CKM matrix and CP violation in SM (I)

- Origin of the Cabibbo-Kobayashi-Maskawa Matrix (CKM)
- Overview of the measurements of the CKM elements
- CP violation in the Standard Model
- Overview of the measurements

Standard Model

$SU(2)_L \times U(1)_Y$

Weak Isospin (symbol L because only the LEFT states are involved)

Weak Hypercharge :

(LEFT and RIGHT states)

			I	I_3	Q	Y
Leptons	doublet L	ν_e	½	½	0	-1
		e_L^-	½	-½	-1	-1
	singlet R	e_R^-	0	0	-1	-2
quarks	doublet L	u_L	½	½	2/3	1/3
		d_L	½	-½	-1/3	1/3
	singlet R	u_R	0	0	2/3	4/3
	singlet R	d_R	0	0	-1/3	-2/3

Idem for the other families

Mass of the Quarks in the Standard Model

- For each generation we have one left-handed SU(2) doublet, and two right-handed singlets

hypercharge $Q-T_3$

SU(3) triplet

$$Q_L^I = \begin{pmatrix} U_L^I \\ D_L^I \end{pmatrix} = (3,2)_{+1/6}, \quad u_R^I = (3,1)_{+2/3}, \quad d_R^I = (3,1)_{-1/3}$$

SU(2) doublet

Eigenstates of weak interactions

- Quarks interact with Higgs field via Yukawa coupling

$$\mathcal{L}_Y = -\mathbf{G}_{ij}\overline{Q^I_{Li}}\phi d^I_{Rj} - \mathbf{F}_{ij}\overline{Q^I_{Li}}\tilde{\phi} u^I_{Rj} + \text{H.c.}$$

Generic complex matrix of yukawa coupling constants

- Quarks acquire mass through because of spontaneous symmetry breaking

$$\mathcal{L}_M = -\sqrt{\frac{1}{2}} v\mathbf{G}_{ij}\overline{d^I_{Li}} d^I_{Rj} - \sqrt{\frac{1}{2}} v\mathbf{F}_{ij}\overline{u^I_{Li}} u^I_{Rj} + \text{H.c.}$$

$$\mathbf{M}_d = \mathbf{G}v/\sqrt{2}, \quad \mathbf{M}_u = \mathbf{F}v/\sqrt{2}.$$

Mass matrices for up and down quarks. Elements are complex!

Weak Interactions and Mass Eigenstates

- Diagonalize mass matrices to obtain mass eigenstates
  - Rotate quark fields by with unitary complex matrices V_{uL}, V_{uR}, V_{dL}, V_{dR}
  - Choose arbitrary phases so that M is diagonal

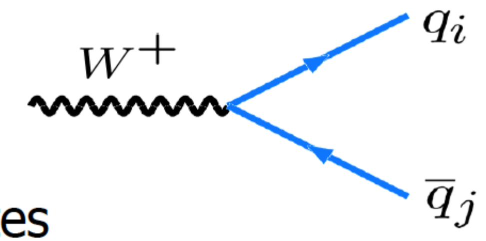
$$\mathbf{M}_d = \mathbf{G}v/\sqrt{2}, \quad \mathbf{M}_u = \mathbf{F}v/\sqrt{2}.$$

$$\mathbf{V}_{dL}\mathbf{M}_d\mathbf{V}^\dagger_{dR} = \mathbf{M}^{\mathrm{diag}}_d, \quad \mathbf{V}_{uL}\mathbf{M}_u\mathbf{V}^\dagger_{uR} = \mathbf{M}^{\mathrm{diag}}_u.$$

Universality of weak interactions: same constant g for all couplings

- Lagrangian for weak interactions of quarks

$$\mathcal{L}_W = -\sqrt{\frac{1}{2}}g\overline{u^I_{Li}}\gamma^\mu \mathbf{1}_{ij} d^I_{Lj} W^+_\mu + \mathrm{h.c.}$$



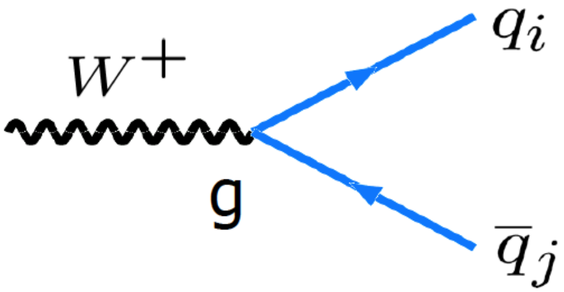
- Lagrangian after going from interaction to mass eigenstates

No more universal coupling constant!

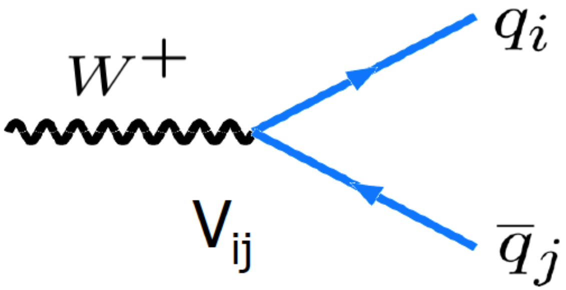
$$\mathcal{L}_W = -\sqrt{\frac{1}{2}}g\overline{u_{Li}}\gamma^\mu \boxed{\overline{\mathbf{V}}_{ij}} d_{Lj} W^+_\mu + \mathrm{h.c.} \qquad \overline{\mathbf{V}} = \mathbf{V}_{uL}\mathbf{V}^\dagger_{dL}$$

No more Universality of Weak Interactions

- In absence of CKM matrix all weak interactions have same coupling
  - This is referred to as universality of weak interactions



- Because of CKM matrix coupling depends on quarks involved in the transition
  - Universality is broken!



