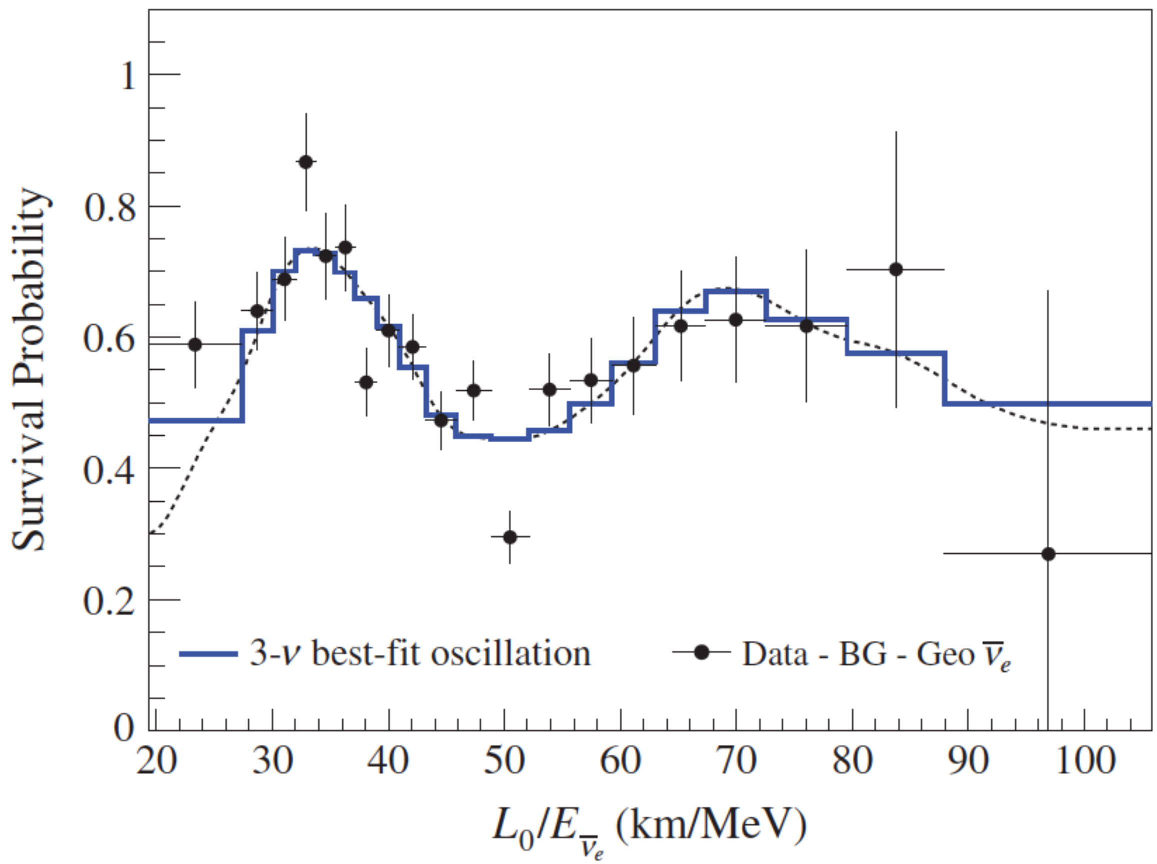
At the long distances of **KamLAND** the oscillations due to the Δm^2_{32} term cannot be resolved, but they average:

$$< \sin^2 \Delta_{32} > = \frac{1}{2}$$

Therefore, neglecting also the $\sin^4 \theta_{13}$ term because small:

$$P(\bar{\nu}_e \to \bar{\nu}_e) \approx \cos^4\theta_{13} \left[1 - \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m^2_{21} L}{4 E_{\bar{\nu}}} \right) \right]$$

Long-baseline experiments with reactors



Clear oscillation signal
Position of the minimum ~ 50 km/MeV gives:

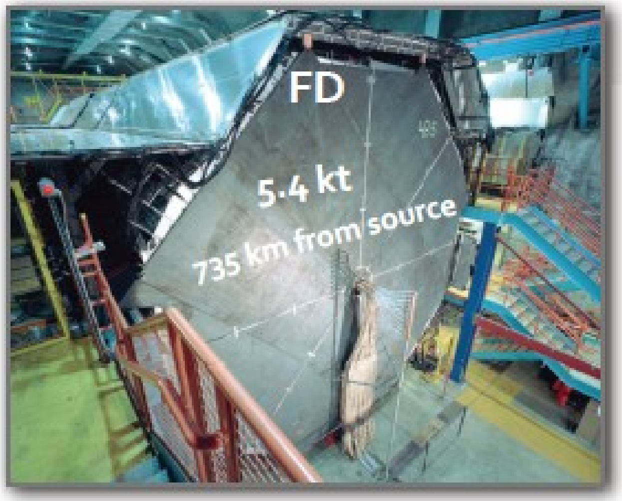
$$\Delta m^2_{21} = (7.6 \pm 0.2) \times 10^{-5}\ eV^2$$

One can mesure also the θ_{12} angle

$$\sin^2 2\theta_{12} = 0.87 \pm 0.04$$

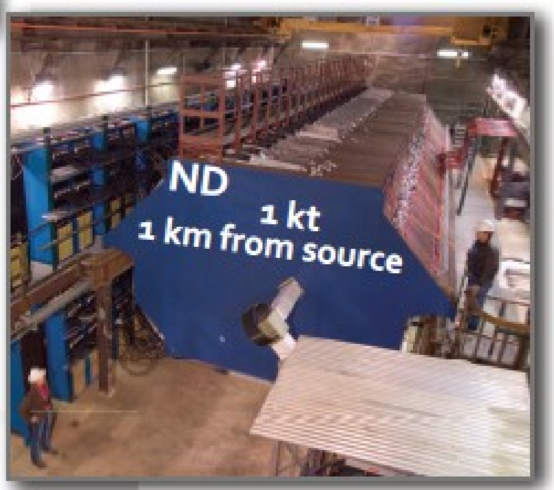
to be compared with the SNO result

Long-baseline experiments with beams

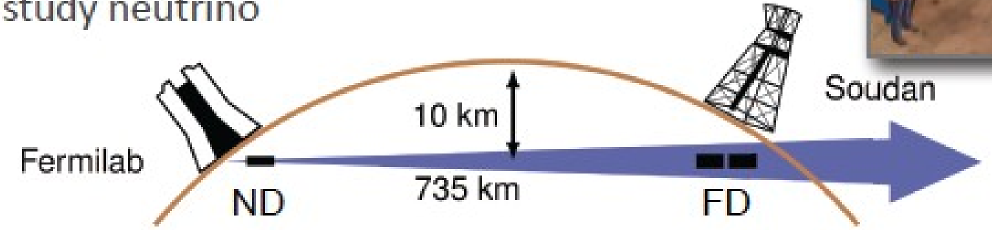


- Long-baseline neutrino oscillation experiment

- Measure NuMI Neutrino beam energy and flavor composition with two detectors over 735 km
  - L/E ~ 500 km/GeV



- Near Detector at Fermilab
- Far Detector at Soudan Underground Lab, MN
- Compare Near and Far measurements to study neutrino mixing



Long-baseline experiments with beams

- MINOS studies the neutrino oscillations using a pure beam of ν_μ
- Check of the Super-Kamiokande result using neutrinos coming from accelerators
- Due to the fact that the θ_{13} angle is small, the $\nu_\mu \rightarrow \nu_\tau$ oscillations are dominant
- L is fixed, the oscillations are seen as distortion of the observed spectrum
- The first minimum is at 1.3 GeV
- Having a beam of low energy (from 1 to 5 GeV, with a peak at 3 GeV), we are under the threshold for the τ appearance. **Disappearance experiment:** measurement of $|\Delta m^2_{32}|$ and of θ_{23}
- Using the approximation $\Delta_{32} \approx \Delta_{31}$

$$P(\nu_\mu \rightarrow \nu_\mu) \approx 1 - 4|U_{\mu 1}|^2 |U_{\mu 2}|^2 \sin^2 \Delta_{21} - 4|U_{\mu 3}|^2 [1 - |U_{\mu 3}|^2] \sin^2 \Delta_{32}$$

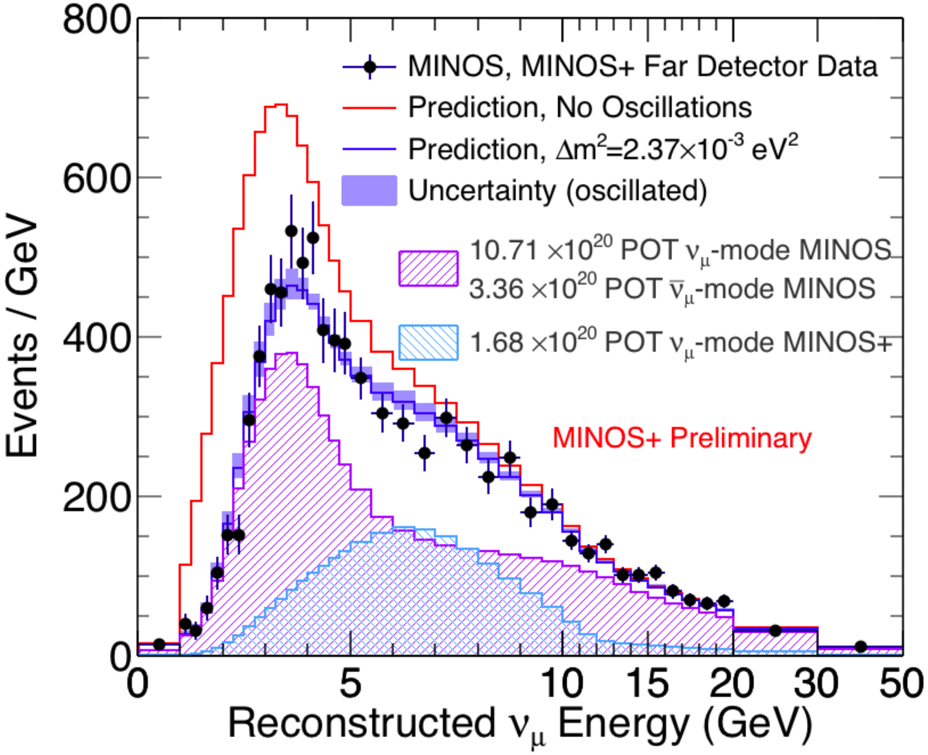
- For MINOS the term with the long wave length is negligible therefore:

