The CKM Matrix and the CP violation in the SM

- Origin of the Cabibbo-Kobayashi-Maskawa Matrix (CKM)
- Overview of the measurements of the CKM elements
- CP violation in the Standard Model
- Overview of the measurements

Barionic asymmetry of the Universe

- Universe is very empty but in a biased way

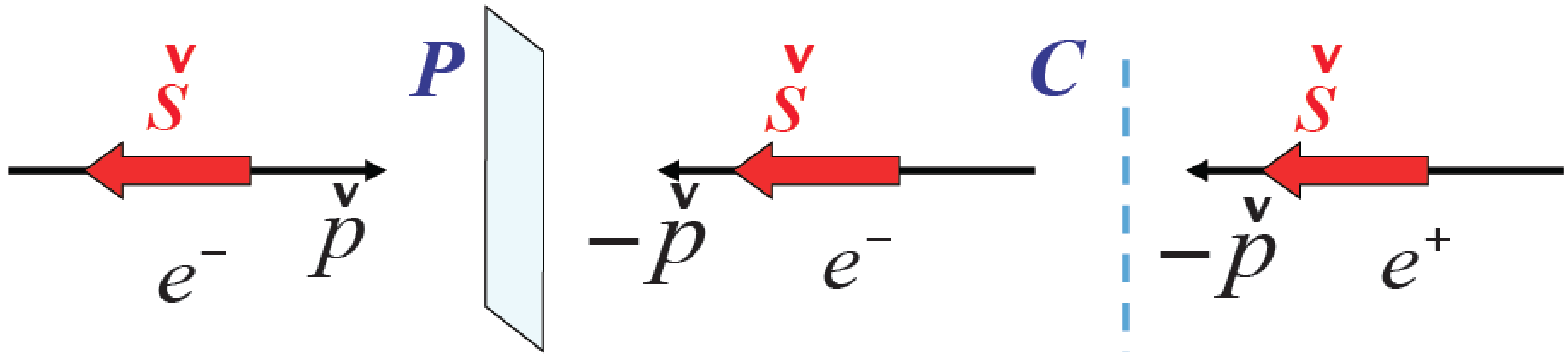
$$\frac{n_{baryon}}{n_{photons}} = 6.14(19) \cdot 10^{-10} \quad \frac{\mathrm{N(anti\text{-}baryon)}}{\mathrm{N(baryon)}} \leq \ 10^{-4}\text{-}10^{-6}$$

  - Absence of anti-nuclei amongst cosmic rays in our galaxy
  - Absence of intense γ–ray emission due to annihilation of distant galaxies in collision with antimatter galaxies

- The early universe believed to have equal amount of matter and anti-matter
  - What happened to the anti-matter?

- CP Violation is one of the three ingredients required to generate such an asymmetry after the Big Bang (A. Sakharov, 1967)
  - Baryon-number violating processes
  - Non-equilibrium state during expansion
  - C and CP Violation

P and C Symmetries and the Fundamental Interactions



- Parity, *P*
  - Parity reflects a system through the origin. Converts right-handed coordinate systems to left-handed ones.
  - Vectors change sign but axial vectors remain unchanged
    - $\boldsymbol{x} \rightarrow -\boldsymbol{x}$, $\boldsymbol{L} \rightarrow \boldsymbol{L}$
- Charge Conjugation, *C*
  - Charge conjugation turns a particle into its anti-particle
    - $e^+ \rightarrow e^-$, $K^- \rightarrow K^+$, $\gamma \rightarrow \gamma$

CP Symmetry, particles and antiparticles

- CP symmetry transforms a particle in its anti-particle

- CP is violated IF particles and anti-particles behave differently!

Weak Interactions and Symmetry Violations

- P and C are good symmetries of the strong and electromagnetic interactions

- Parity violation observed in 1957
  - Asymmetry in β decays of $^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni} + e^{-} + \nu$
  - Electrons produced mostly in one hemisphere

- Charge-conjugation violation 1958
  - Only left-handed neutrinos and right-handed anti-neutrinos

- CP believed to be a good symmetry, but …

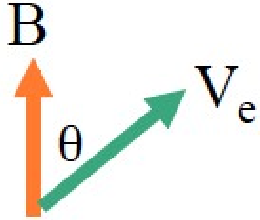
The weak interactions violates the Parity