Cabibbo-Kobayashi-Maskawa Matrix

$$V_{CKM} = V_{uL}^{\dagger} V_{dL} \qquad \mathbf{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Origin of CKM matrix is the difference between mass eigenstates and weak interaction eigenstates
- Lagrangian of Standard Model is diagonal in weak eigenstates with universal coupling constant
- Universality is broken when moving from interaction basis to mass basis necessary to obtain Lagrangian for mass terms after spontaneous symmetry breaking
- V_{CKM} is a unitary complex matrix

Properties of CKM Matrix

M(diag) is unchanged if $V'^f_L = P^f V^f_L$; $V'^f_L = P^f V^f_R$ $\qquad V(CKM) = P^u (CKM) P^{*d}$

P^f = phase matrix

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \rightarrow \begin{pmatrix} e^{-i\varphi_1} & 0 \\ 0 & e^{-i\varphi_2} \end{pmatrix} \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \begin{pmatrix} e^{-i\chi_1} & 0 \\ 0 & e^{-i\chi_2} \end{pmatrix} = \begin{pmatrix} V_{11} e^{-i(\varphi_1-\chi_1)} & V_{12} e^{-i(\varphi_1-\chi_2)} \\ V_{21} e^{-i(\varphi_2-\chi_1)} & V_{22} e^{-i(\varphi_2-\chi_2)} \end{pmatrix}$$

$$(\varphi_2 - \chi_2) = (\varphi_2 - \chi_1) + (\varphi_1 - \chi_2) - (\varphi_1 - \chi_1)$$

Among 4 phases, only 3 can be arbitrarly chosen and removed (so 2n-1)

Generally for a rotation matrix in complex plane

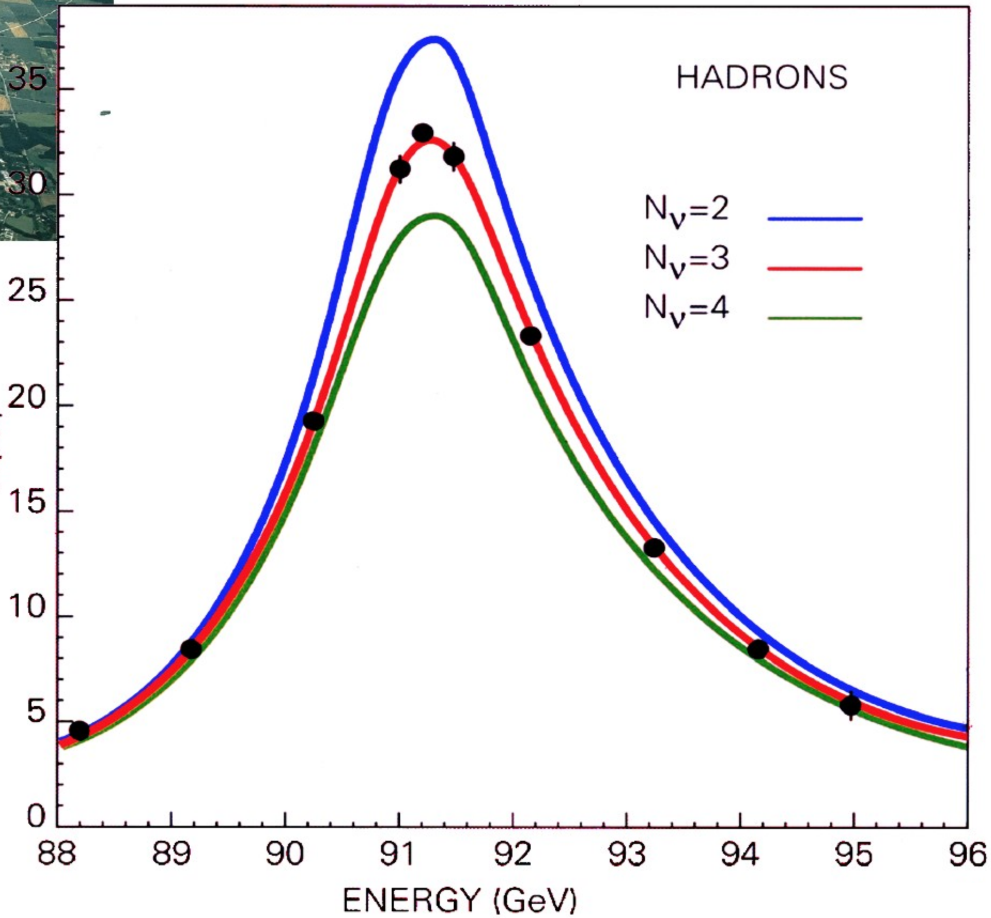
Quark families	# Angles	# Phases	# Irreducible Phases
n	n(n-1)/2	n(n+1)/2	n(n+1)/2-(2n-1)=(n-1)(n-2)/2
2	1	3	0
3	3	6	1
4	6	10	3

Necessary for CP Violation in SM

- Today we know there are three flavors, or generations of quarks
- But this was not the case when CKM matrix was first proposed in 1973!

How do we know there are only 3 generations of matter?

Number of neutrino families from LEP @ CERN



Families of matter known in 1972

Three Quarks for Muster Mark !…*Joyce*

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} ? \\ s \end{pmatrix}$$

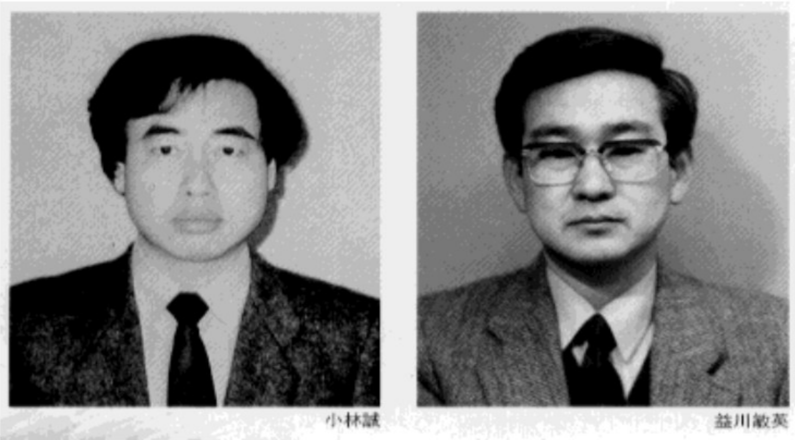
$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}$$

Only 2 families were known

Charm quark not even observed yet!