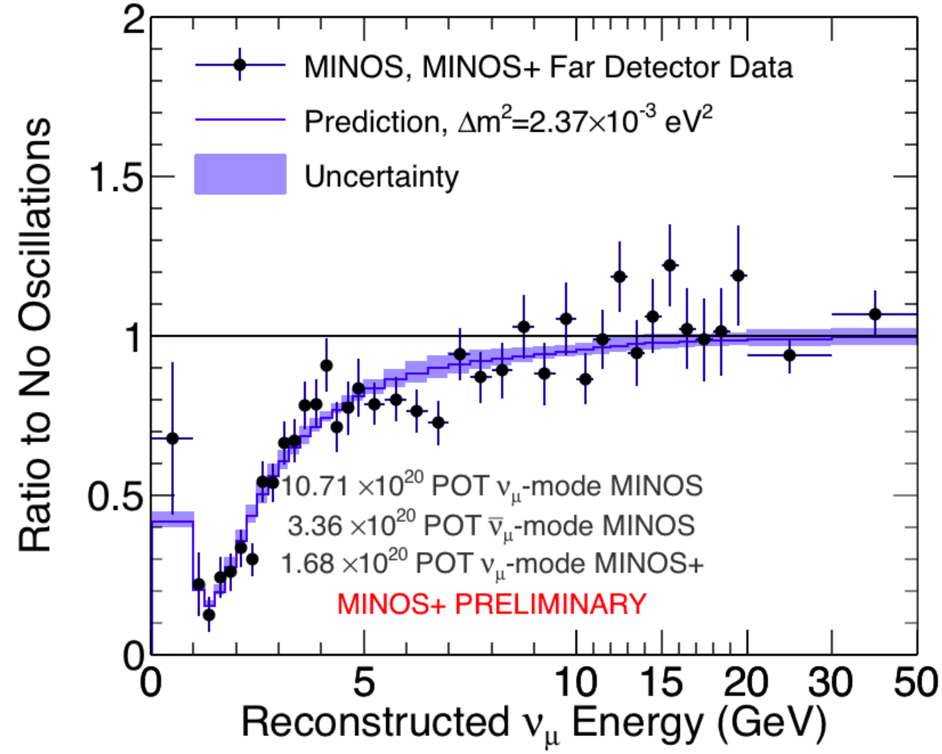
$$P(\nu_\mu \rightarrow \nu_\mu) \approx 1 - 4|U_{\mu 3}|^2 [1 - |U_{\mu 3}|^2] \sin^2 \Delta_{32}$$

that is:

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_\mu) &= 1 - 4 \sin^2 \theta_{23} \cos^2 \theta_{13} \left[1 - \sin^2 \theta_{23} \cos^2 \theta_{13}\right] \sin^2 \Delta_{32} \\ &= 1 - \left[\sin^2 \theta_{23} \cos^2 \theta_{13} + \sin^2 \theta_{23} \sin^2 2\theta_{13}\right] \sin^2 \Delta_{32} \\ &\approx 1 - A \sin^2 \left(\frac{\Delta m^2_{32} L}{4 E_\nu} \right) \end{aligned}$$

Long-baseline experiments with beams





$|\Delta m^2_{32}| = (2.3 \pm 0.1) \times 10^{-3}\ \mathrm{eV}^2$

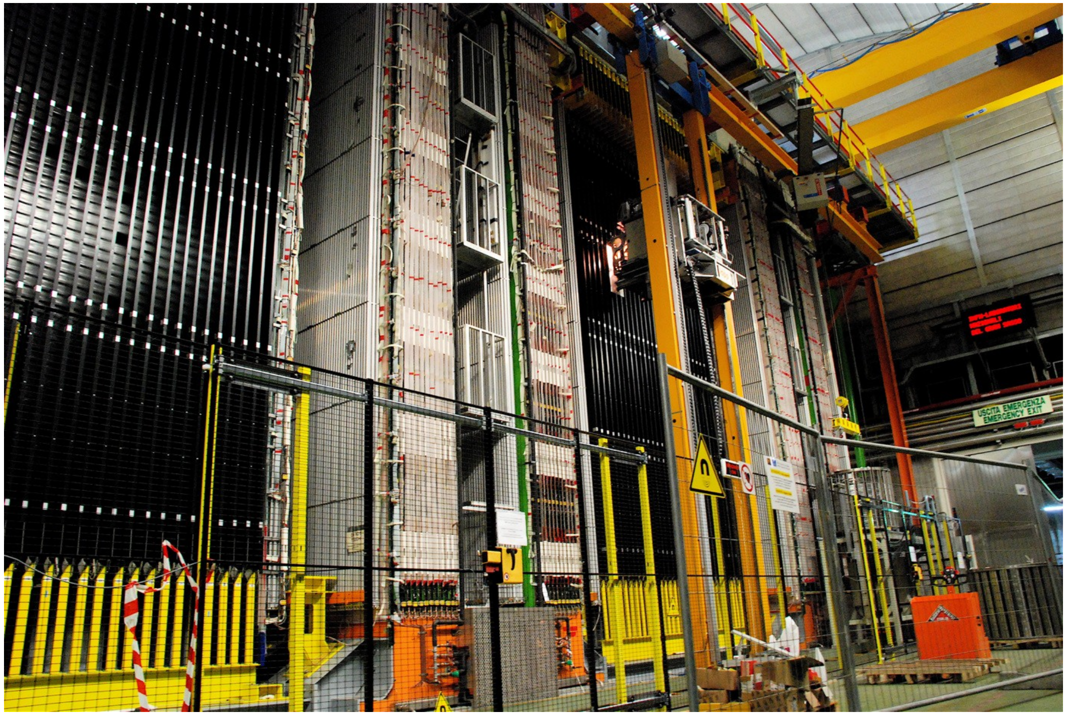
$\sin^2 2\theta_{23} > 0.90$

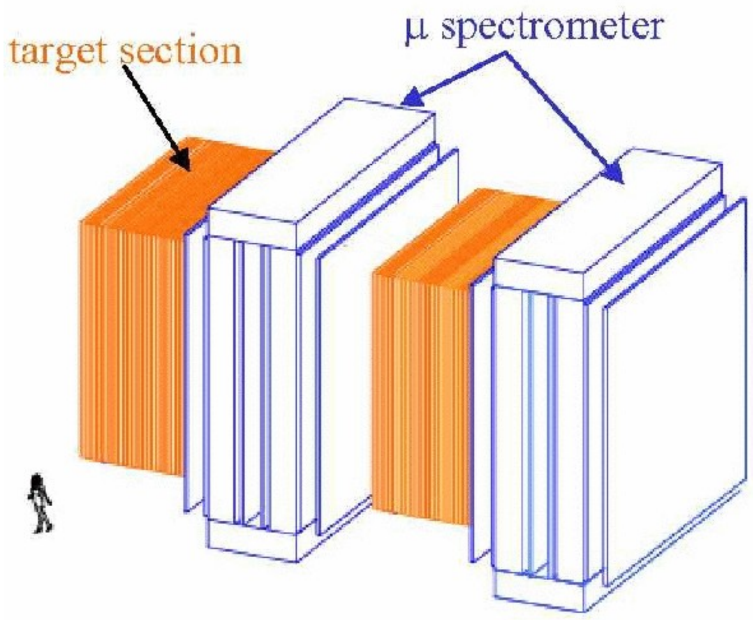
CNGS. CERN to Gran Sasso Neutrino Project

- It is necessary to verify experimentally if the ν_μ disappearance observed in the atmospheric neutrinos is followed by the **ν_τ appearance**
- **OPERA: experiment at LNGS optimized for the τ appearance (2007 – 2012)**
- **The reaction $\nu_\tau N \rightarrow \tau\ N'$ requires $E > 10$ GeV**