1956

Observation of a spatial asymmetry in the β-decay electrons from $^{60}Co \rightarrow {}^{60}Ni + e^- + \nu$

- Cold ^{60}Co inside a Solenoidal B Field
- ^{60}Co nuclei spin aligned with B field direction
- ^{60}Co undergoes β decay ....electron emitted
- Measure electron intensity w.r.t B field dir.
- Result:Electrons preferentially emitted in opposite spin direction

$$I(\theta) = 1 - \frac{v_e}{c}\cos\theta$$



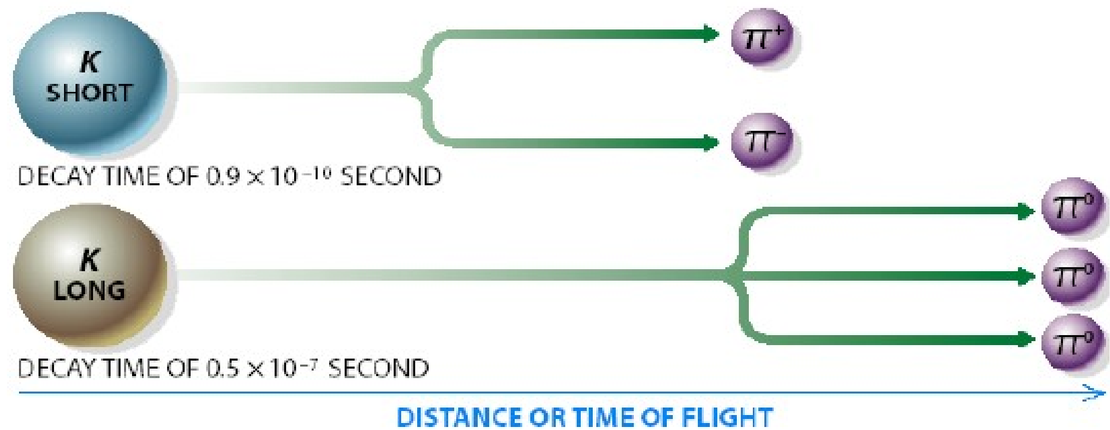
asymmetry of intensity → Weak interaction violated Parity

Kaons CP violation

- CP conservation implies

CP = +1

CP = −1



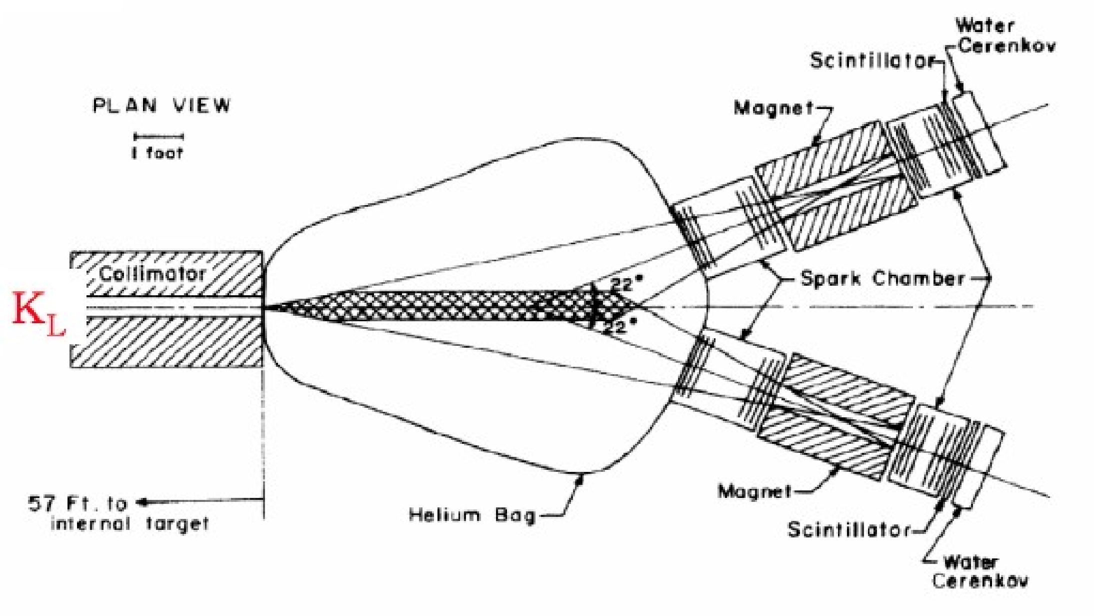
- CP violation in kaons observed in 1964