Kobayashi-Maskawa Mechanism of CP Violation

1972



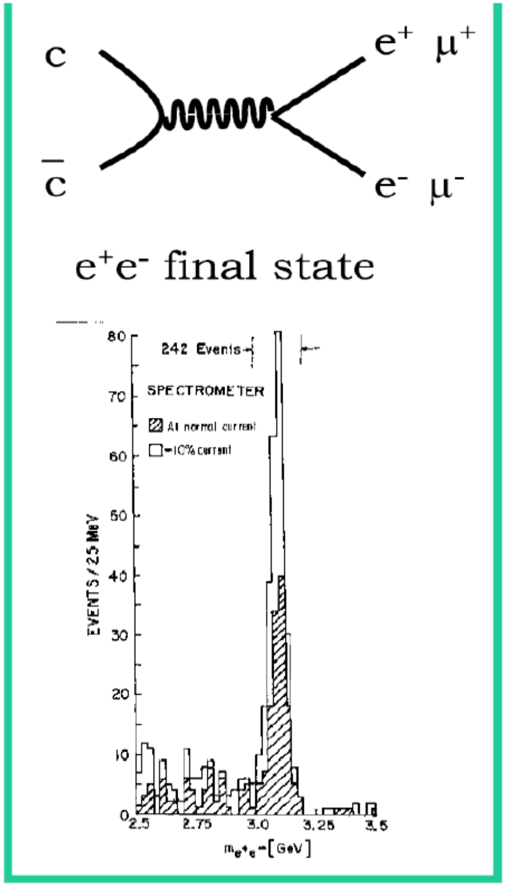
Two Young
Postdocs at that
time !

- Proposed a daring explanation for CP violation in K decays
- CP violation appears only in the charged current weak interaction of quarks
- There is a single source of CP Violation ⇒ Complex Quantum Mechanical Phase δ_{KM} in inter-quark coupling matrix
- Need at least **3 Generation of Quarks** (then not known) to facilitate this
- **CP is NOT an approximate symmetry**, $\delta_{KM} \cong 1$, it is MAXIMALLY violated !

1974: Discovery of charm in J/psi

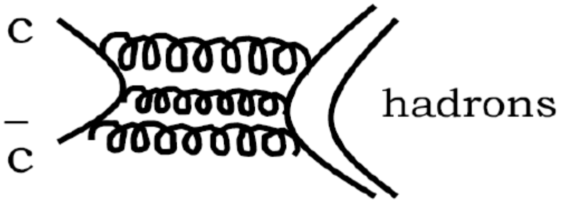
Seen as a resonance m~3.1 GeV Γ~10-100KeV

- Brookhaven (p on Be target)



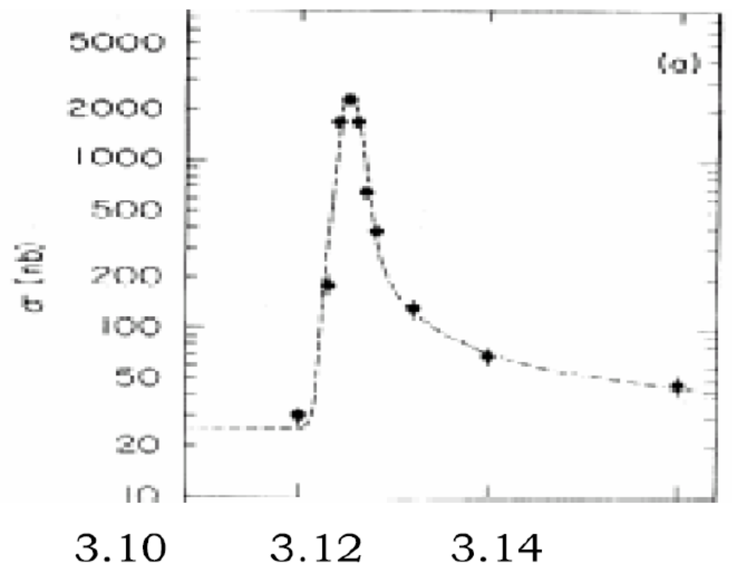
e^+e^- final state

Γ(ee)~5 KeV
Γ(μμ)~5 KeV



Γ~70 KeV

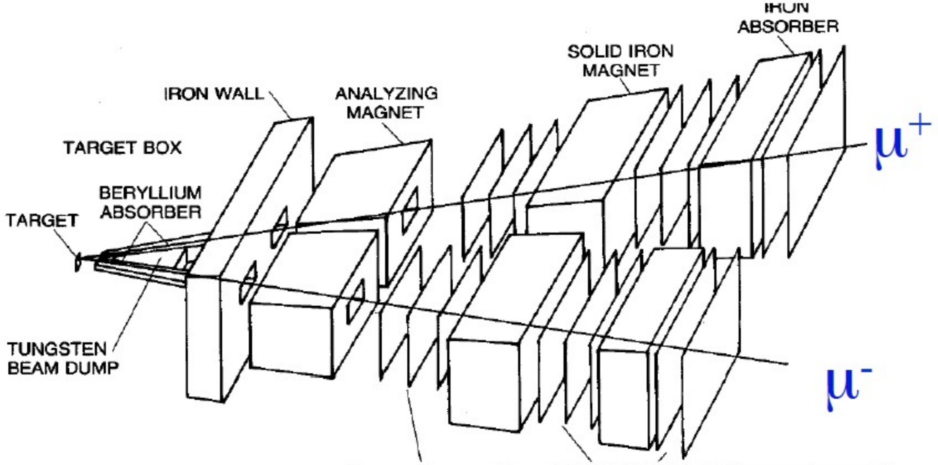
SLAC (e+e-)