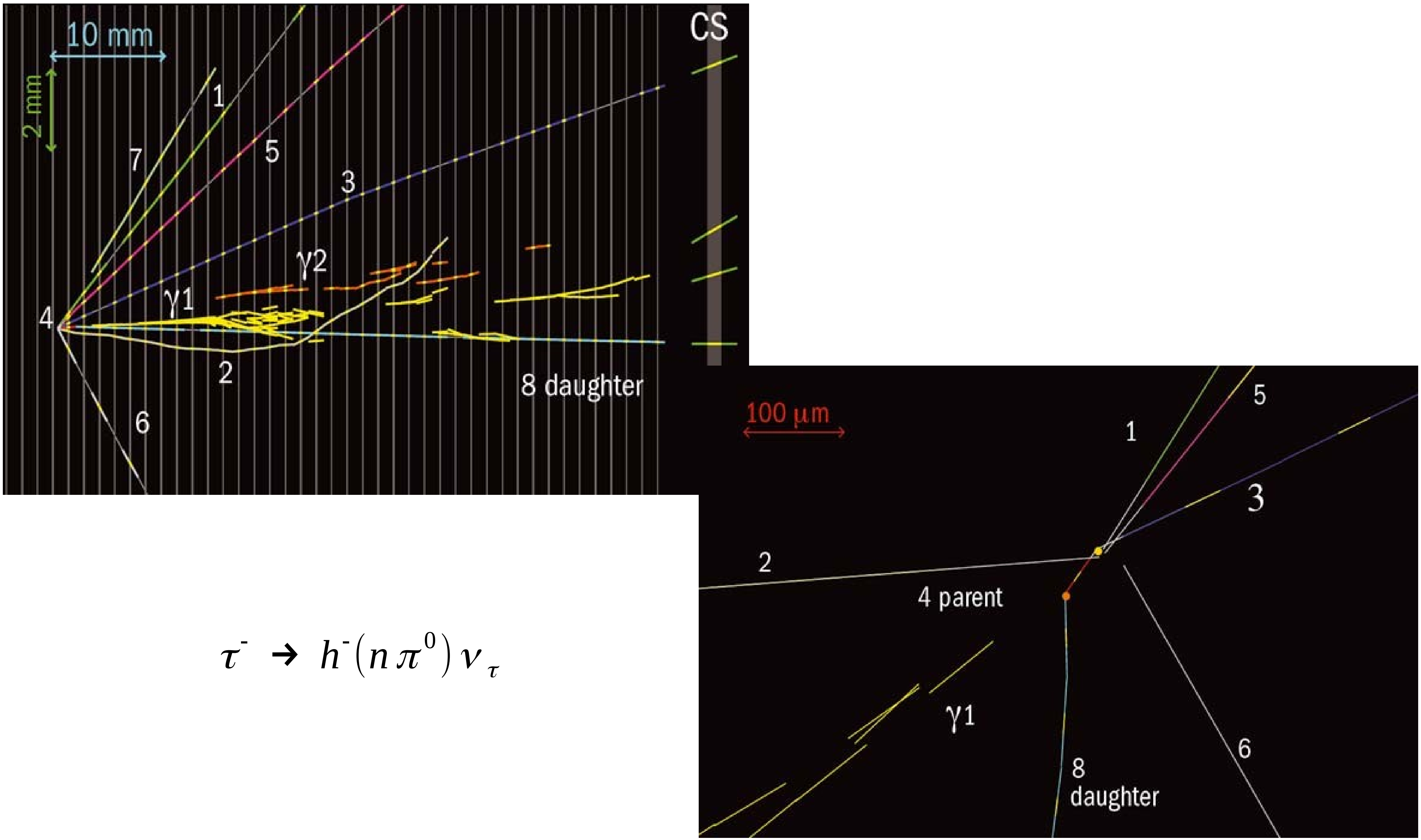
  The small cross section requires a sensible mass of 2-3 kt

  To observe $\tau \Rightarrow$ *high spatial resolution* ≈ 10 μm (photographic emulsions)





CNGS. CERN to Gran Sasso Neutrino Project



$$\tau^- \rightarrow h^-(n\pi^0)\nu_\tau$$

The neutrinos in the matter

In the **Sun** ν_e are produced by termonuclear reactions (with energies of ~ MeV) near its centre; to get out they go through material with variable density from $\rho \approx 10^5$ kg m^{-3} $\Rightarrow \rho = 0$

Analogy:

a light wave in the matter has a speed different respect to when it is in the vacuum, the refractive index $n \neq 1$, that is the photons have an effective mass $\neq 0$

reason ⇒ interaction of the photon with the matter ⇒ coherent diffusion in the forward direction

The effect is proportional to the diffusion amplitude in the forward direction

The neutrinos:

the neutrinos in the matter have $n \neq 1$, that is the neutrinos in the matter have masses ≠ in the vacuum

All types of neutrinos interact with the matter through NC, only the ν_e interact with the electrons of the matter through CC. The difference of the interaction energies at the place r is:

$$\Delta V(r) = V_e(r) - V_{\mu,\tau}(r) = \sqrt{2}\, G_F\, N_e(r)$$

⇒ the index of refraction of the ν_e is different

The ν_e have an "effective mass" different that in the vacuum, dependent on the electron density in that particular point of the space

- **It happens a crossing of the levels and a resonant transition**
- **effect ∝ forward amplitude in the forward direction ∝ G_F (≠ ∝ G_F^2 of the cross section) ⇒ it is a big effect**

Mikheev-Smirnov-Wolfenstein effect

- We study the mixing between the two neutrinos ν_e , ν_α where $\alpha = \mu,\tau$.
- For oscillation, we are only interested in the terms in the Hamiltonian that are different for electron neutrinos compared with other flavours of neutrinos
- **In vacuum** the interesting part of the Hamiltonian is:.

$$H_V = \frac{\delta m^2}{4E}\begin{pmatrix} -\cos 2\theta_{12} & \sin 2\theta_{12} \\ \sin 2\theta_{12} & \cos 2\theta_{12} \end{pmatrix}$$

where $\delta m^2 \equiv m^2{}_2 - m^2{}_1$ and θ_{12} is the mixing angle in the vacuum.

The time-independent Schrödinger equation is:

$$H_V \begin{pmatrix} \nu_e(t) \\ \nu_\alpha(t) \end{pmatrix} = E \begin{pmatrix} \nu_e(t) \\ \nu_\alpha(t) \end{pmatrix}$$

The difference in energies of the two eigenstates is given by $\Delta E = \delta m^2/2E$

Mikheev-Smirnov-Wolfenstein effect

- The effect of the CC coherent forward scattering is to change the effective potential for electron neutrinos by

$$V_e = \pm\sqrt{2}\,G_F N_e$$

(for electron-antineutrinos it is necessary to put a minus (-) sign in front of the formula).

- The evaluation of this effect on the Hamiltonian can be done using $E^2 - p^2 = m^2$ and assuming that the neutrinos are ultrarelativistic and $V_e \ll E$:

$$m_\nu^2 = \left(E + V_e\right)^2 - p^2 \approx m^2 + 2EV_e$$

Therefore the change in m^2 for the electron neutrino is given by:

$$\Delta m_{\nu_e}^2 = 2\sqrt{2}\,G_F N_e E$$

- Assuming the neutrinos ultrarelativistic, the contribution from matter to the Hamiltonian is:

$$\Delta H_M = \sqrt{2}\,G_F N_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Mikheev-Smirnov-Wolfenstein effect

- Any term proportional to the unit matrix can be dropped discussing oscillations. So it is possible to rewrite the previous term in the following manner:

$$\Delta H_M = \frac{\sqrt{2}\, G_F N_e}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Combining the vacuum and the matter terms:

$$H_M = \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_{12} & \sin 2\theta_{12} \\ \sin 2\theta_{12} & \cos 2\theta_{12} \end{pmatrix} + \frac{\sqrt{2}\, G_F N_e}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- It is conventional to define

$$A = \pm \frac{2\sqrt{2}\, G_F N_e E}{\delta m^2}$$

so the previous expression simplifies in the following manner:

$$H_M = \frac{\delta m^2}{4E} \begin{pmatrix} -\cos 2\theta_{12} + A & \sin 2\theta_{12} \\ \sin 2\theta_{12} & \cos 2\theta_{12} - A \end{pmatrix}$$

Mikheev-Smirnov-Wolfenstein effect

- The solution of the corresponding Schroedinger equation is simple in the case where **the matter density is constant**.
- It is possible to define an effective mixing angle in the presence of matter as θ_m and an effective difference of squared masse δm^2_m

$$H_M = \frac{\delta m_m^2}{4E}\begin{pmatrix} -\cos 2\theta_m & \sin 2\theta_m \\ \sin 2\theta_m & \cos 2\theta_m \end{pmatrix}$$

- which leads to the usual functional dependence of the oscillation probability:

$$P(\nu_e \rightarrow \nu_\alpha) = \sin^2 2\theta_m \sin^2\left(\frac{\delta m_m^2 L}{4E}\right)$$

- While comparing the two Hamiltonian one derives the expression for the effective parameters in matter:

$$m^2_{1m,2m} = \frac{1}{2}\left[(m_2^2 + m_1^2 + A) \mp \delta m^2 \sqrt{(\cos 2\theta_{12} - A)^2 + \sin^2 2\theta_{12}}\right]$$

$$\delta m_m^2 = \delta m^2 \sqrt{(\cos 2\theta_{12} - A)^2 + \sin^2 2\theta_{12}}$$

$$\tan 2\theta_m = \frac{\tan 2\theta_{12}}{1 - A \sec 2\theta_{12}}$$

Mikheev-Smirnov-Wolfenstein effect

- If $A = 0$ one returns back to the formalism of the vacuum evolution: $\theta_m = \theta_{12}$
- If $A \rightarrow \infty$ (as at the centre of the sun where N_e is very very large) then $\tan 2\theta_m \rightarrow 0$ and then $\theta_m \rightarrow \pi/2$
- $\tan 2\theta_m \rightarrow \infty$ that is $\theta_m = \pi/4$ (resonance) and the mixing becomes maximum even if θ_{12} is small for an electronic density equal to:

$$N_e = \frac{1}{E} \frac{\delta m^2 \cos 2\theta_{12}}{2\sqrt{2}\, G_F}$$

It has to be $N_e > 0$, this resonance condition is satisfied if:

1) $\cos 2\theta_{12} > 0$ that is the angle is in the first octant

2) $\delta m^2 = m^2_2 - m^2_1 > 0$ that is $m_2 > m_1$

In particular there is no resonance if $\theta_{12} = \pi/4$

Mikheev-Smirnov-Wolfenstein effect

A variable density causes a dependence of the mass eigenstates on A (N_e).