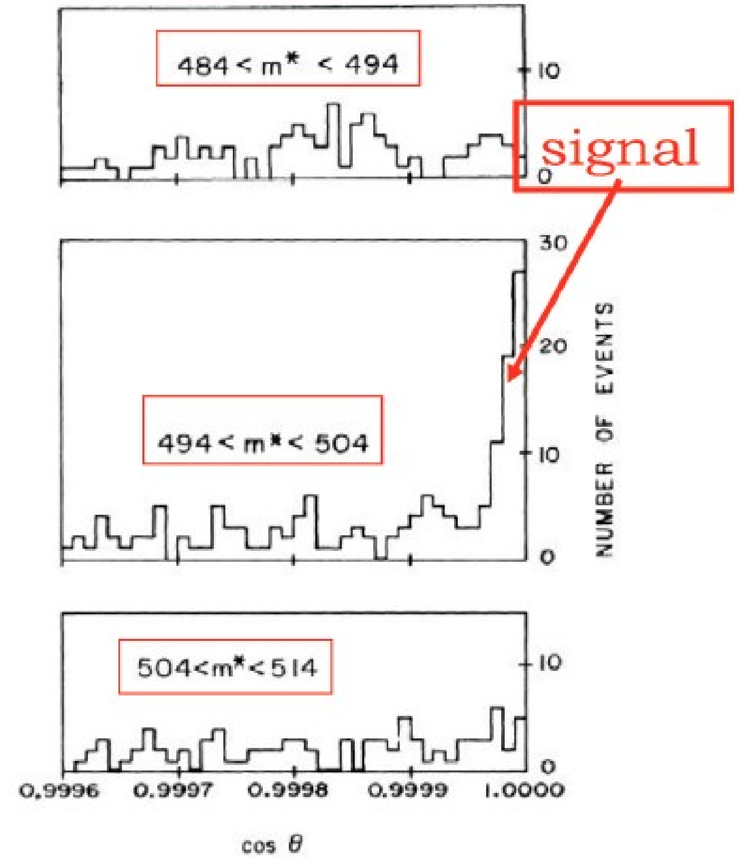
0.2% of the time!

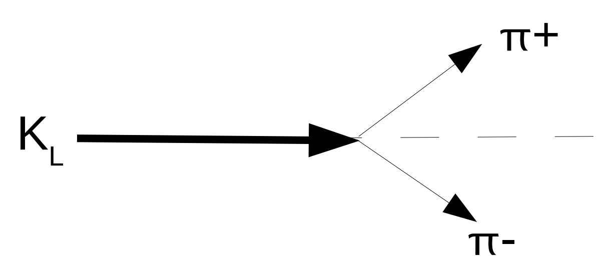
- No theoretical explanation!

Observation of the CP violation of the Kaons

$$\frac{A\left(\left|K_L^0\right\rangle \to 2\pi\right)}{A\left(\left|K_s^0\right\rangle \to 2\pi\right)} = (2.27 \pm 0.02)10^{-3}$$







Complex coupling constant - CP violation

Fermion bilinear	Boson field F	$\mathbf{P}\ F\ \mathbf{P}^\dagger$	$\mathbf{C}\ F\ \mathbf{C}^\dagger$	$\mathbf{CP}\ F\ \mathbf{CP}^\dagger$
$\overline{\psi}\psi$	Scalar $S^+(t,\vec{x})$	$S^+(t,-\vec{x})$	$S^-(t,\vec{x})$	$S^-(t,-\vec{x})$
$\overline{\psi}\gamma^5\psi$	Pseudoscalar $P^+(t,\vec{x})$	$-P^+(t,-\vec{x})$	$P^-(t,\vec{x})$	$-P^-(t,-\vec{x})$
$\overline{\psi}\gamma_\mu\psi$	Vector $V_\mu^+(t,\vec{x})$	$V_\mu^+(t,-\vec{x})$	$-V_\mu^-(t,\vec{x})$	$-V_\mu^-(t,-\vec{x})$
$\overline{\psi}\gamma_\mu\gamma^5\psi$	Axial $A_\mu^+(t,\vec{x})$	$-A_\mu^+(t,-\vec{x})$	$A_\mu^-(t,\vec{x})$	$-A_\mu^-(t,-\vec{x})$

Table 2.1: Properties of charged boson fields and corresponding fermion bilinear terms under **P**, **C**, and **CP**. γ^5 and γ^μ are the Dirac matrices.