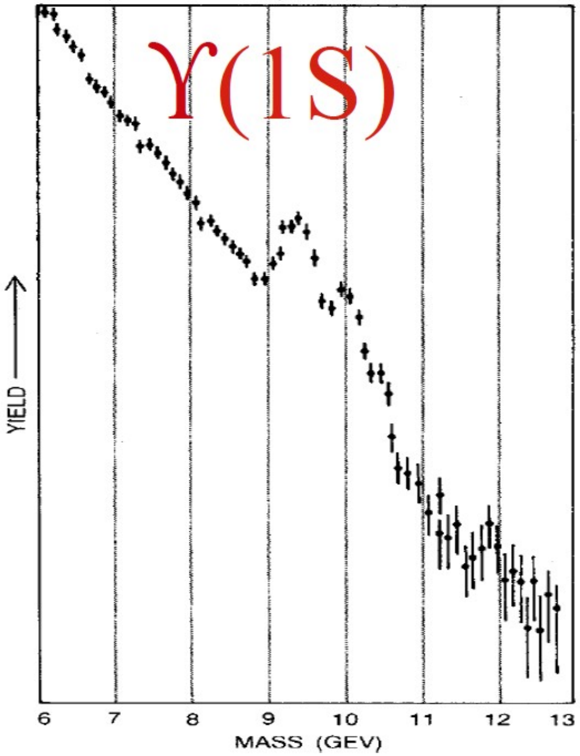
hadronic final state

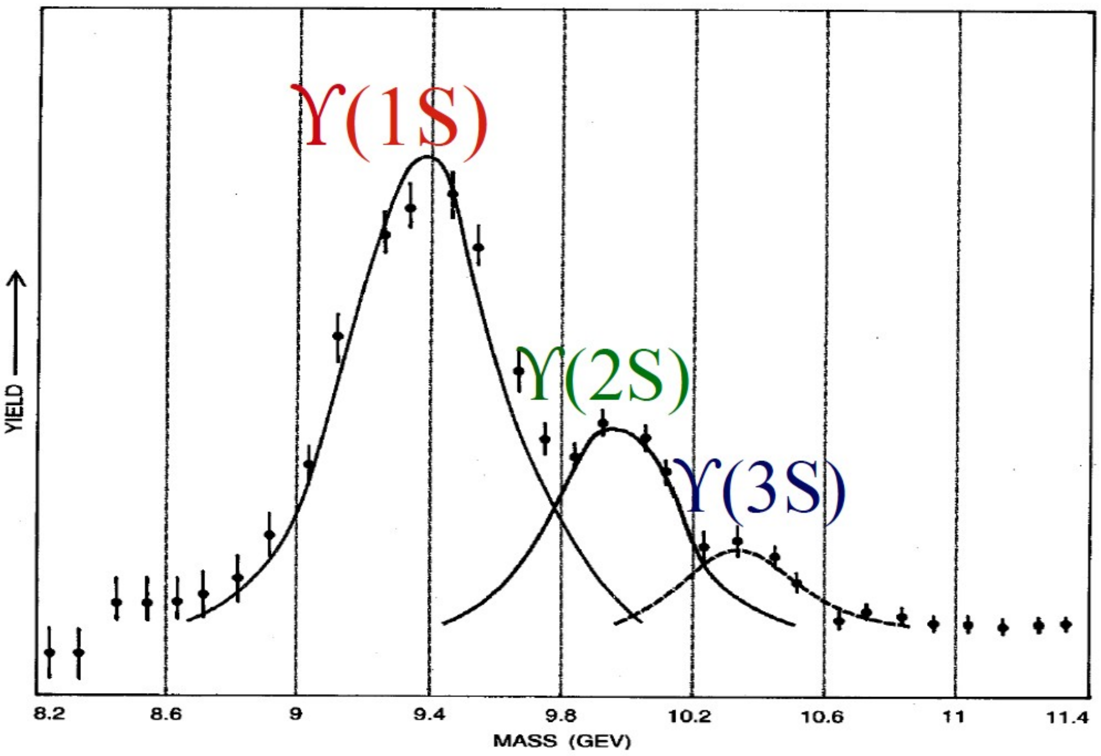
The decay through strong interaction is so suppressed that the electromagnetic interaction becomes important

1977: Discovery of bottom in Upsilon(1S) @ FNAL



PRL **39**
252(1977)