Assume the case $m^2{}_1 \approx 0$ and $m^2{}_2 > 0$ which implies $\delta m^2 \approx m^2{}_2$

For $\theta = 0$, $\theta_m = 0$ for all A:

$$\begin{aligned} \nu_{1m} &= \nu_1 = \nu_e \quad \text{with} \quad m^2_{1m} = A \\ \nu_{2m} &= \nu_2 = \nu_\mu \quad \text{with} \quad m^2_{2m} = m^2_2 \end{aligned}$$

For small $\theta > 0$, now for $A = 0$ the angle $\theta_m = \theta$ which is small and implies:

$$\begin{aligned} \nu_{1m} &= \nu_1 \approx \nu_e \quad \text{with} \quad m^2_{1m} = 0 \\ \nu_{2m} &= \nu_2 \approx \nu_\mu \quad \text{with} \quad m^2_{2m} = m^2_2 \end{aligned}$$

For large A there is $\theta_m \approx 90^o$ and the states are given by:

$$\begin{aligned} \nu_{1m} &\approx -\nu_\mu \quad \text{with} \quad m^2_{1m} \approx m^2_2 \\ \nu_{2m} &\approx \nu_e \quad \text{with} \quad m^2_{2m} \approx A \end{aligned}$$

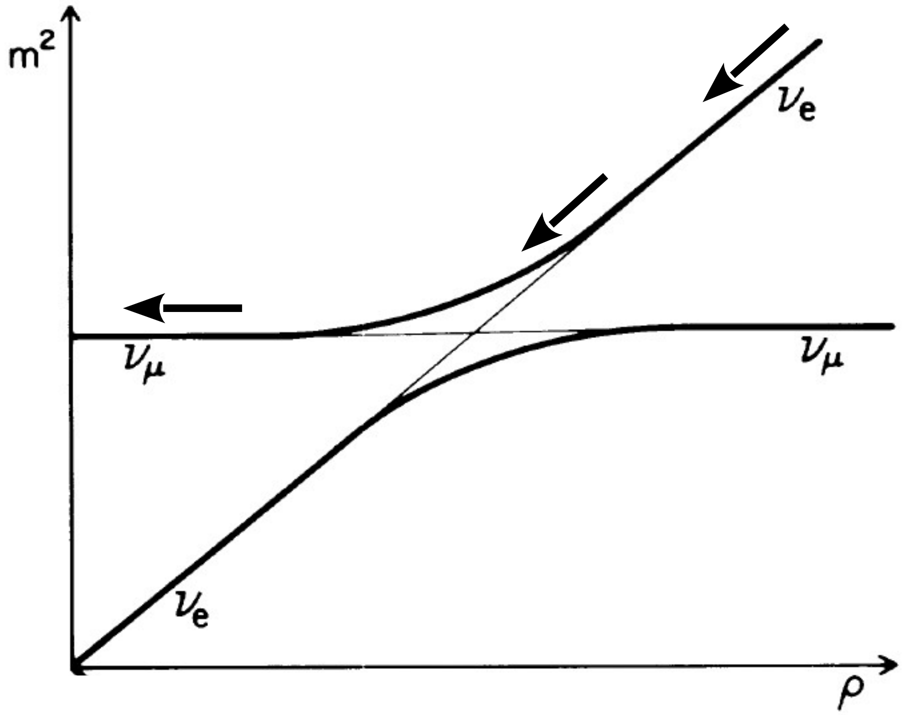
opposite to the $\theta = 0$ case. **This implies an inversion of the neutrino flavour.**

Mikheev-Smirnov-Wolfenstein effect

While in vacuum ν_{1m} is more or less ν_e, at high electron density it corrisponds to ν_μ.

The opposite is valid for ν_{2m}

This flavor flip is produced by the resonance where maximal mixing is possible.

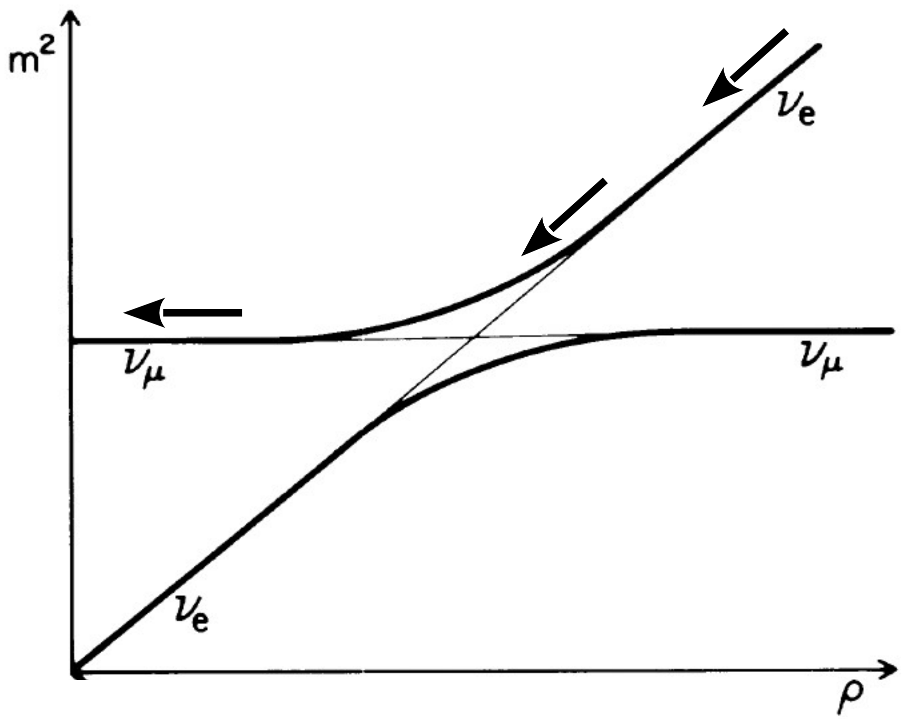


Mikheev-Smirnov-Wolfenstein effect

Solar neutrinos are produced in the interior of the Sun, where the density is rather high, therefore $\theta_m \approx 90^o$, the produced ν_e are basically identical to ν_{2m} , the heavier mass eigenstate.

A ν_e produced in the interior of the Sun , therefore, moves along the upper curve and passes a layer of matter where the resonance condition is fulfilled. Here maximal mixing occurs, $\theta_m \approx 45^o$, and

$$\nu_{2m} = \frac{1}{\sqrt{2}}(\nu_2 + \nu_\mu)$$



Mikheev-Smirnov-Wolfenstein effect

Passing the resonance from right to left and remianing on the upper curve, the state ν_{2m} at the edge of the Sun is now associated with ν_μ

The average probability for a ν_e produced in the solar interior, passes the resonance and leaves the Sun still in ν_e is given by:

$$P(\nu_e \to \nu_e) = \frac{1}{2}\left(1+\cos 2\theta_m \cos 2\theta_{12}\right) \approx \sin^2\theta_{12}$$

with θ_m as the mixing angle at the place of neutrino production: $\theta_m \approx 90^o$

The conversion is therefore:

$$P(\nu_e \to \nu_\mu) = \frac{1}{2}\left(1-\cos 2\theta_m \cos 2\theta_{12}\right) \approx \cos^2\theta_{12}$$

The smaller the vacuum mixing angle is, the larger the flavour transition probability becomes.

Resonance?

Electron density in the centre of the sum (the highest): $N_0 \approx 6\times10^{33} m^{-3}$

For the resonance, the energy must satisfy the relationship:

$$E > \frac{\delta m^2 \cos 2\theta_{12}}{2\sqrt{2} G_F N_0} \approx \delta m^2 \cos 2\theta_{12} \times 6.7\times10^{10} eV \approx 2\, MeV$$

If $E < 2$ MeV, the neutrinos doesn't meet the resonance, they are in the vacuum. They oscillate with a maximum excursion of:

$$A(\nu_e \rightarrow \nu_\alpha) = \sin^2 2\theta_{12}$$

The observation is averaged on many periods, so the oscillating term is equal to ½. The survival probability is:

$$P(\nu_e \rightarrow \nu_e) = 1 - \frac{1}{2}\sin^2 2\theta_{12} \qquad E < \approx 2\, MeV$$

Resonance?

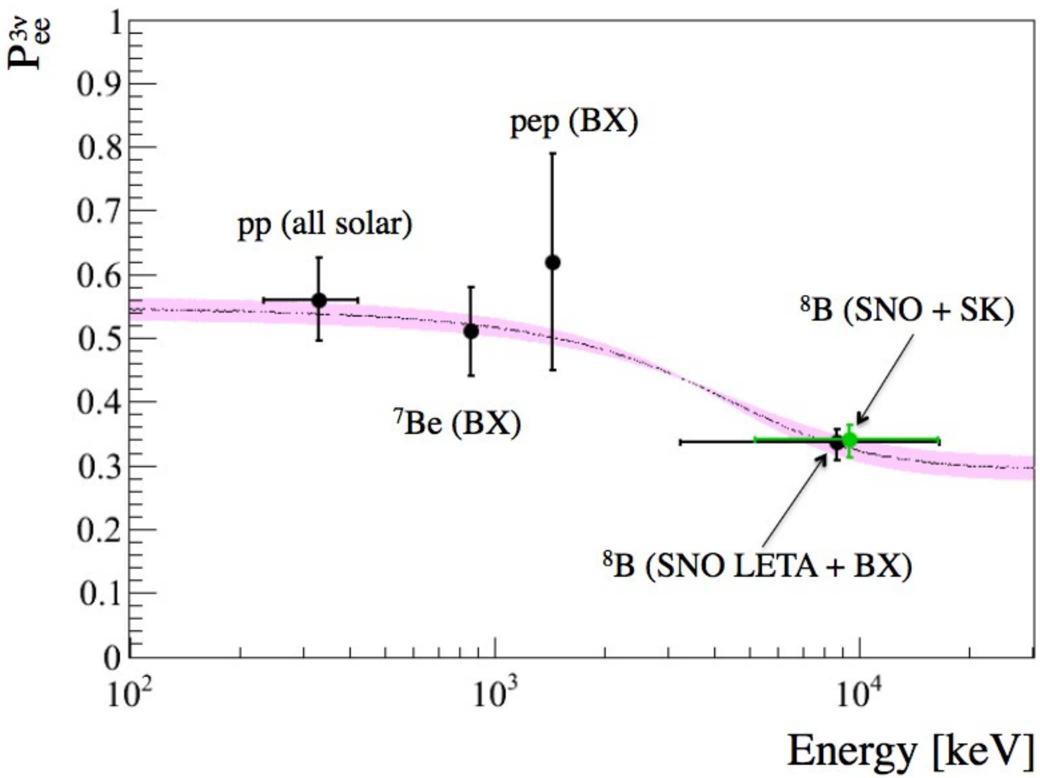


FIG. 84. Electron neutrino survival probability as a function of neutrino energy according to MSW–LMA model. The band is the same as in Fig. 83, calculated for the production region of ^{8}B solar neutrinos which represents well also other species of solar neutrinos. The points represent the solar neutrino experimental data for ^{7}Be and *pep* mono–energetic neutrinos (Borexino data), for ^{8}B neutrinos detected above 5000 keV of scattered-electron energy T (SNO and Super-Kamiokande data) and for $T > 3000$ keV (SNO LETA + Borexino data), and for *pp* neutrinos considering all solar neutrino data, including radiochemical experiments.