Generic interaction lagrangian with vector and axial fields

$$\begin{aligned}\mathcal{L} \;&=\; a\, V_\mu^+(t,\vec{x})V^{\mu-}(t,\vec{x}) + b\, A_\mu^+(t,\vec{x})A^{\mu-}(t,\vec{x}) + \\ &\quad c\, V_\mu^+(t,\vec{x})A^{\mu-}(t,\vec{x}) + c^*\, A_\mu^+(t,\vec{x})V^{\mu-}(t,\vec{x})\end{aligned}$$

a, b: real constants
c: complex constant

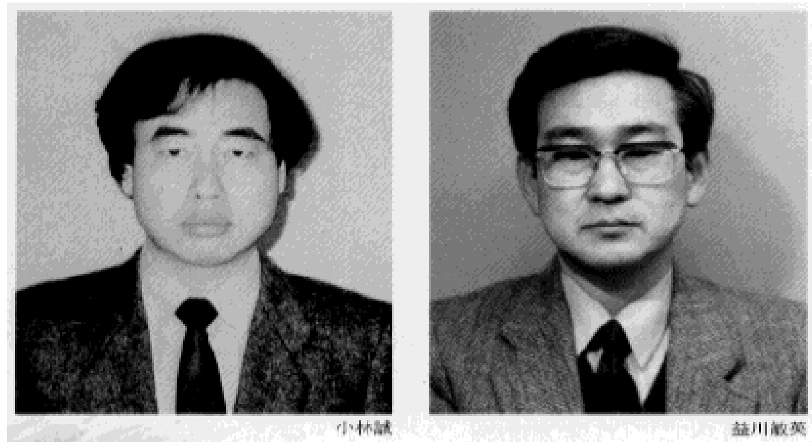
Lagrangian after CP transformation

$$\begin{aligned}\mathbf{CP}\mathcal{L}\mathbf{CP}^\dagger \;&=\; a\, V_\mu^-(t,-\vec{x})V^{\mu+}(t,-\vec{x}) + b\, A_\mu^-(t,-\vec{x})A^{\mu+}(t,-\vec{x}) + \\ &\quad c\, V_\mu^-(t,-\vec{x})A^{\mu+}(t,-\vec{x}) + c^*\, A_\mu^-(t,-\vec{x})V^{\mu+}(t,-\vec{x})\ .\end{aligned}$$