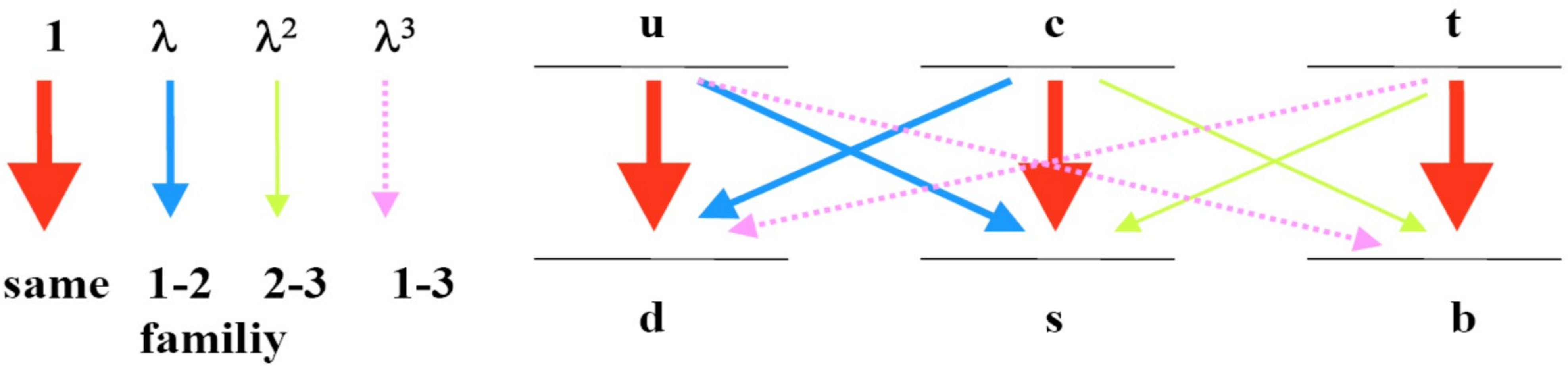
Standard Parameterization of CKM matrix

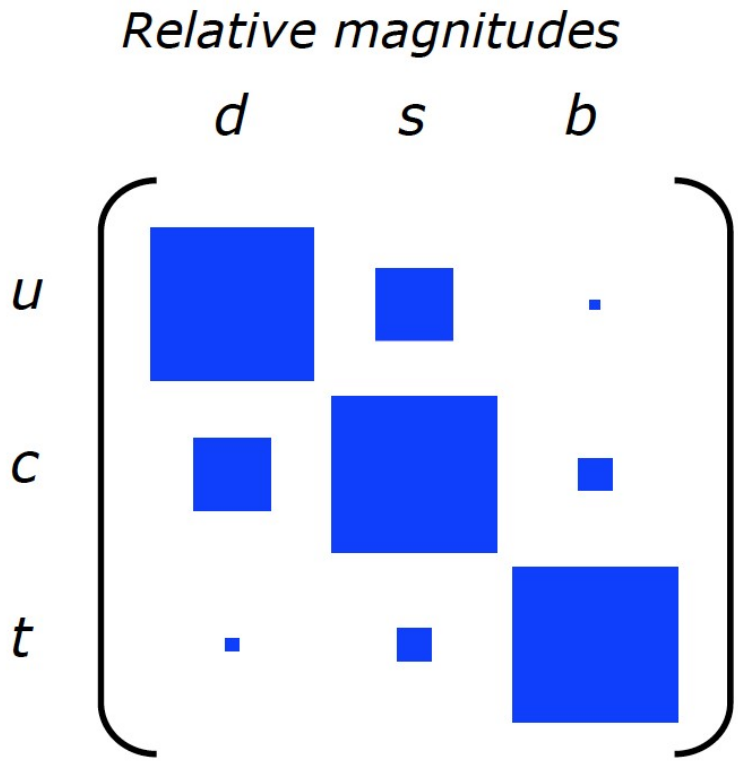
3 mixing angles and one CP-violating KM phase.
The angles θ_{ij} in the first quadrant, so s_{ij}, $c_{ij} \geq 0$

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix}$$

Features of CKM Matrix





Diagonal elements ~ 1

V_{cb} , V_{ts} $\sim 4\times 10^{-2}$

V_{us} , V_{cd} ~ 0.2

V_{ub} , V_{td} $\sim 4 \times 10^{-3}$

Wolfenstein Parameterization of CKM Matrix

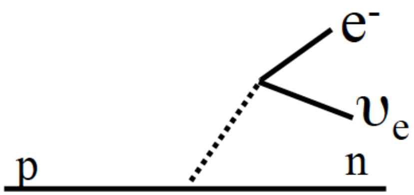
- Wolfenstein first saw a pattern with 4 parameters

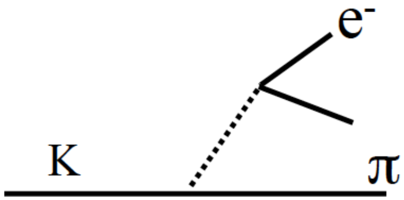
Cabibbo angle
with 2 generations

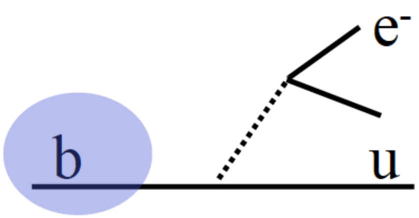
$$\mathbf{V}_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

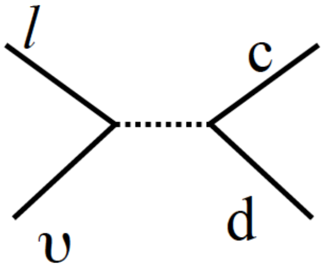
$$\begin{aligned} \lambda &= \left|V_{us}\right| &&\approx 0.22 \\ A &= \left|V_{cb}\right|/\lambda^2 &&\approx 0.80 \\ \sqrt{\rho^2+\eta^2} &= \left|V_{ub}\right|/\left(\lambda\left|V_{cb}\right|\right) &&\approx 0.35 \\ \eta/\rho &= \tan\left[\arg\left(V_{ub}\right)\right] &&\approx 2.5 \end{aligned}$$