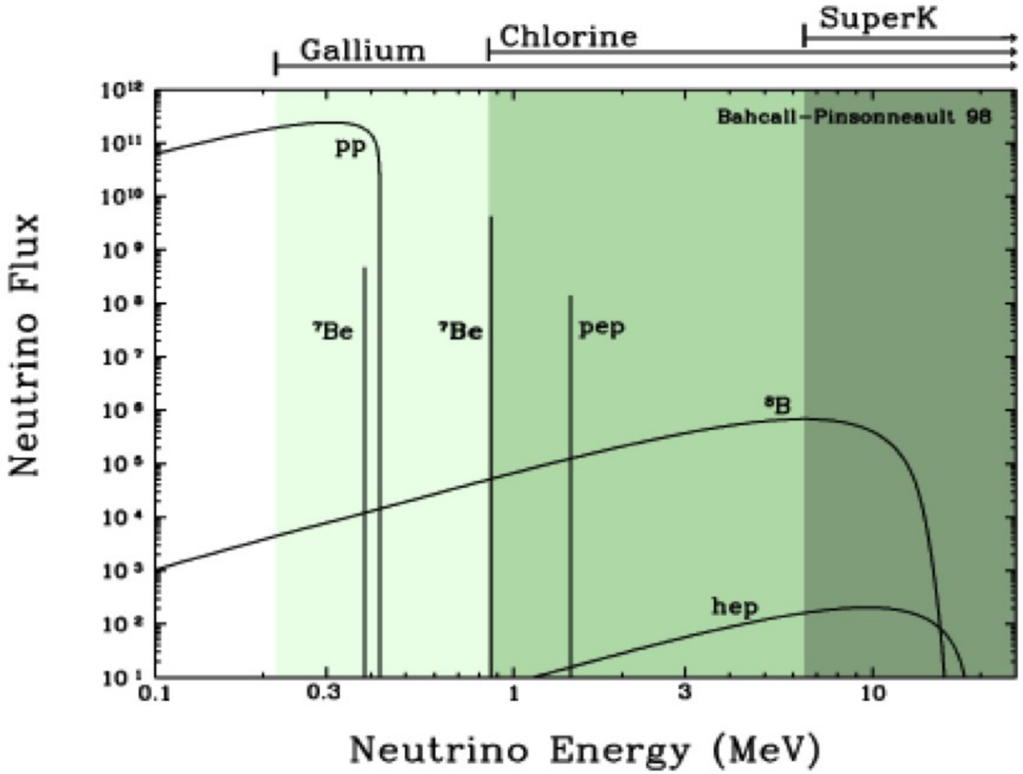
Large part of the points come from the BOREXINO experiment at LNGS

Solar neutrinos

- **The Sun is a sphere of gaseous Hydrogen (*R* = 700 000 km) of high density (at the center the density is similar to that of lead) and at high temperature. It burns Hydrogen, 600 Mt/s**
- **The global reaction producing ~95% of the Sun energy is:**

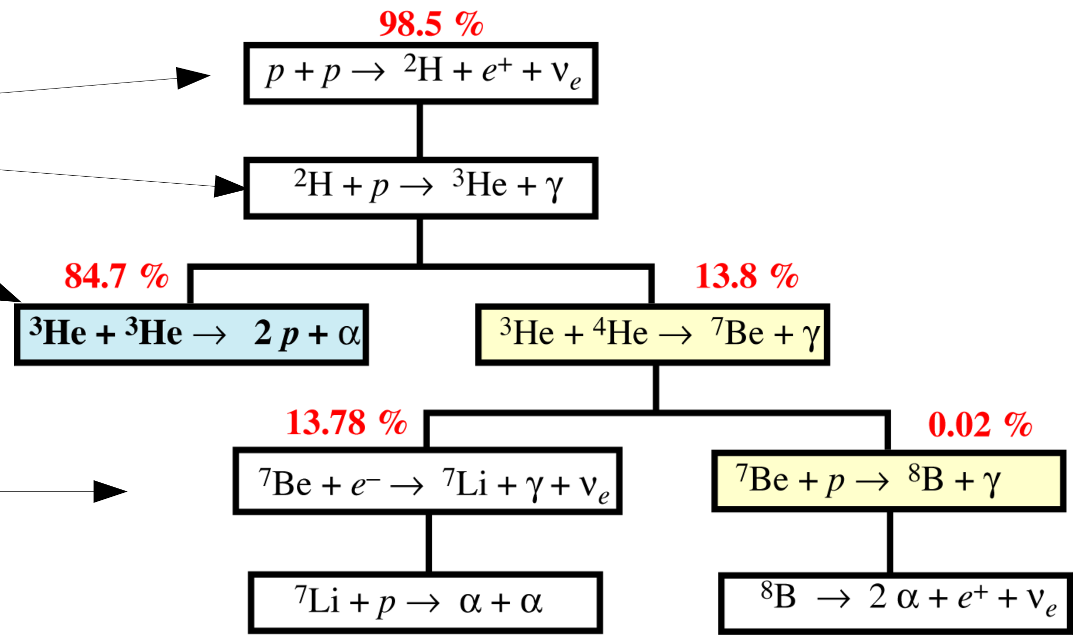
$$4p \rightarrow {}^4\text{He} + 2e^+ + 2\,\nu_e + 26.7\ \text{MeV}$$

- The electromagnetic energy reaches the Sun surface after 400 000 years and then it is irradiated.
- The neutrinos arrives directly from the central part of the Sun. **Flux to the Earth = 60 billions/s cm²**



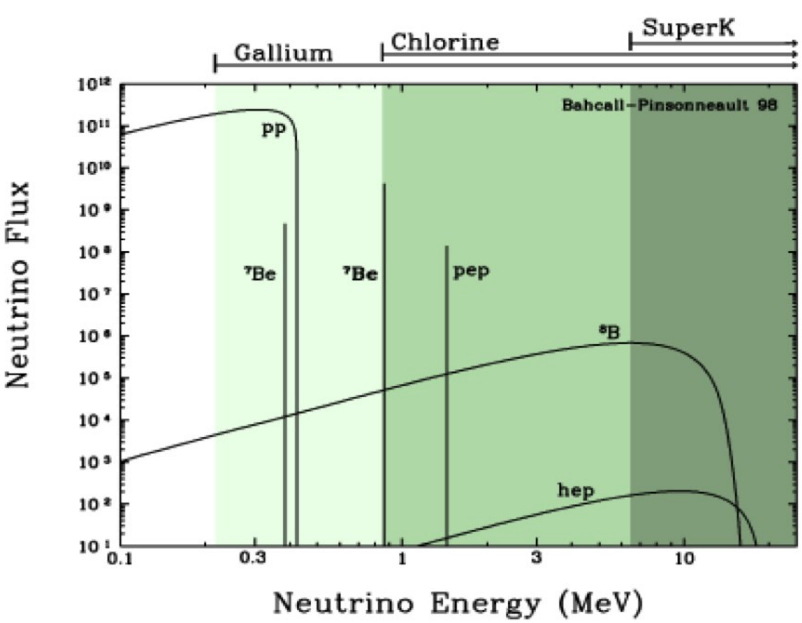
Solar neutrinos

pp cycle
The produced neutrinos have low energy (< 0.5 MeV) very difficult to detect



neutrinos from ^{7}Be
they have two distinct peaks at 0.39 MeV and at 0.86 MeV

neutrinos from ^{8}B
they have energy up to 15 MeV



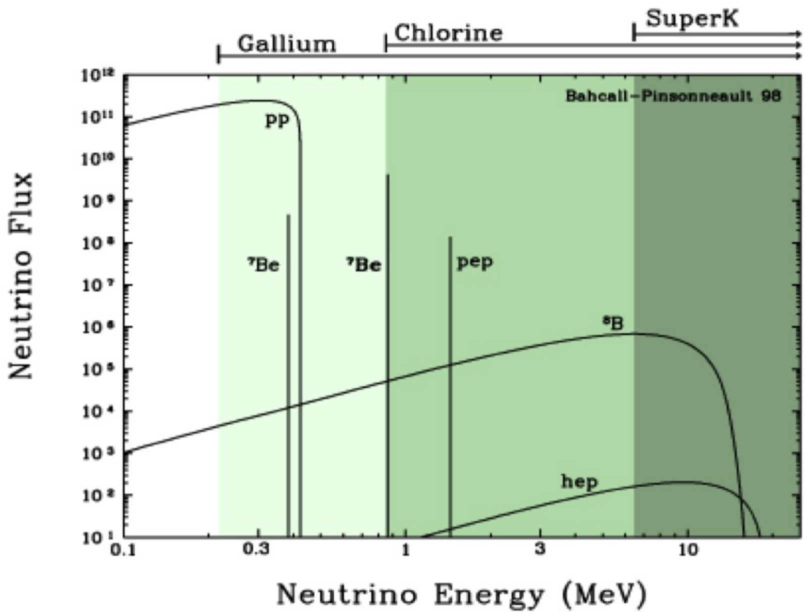
Solar neutrinos

Radiochemical experiments using solar neutrinos:

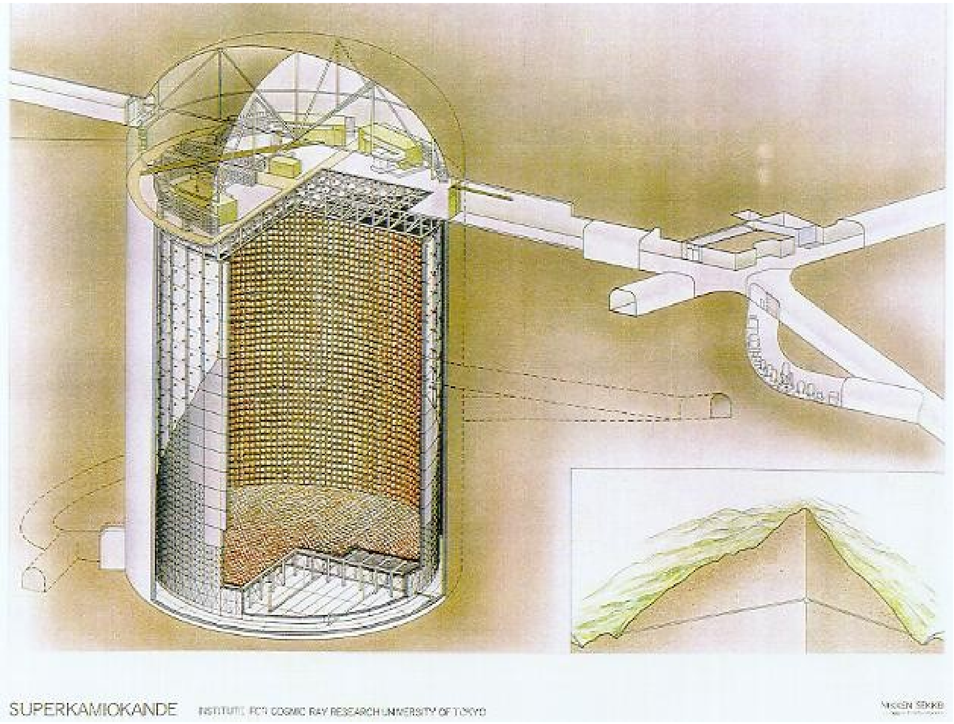
- **Homestake experiment in South Dakota (USA)**
  1964 The discovery. R. Davis. $\nu_e + {}^{37}\mathrm{Cl} \rightarrow e^- + {}^{37}\mathrm{Ar}$
  615 t of tetracloroethylene (expected < 1 /day)
  Ratio: measured/expected from Solar Standard Model: $R = 0.301 \pm 0.027$

- **GALLEX (LNGS).** $\nu_e + {}^{71}\mathrm{Ga} \Rightarrow e^- \; {}^{71}\mathrm{Ge}$
  1997. low energy neutrinos, *pp*, flux known from the luminosity
  $R = 0.529 \pm 0.042$

Solar neutrino problem

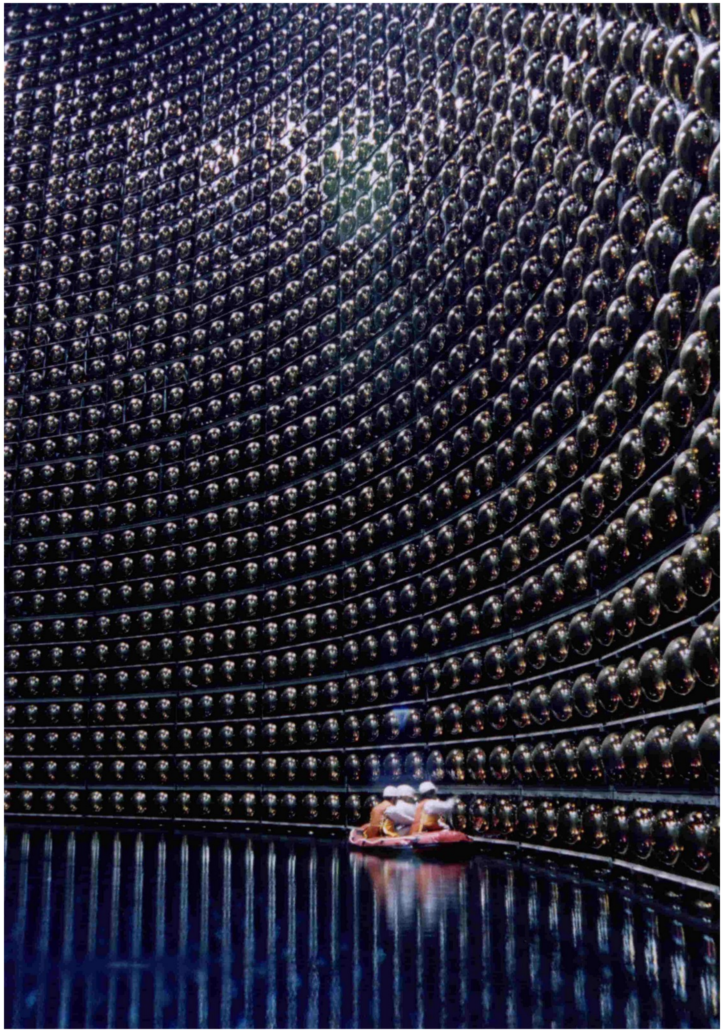


Super-Kamiokande experiment



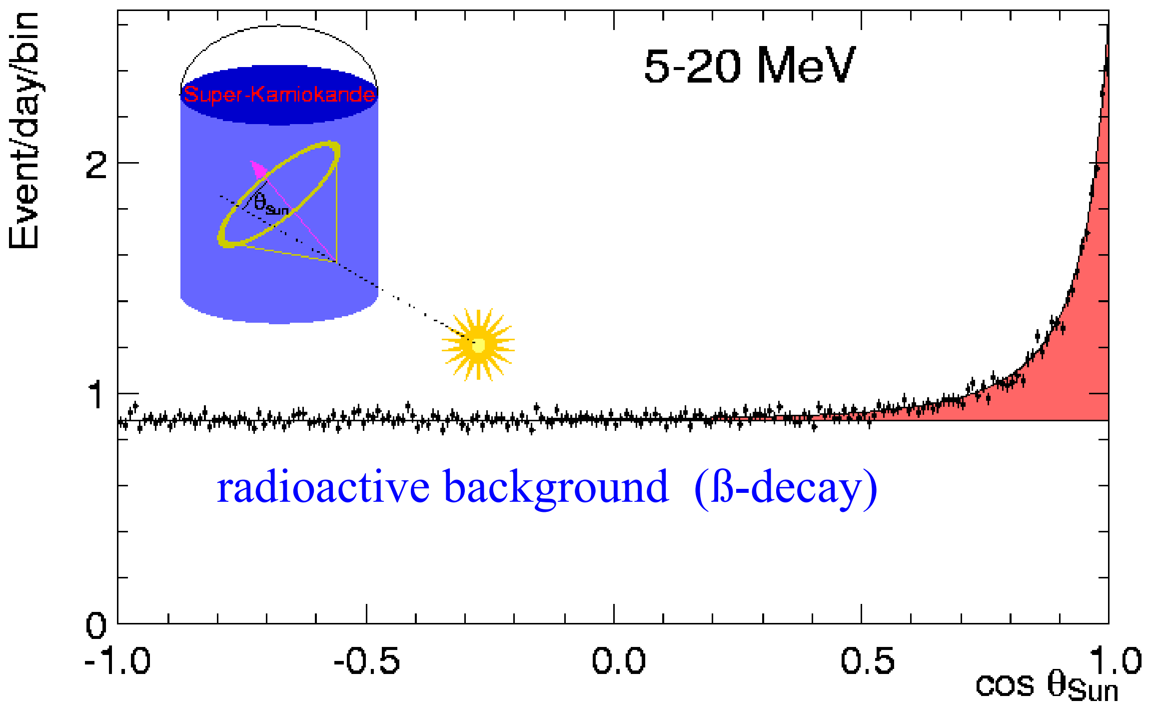
cylindric detector containing 50kton of ultrapure water
The Cherenkov radiation is used to detect the vertices of the event, estimate the energy, discriminate the type of particle (e=like, muon-like)

Controls during the filling



Super-Kamiokande experiment

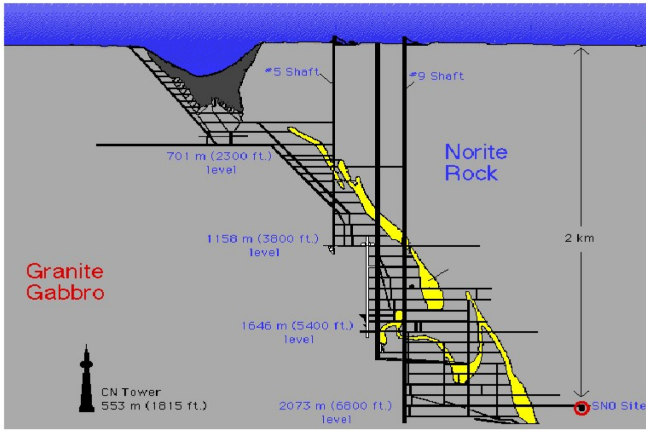
- The oxygen is a nucleus very stable. The process $\nu_e + {}^{16}_{8}O \rightarrow {}^{16}_{9}F + e^-$ is not kinematically permitted due to the available energies.
- The solar neutrinos are seen through the elastic process: $\nu_e + e^- \rightarrow \nu_e + e^-$
- The relativistic electron of the final state can be seen from the emitted Cherenkov photons: the number of detected photons gives a measurement of the energy of the neutrino and the direction of the electron can be measured from the orientation of the Cherenkov rings.
- Threshold at 5 MeV, below the radioactive backgrounds are dominant. Sensitivity to the ^{8}B neutrinos



peak of the solar neutrinos

Ratio: measured/expected from Solar Standard Model: $\boldsymbol{R = 0.406 \pm 0.014}$