Lagrangian invariant under CP IF AND ONLY IF c=c*! c must be real

KM mechanism for the CP violation

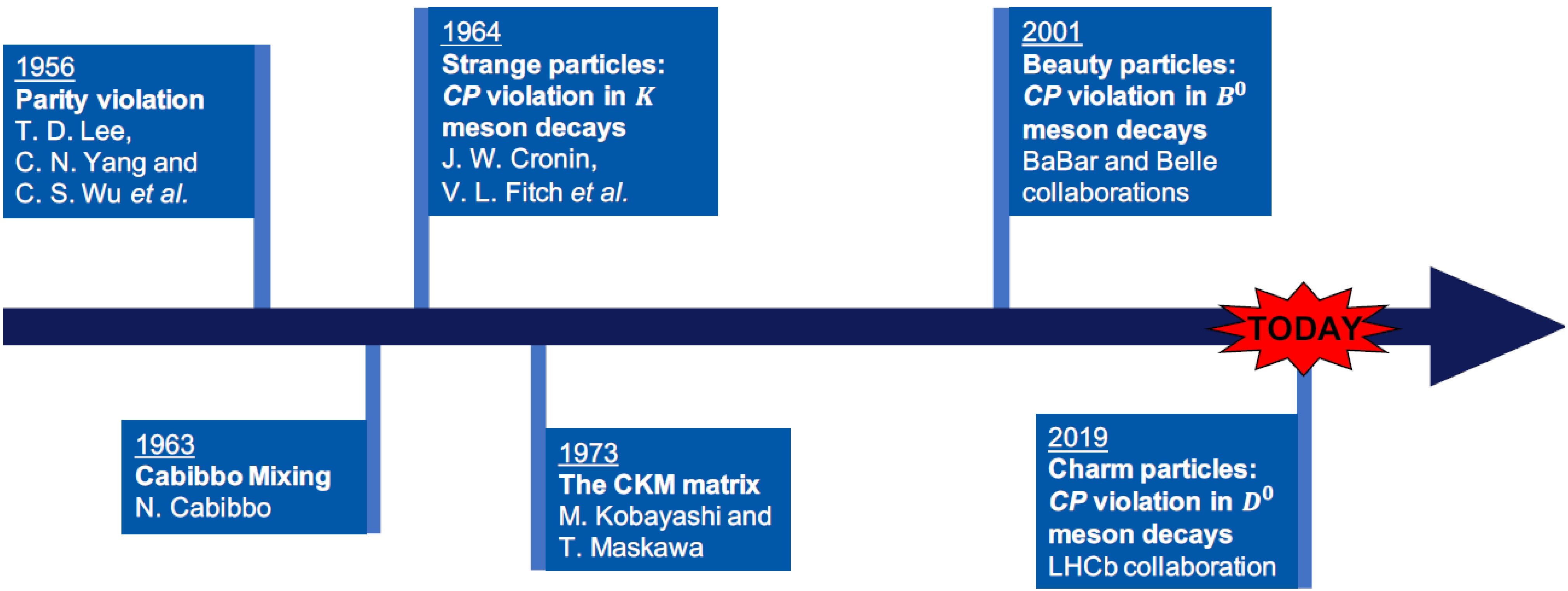
1972



Two Young Postdocs at that time !

- Proposed a daring explanation for CP violation in K decay:
- CP violation appears only in the charged current weak interaction of quarks
- There is a single source of CP Violation ⇒ Complex Quantum Mechanical Phase δ_{KM} in inter-quark coupling matrix
- Need at least **3 Generation of Quarks** (then not known) to facilitate this
- **CP is NOT an approximate symmetry**, $\delta_{KM} \cong 1$, it is MAXIMALLY violated !