Measurements of CKM Element Magnitudes

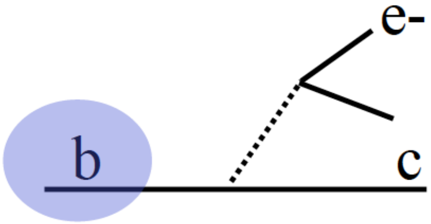


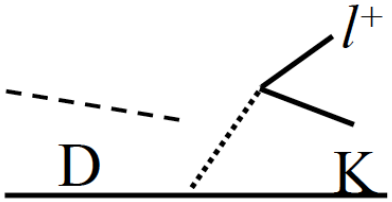


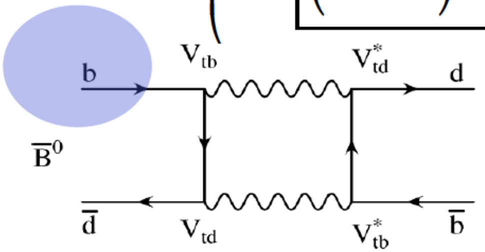


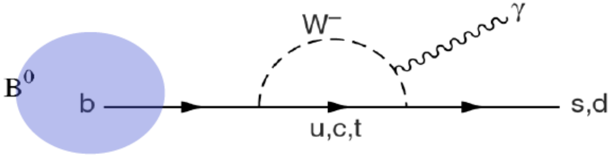


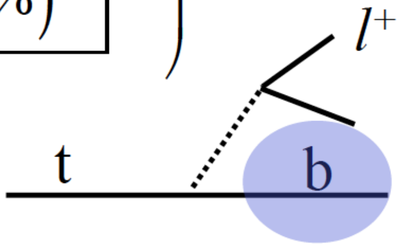
$$\begin{pmatrix} V_{ud}=0.97425\ (\pm 0.02\%) & V_{us}=0.2252\ (\pm 0.4\%) & V_{ub}=0.004\ (\pm 6\%) \\ V_{cd}=.230\ (\pm 2\%) & V_{cs}=0.987\ (\pm 1\%) & V_{cb}=0.041\ (\pm 3\%) \\ V_{td}\sim 0.008\ (\pm 4\%) & V_{ts}\sim 0.039\ (\pm 3\%) & V_{tb}=1.01\ (\pm 3\%) \end{pmatrix}$$











PDG review of CKM

b quark plays a special role in determination of CKM elements!

Measuring CKM Elements

- Measurements related to first 2 generations briefly discussed here
  - Most measurements established since a while

- Mostly focus on decays of B mesons and related measurements because
  - B factories at SLAC and KEK since 1999 have allowed a detailed study of many B decays that were not available previously

  - B mesons are an excellent laboratory to study CP Violation
    - observations of 2 different types of CP violation in B mesons since 2001!
    - First observation in 1964 with neutral Kaons

- Redundant measurements of same observables in different processes allow to verify CKM paradigm
  - Discrepancies could be a sign of New Physics beyond Standard Model
  - For example: use measurements to verify unitarity of CKM matrix

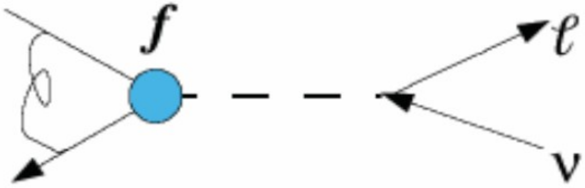
From Hadrons to Quarks

- CKM matrix elements describe processes at quark level but processes observed experimentally involve hadrons
- Theory is used to relate measurements with hadrons to quantities defined for quarks
  - HQET, OPE, Lattice QCD
- Ultimately must verify theories with measurements
- When models are used to interpret data this should be described clearly and some kind of error assigned to the model-dependency

Typology of Tree Decay Amplitudes

Leptonic

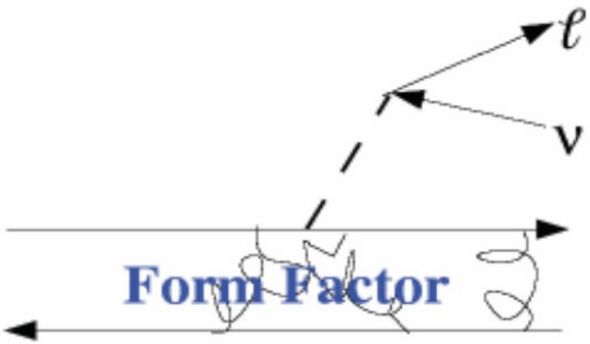
Hadronic & Leptonic current do factorize



- Low energy QCD: decay constant f
- Lattice QCD starts to get precise

Semileptonic

(In most cases best way to extract $|V_{ij}|$)



<u>Exclusive Decays:</u>

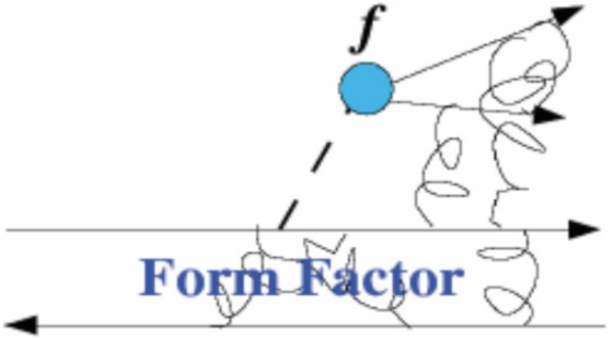
- FF: Symmetries (χ & HQS)
- FF: Lattice QCD, Sum Rules; ...

<u>Inclusive Decays:</u>

- Operator Product Expansion

Hadronic

No factorization in naïve sense due to gluon exchange



Theoretical developments:
e.g. QCD Factorisation approach
Not used for $|V_{ij}|$ extraction (yet)

CKM Elements in First Two Generations

$$\mathbf{V}_{CKM} = \left(\begin{array}{|cc|c} \hline V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ \hline V_{td} & V_{ts} & V_{tb} \end{array}\right)$$

$$\mathbf{V}_{CKM} = \left(\begin{array}{|cc|c} \hline 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ \hline A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{array}\right) + O(\lambda^4)$$

Measuring $|V_{ux}|$ and $|V_{cx}|$

$|V_{ud}|$: 1) Super-allowed nuclear β-decays

2) Neutron β-decay

3) Pionic β-decay

$|V_{us}|$: 1) Semileptonic Kaon decays

2) Leptonic Kaon & Pion decay

$|V_{cd}|$, $|V_{cs}|$: 1) Dimuon production from neutrinos on nuclei

2) Semileptonic D-meson decays

$|V_{ud}|$: β Decays

Fermi-transitions: $0^+ \rightarrow 0^+$ within same isospin multiplet
pure vector-current (take advantage of CVC)

$$|V_{ud}|^2 = \frac{2\,\pi^3 \ln 2}{m_e^5} \cdot \frac{1}{2\,G_F^2\,(1+\Delta_R)\,Ft}\,, \qquad Ft = f \cdot t_{1/2} \cdot (1+\delta_R) \cdot (1-\delta_C)$$

Radiative Correction
(nucleus-independent)
$\Delta_R = (2.40 \pm 0.08)\%$

1) PS Integral (~ E_0^5)
2) Radiative Correction
(nucleus-dependent)
3) Isospin-symmetry breaking