SNO experiment



Sudbury Neutrino Observatory

1000 tonnes D_2O

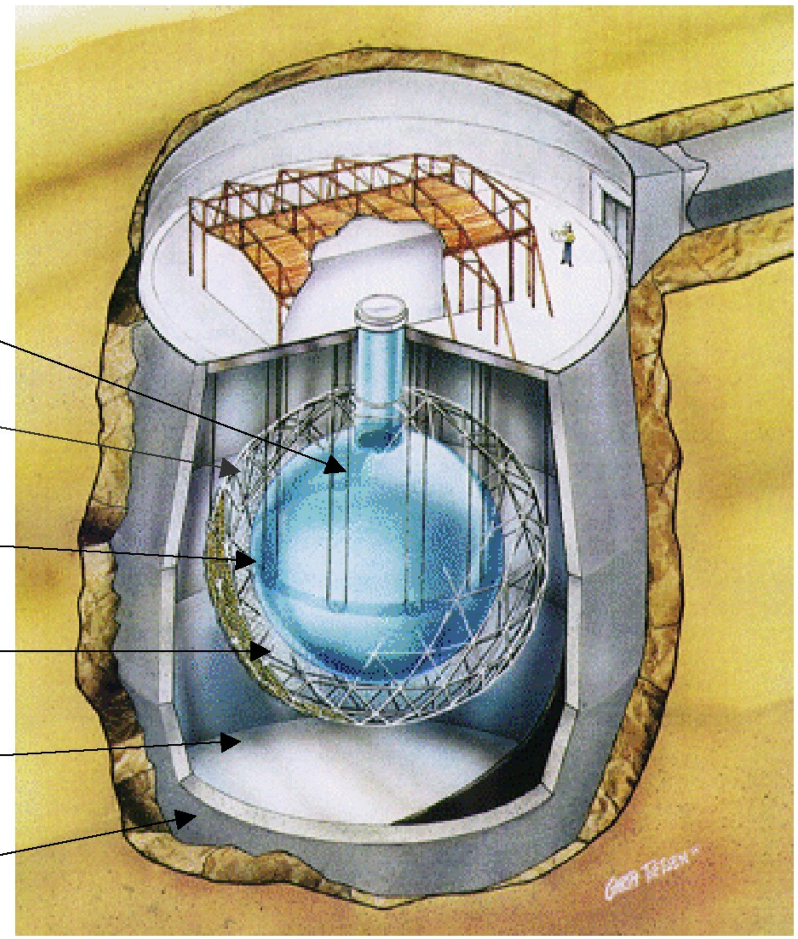
Support Structure for 9500 PMTs, 60% coverage

12 m Diameter Acrylic Vessel

1700 tonnes Inner Shielding H_2O

5300 tonnes Outer Shield H_2O

Urylon Liner and Radon Seal



SNO experiment

ν Reactions in SNO

CC $\nu_e + d \Rightarrow p + p + e^-$

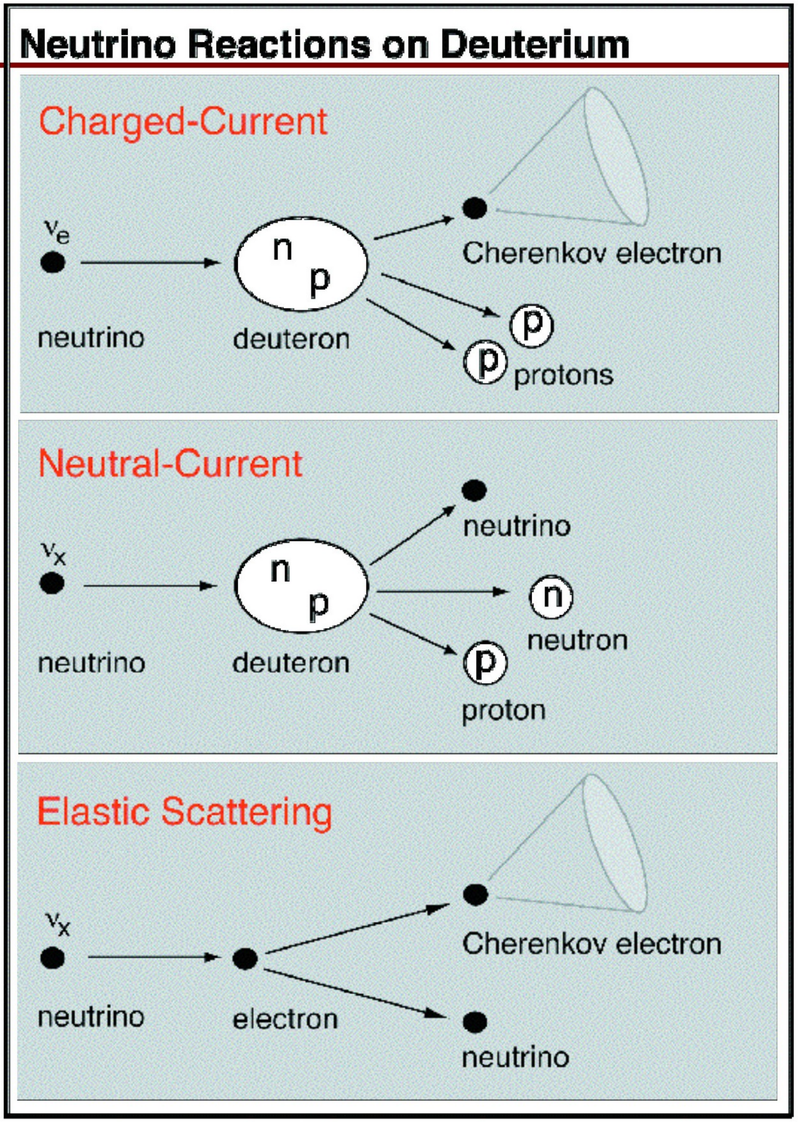
-Gives ν_e energy spectrum
-Weak direction sensitivity ∝ 1-1/3cos(θ)
- ν_e only.

NC $\nu_x + d \Rightarrow p + n + \nu_x$

-Measure total ^8B ν flux from the sun.
- Equal cross section for all ν types

ES $\nu_x + e^- \Rightarrow \nu_x + e^-$

-Low Statistics
-Mainly sensitive to ν_e, some
-sensitivity to ν_μ and ν_τ
-Strong direction sensitivity



SNO experiment

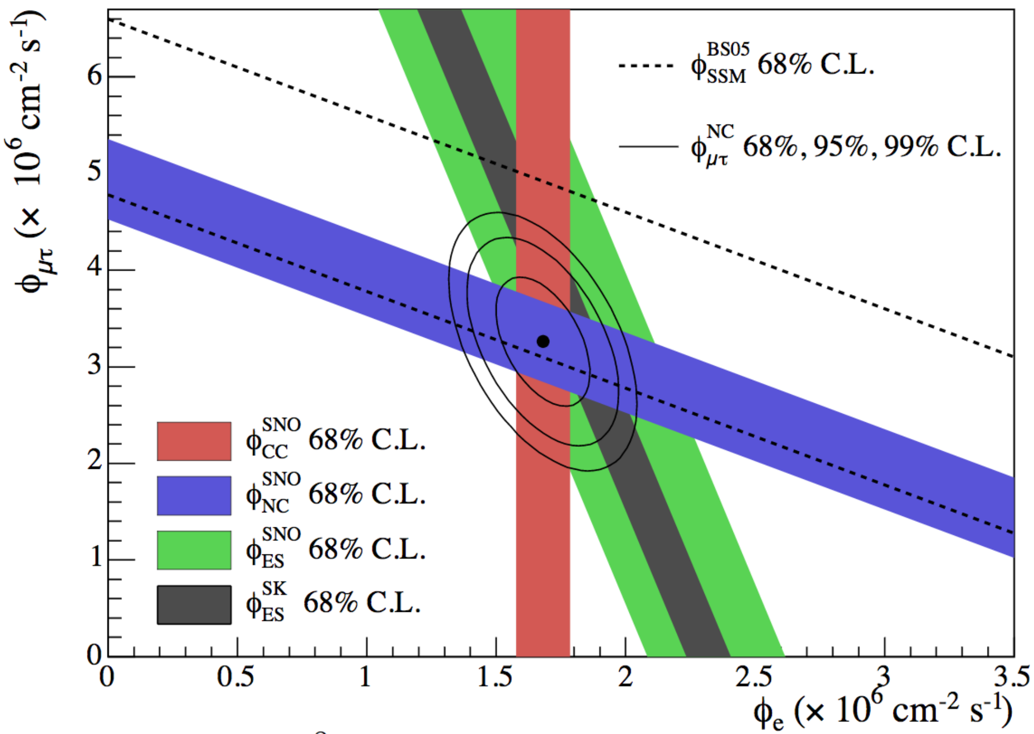


Figure 14.8: Fluxes of ^{8}B solar neutrinos, $\phi(\nu_e)$, and $\phi(\nu_{\mu,\tau})$, deduced from the SNO's CC, ES, and NC results of the salt phase measurement [192]. The Super-Kamiokande ES flux is from Ref. 237. The BS05(OP) standard solar model prediction [98] is also shown. The bands represent the 1σ error. The contours show the 68%, 95%, and 99% joint probability for $\phi(\nu_e)$ and $\phi(\nu_{\mu,\tau})$. The figure is from Ref. 192.