CP violation history



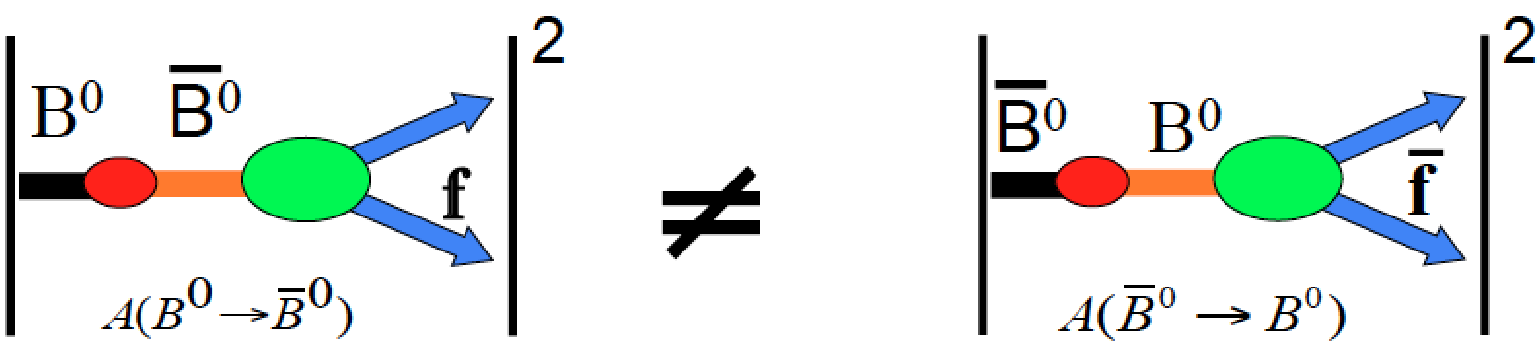
CP Violation

- *CP* violation is observed only in the weak interactions. Mainly (but not only) in the decay of the *K, B, B_s* and *D* (2019!) neutral mesons.
- There are three manners of *CP* violation
- **Violation in the wave function (violation in the mixing)**.
  It happens when the wave functions of H_{free} are not eigenstates of *CP*. It was observed only in the neutral kaons. The effect is small but important. Experiment of Christenson *et al*. The state with short lifetime is not exactly $K^0{}_1$ (eigenstate of *CP* with eigenvalue +1), but it containes a small component of $K^0{}_2$ (eigenstate of *CP* with eigenvalue -1) and viceversa.
- **Violation in decays (direct CP violation)**.
  M is a meson and f is a final state of an its possible decay. $\overline{M}$ is its antimeson and $\overline{f}$ the coniugate state of f. If *CP* is conserved the two decay amplitude are equal: $A(M \rightarrow f) = A(\overline{M} \rightarrow \overline{f})$
  The equality is true also for the absolute values, namely for the decay probability and for the phases.
  The phase is observable from the interference between different amplitudes which contribute to the matrix element. In principle this violation can appear in the decays of both charged and neutral mesons. Up to now it was seen in the following systems: K^0, D^0, $B^0{}_s$, B^0 e $B^{\pm}$.
- **Violation in the interference between mixing and decay**
  Interference between a decay without mixing, $M^0 \rightarrow f$, and a decay with mixing, $M^0 \rightarrow \overline{M}^0 \rightarrow f$
- This effect happens only in the decays into common final states to M^0 and $\overline{M}^0$, including all the *CP* eigenstates. Up to now observed only in the B^0 system.