$|V_{ud}| = 0.97373 \pm 0.00031$

<u>Neutron β-decays</u>: $n \rightarrow p\, e^-\, \nu_e$

Vector transition: $G_V = g_V\, G_F\, |V_{ud}|$ (CVC <=> Isospin Cons.: $g_V = 1$)

Axial-V. transition: $G_A = g_A\, G_F\, |V_{ud}|$ (PCAC: $g_A/g_V \equiv \lambda \neq 1$)

$$\frac{1}{\tau_n} = \frac{(m_e c^2)^5 \cdot G_F^2 \cdot |V_{ud}|^2}{2\,\pi^3\,\hbar\,(\hbar c)^6} \cdot (1+3\lambda^2) \cdot f\,(1+\delta_R) \cdot (1+\Delta_R)$$

=> Measure τ_n and λ not well known

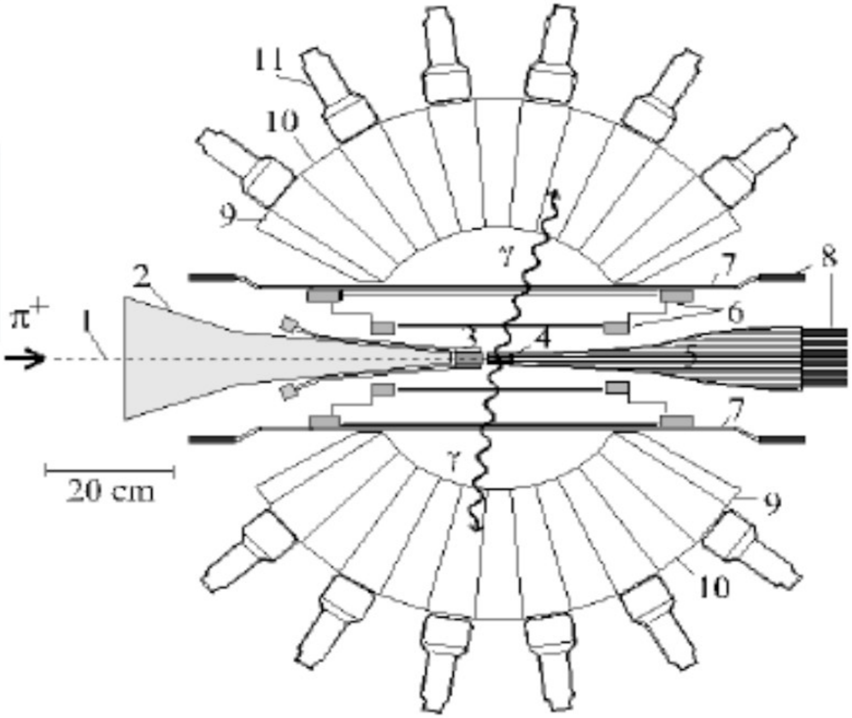
Gamov-Teller-transition => g_A

Fermi-transition => g_V

Super-allowed pion β decay

$\pi^+ \to \pi^0 e^+ \nu_e$ Pure Vector transition

$$|V_{ud}|^2 = \frac{(K/\ln 2)\, Br(\pi^+ \to \pi^0 e^+ \nu_e)}{2 G_F (1+\Delta_R) f_1 f_2 f (1+\delta_R) \tau_\pi}$$



Best experiment:

* **PIBETA experiment at PSI**

* **Stopped π^+**

* **Detection of π^0 in CsI ball,**

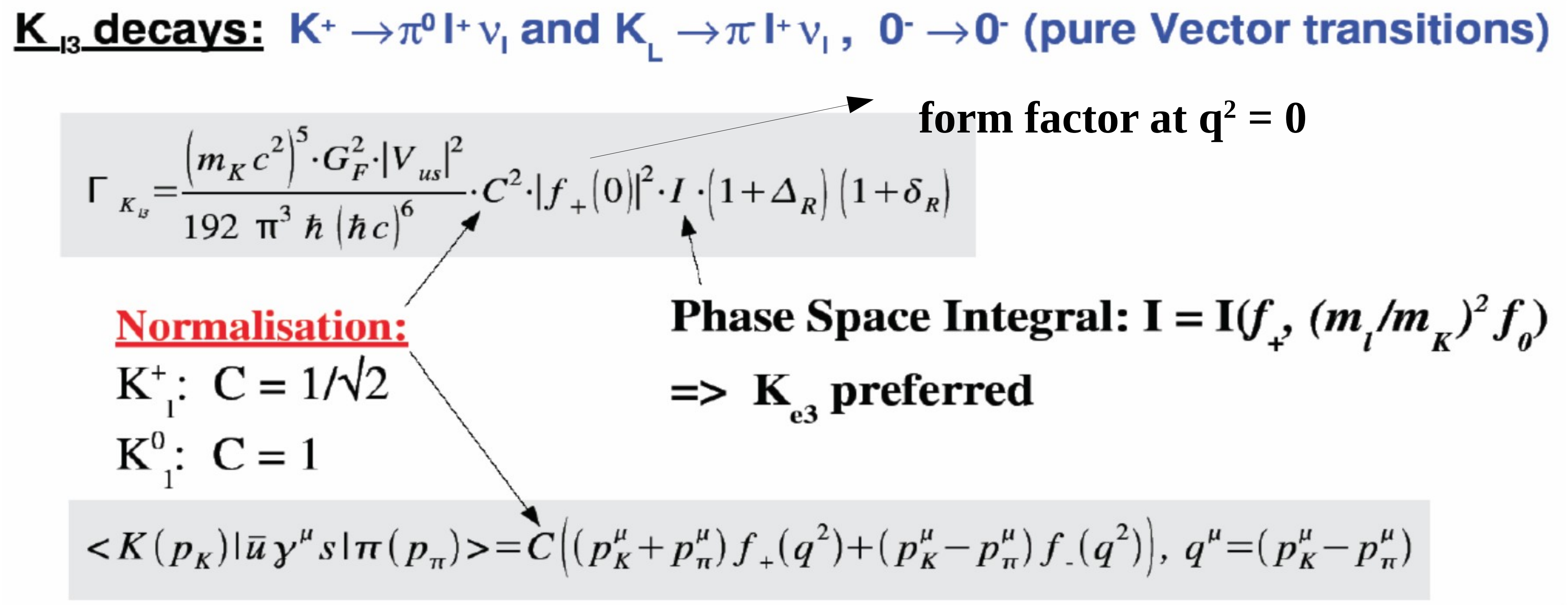
* **Normalisation with $\pi^+ \to e^+ \nu_e$**