$$\Phi_{CC} = \Phi_e$$
$$\Phi_{ES} = \Phi_e + r\,\Phi_{\mu\tau}$$
$$\Phi_{NC} = \Phi_e + \Phi_{\mu\tau}$$

$$\Phi_{SNO}^{CC} = (1.68 \pm 0.06^{+0.08}_{-0.09}) \times 10^6\, cm^{-2} s^{-1}$$
$$\Phi_{SNO}^{ES} = (2.35 \pm 0.22 \pm 0.15) \times 10^6\, cm^{-2} s^{-1}$$
$$\Phi_{SNO}^{NC} = (4.94 \pm 0.21^{+0.38}_{-0.34}) \times 10^6\, cm^{-2} s^{-1}$$

Flux of neutrinos from ^{8}B predicted by SSM: $\Phi_{SM}^{NC} = 5.05^{+1.01}_{-0.81}$

From the experimental results one obtains the following flux for the active non-ν_e neutrinos:

$$\Phi(\nu_{\mu\ or\ \tau}) = (3.26 \pm 0.25^{+0.40}_{-0.35}) \times 10^6\, cm^{-2} s^{-1}$$

Final summary

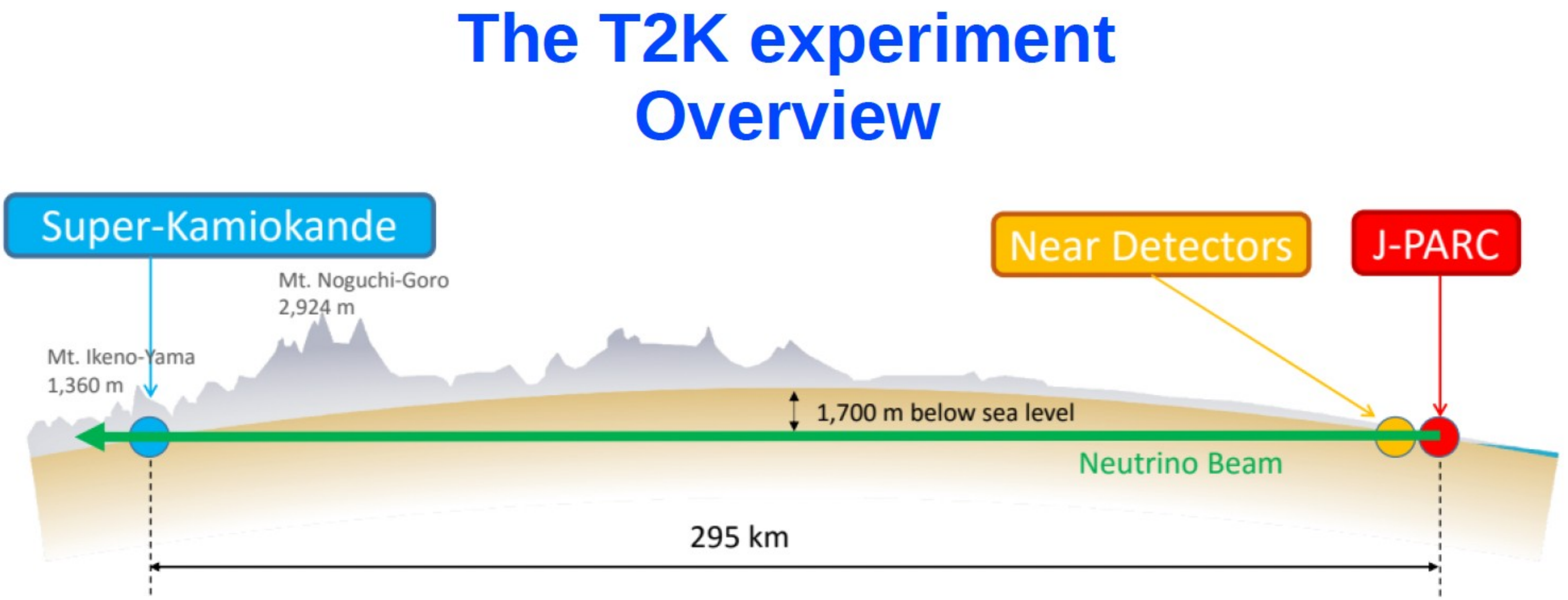
$m^2_2 - m^2_1 = (7.6 \pm 0.2) \times 10^{-5}$ eV^2
$|m^2_3 - m^2_2| = (2.3 \pm 0.1) \times 10^{-3}$ eV^2

$\sin^2 2\theta_{12} >= 0.87 \pm 0.04$
$\sin^2 2\theta_{23} > 0.92$
$\sin^2 2\theta_{13} = 0.10 \pm 0.01$

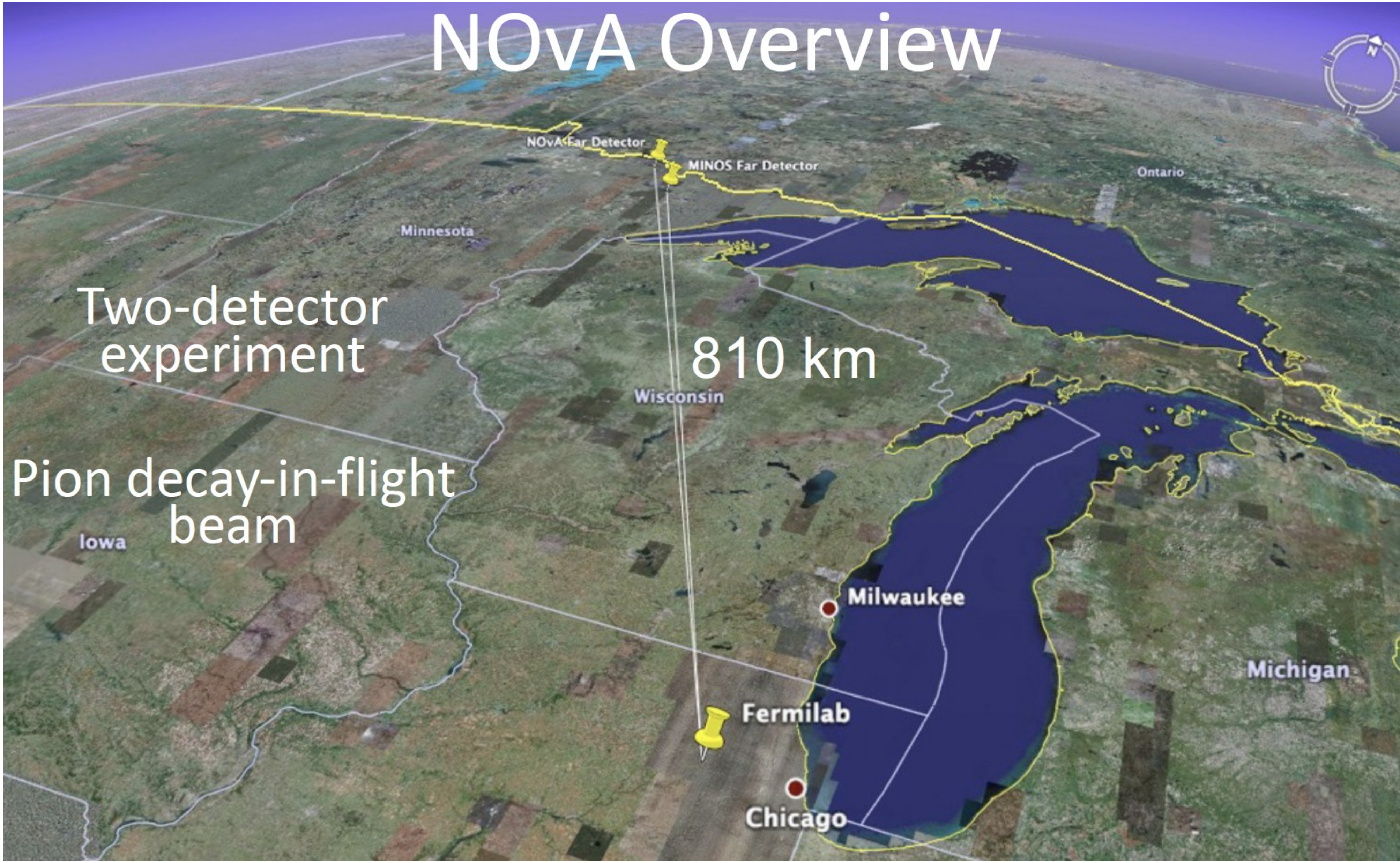
$$\begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{pmatrix} \sim \begin{pmatrix} 0.85 & 0.50 & 0.17 \\ 0.35 & 0.60 & 0.70 \\ 0.35 & 0.60 & 0.70 \end{pmatrix}$$

- **δ is not known** — **T2K**, NOνA, DUNE, HYPERK,......
- **the mass hierarchy not known** — **JUNO, T2K,** NOνA, DUNE, HYPERK,...
- **the absolute masses are not known** — **GERDA, CUORE, LEGEND-200,** KATRIN, ...
- **not known if the neutrino is a Dirac or a Majorana particle** — **GERDA, CUORE, LEGEND-200,...**

Final summary… CP phase



Final summary… CP phase



Final summary… CP phase

CP phase

- ~270° (-90°) seems slightly favored by many exp.s (< 3σ)
- Combined analysis may give more preference, but not stable yet
- DUNE & HyperK can give a more definite answer
- Further improvement may come from KNO, ESSnuSB, and THEIA