PRL 93, 181803 (2004):

BF ($\pi^+ \to \pi^0 e^+ \nu_e$) = $(1.036 \pm 0.004_{stat} \pm 0.004_{sys} \pm 0.003_{\pi e2})\ 10^{-8}$

$|V_{ud}|_\pi$ = 0.9739 ± 0.0029

$|V_{us}|$: Semileptonic K Decays



K_{l3} decays: $K^+ \to \pi^0 l^+ \nu_l$ and $K_L \to \pi^- l^+ \nu_l$, $0^- \to 0^-$ (pure Vector transitions)

$$\Gamma_{K_{l3}} = \frac{(m_K c^2)^5 \cdot G_F^2 \cdot |V_{us}|^2}{192\ \pi^3\ \hbar\ (\hbar c)^6} \cdot C^2 \cdot |f_+(0)|^2 \cdot I \cdot (1+\Delta_R)\,(1+\delta_R)$$

form factor at $q^2 = 0$

Normalisation:

K^+_l: $C = 1/\sqrt{2}$

K^0_l: $C = 1$

Phase Space Integral: $I = I(f_+, (m_l/m_K)^2 f_0)$

=> K_{e3} preferred

$$<K(p_K)|\bar{u}\gamma^\mu s|\pi(p_\pi)> = C\left((p_K^\mu + p_\pi^\mu) f_+(q^2) + (p_K^\mu - p_\pi^\mu) f_-(q^2)\right),\ q^\mu = (p_K^\mu - p_\pi^\mu)$$

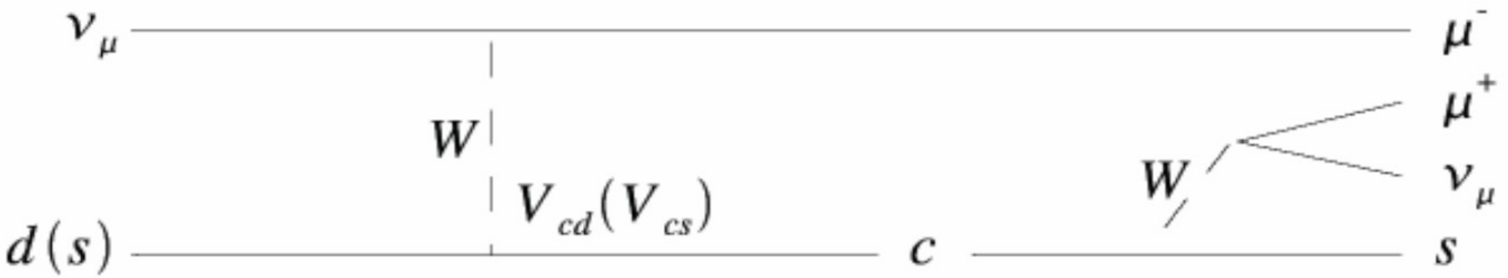
Averaging many experimental results:

$|V_{us}| = 0.2231 \pm 0.0007$ (with $f_+(0)$ coming from lattice QCD calculations)

di-muon Production in Deep Inelastic Scattering

Charm production in Deep Inelastic Scattering of Neutrinos/Anti-Neutrinos on Nucleons:

$$\nu_\mu d(s) \rightarrow \mu^- c \quad (c \rightarrow s \mu^+ \nu_\mu)$$
$$\bar{\nu}_\mu \bar{d}(\bar{s}) \rightarrow \mu^+ \bar{c} \quad (\bar{c} \rightarrow s \mu^- \bar{\nu}_\mu)$$



(Anti-)Neutrino-Nucleon Cross Section (CHDS, CCFR, CHARM II) + quark density distributions:

$$|V_{cd}|^2 BF_c = (4.63 \pm 0.34)\, 10^{-3}$$

Semileptonic BF of charmed hadrons: (produced in DIS fragmentation)

$$BF_c = (0.0919 \pm 0.0094)$$

$$|V_{cd}| = 0.224 \pm 0.014 \quad (6\,\%)$$

In addition:

$$\kappa |V_{cs}|^2 BF_c = (4.53 \pm 0.37)\, 10^{-2}$$

with

$$\kappa = \frac{\int_0^1 dx\,[x\, s(x) + x\, \bar{s}(x)]}{\int_0^1 dx\,[x\, \bar{u}(x) + x\, \bar{d}(x)]} = 0.453 \pm 0.106\,^{+0.028}_{-0.096} \quad (CCFR)$$

$$|V_{cs}| = 1.04 \pm 0.16 \quad (16\%)$$

Semileptonic D and Leptonic D_s Decays

$D \rightarrow K l \nu$

$D_s \rightarrow \mu \nu$

$D_s \rightarrow \tau \nu$

Averaging the determinations from leptonic and semileptonic decays:

$|V_{cs}| = 0.987 \pm 0.011$

Hadronic or Semileptonic ?

- Semileptonic decays are main approach to measurement of these first 4 CKM elements
  - Measure branching fractions and lifetimes
  - One vertex is leptonic → No CKM element
  - One vertex is hadronic → Only 1 CKM element in decay amplitude
  - Extract CKM element for experimental measurement

- Where do we need theory and why
  - Hadronic part of semileptonic decay amplitudes parameterized via form factors
  - Hadronic vertex in leptonic decays parameterized with decay constants
  - Estimate form factors with lattice QCD