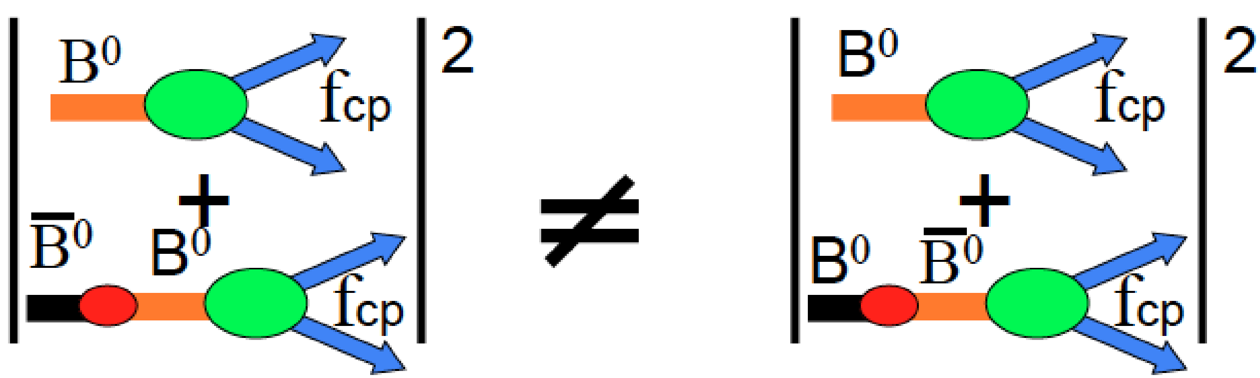
CP Violation

CP Violation in Decay a.k.a. Direct CPV

CP Violation in Mixing



CP Violation in interference between Mixing and Decay

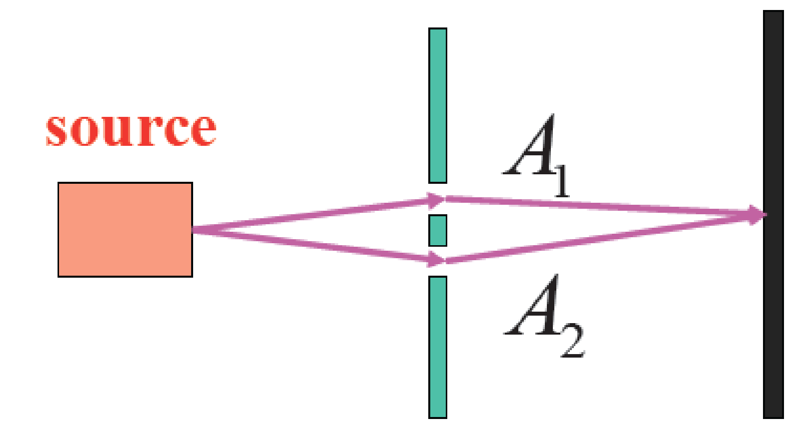


Direct *CP* Violation: decays

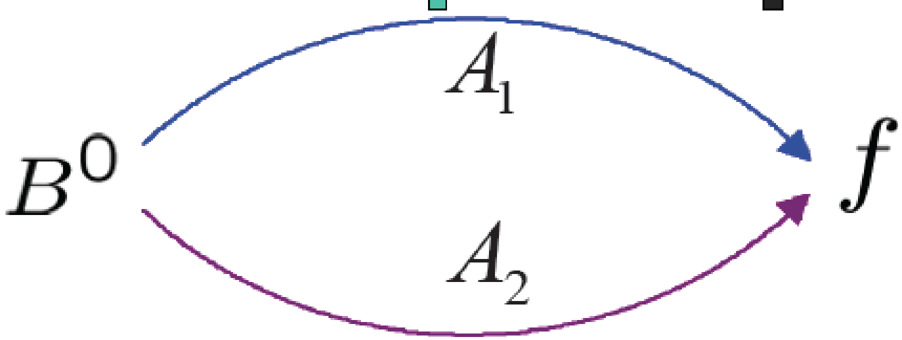
- CP violation can be observed by comparing decay rates of particles and antiparticles

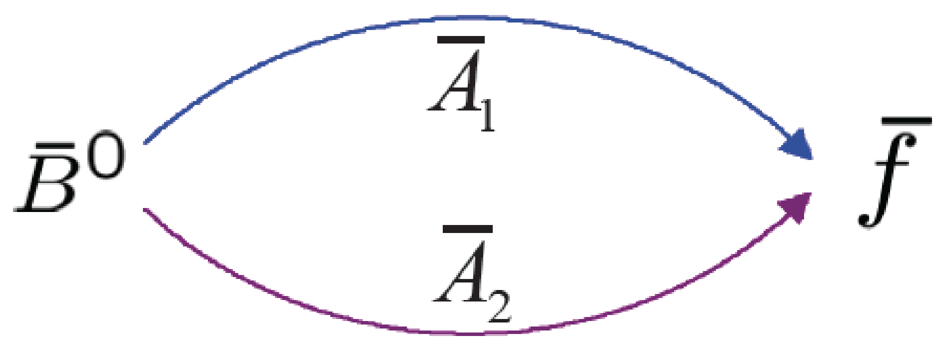
$$\Gamma(a \to f) \neq \Gamma(\bar{a} \to \bar{f}) \Rightarrow \text{CP Violation}$$

- The difference in decay rates arises from a different interference term for the matter vs. antimatter process. Analogy to double-slit experiment:



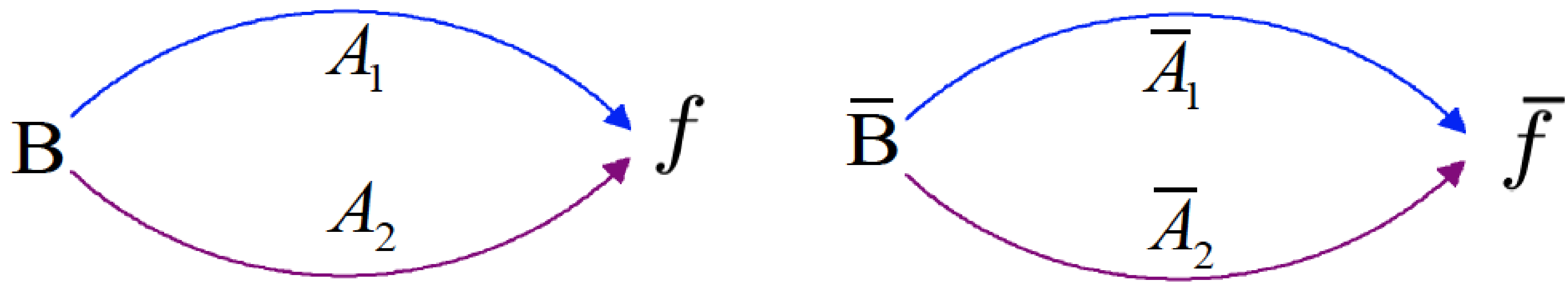
Classical double-slit experiment:
Relative phase variation due to different path lengths: interference pattern in space





CP Violation of the B mesons

Identify B final states which are arrived at by two paths



In B^0 system, $B^0 \rightarrow \bar{B}^0$ oscillation provides one path with

the other path(s) come from weak decay of B hadron

In $B^{\pm}$system $\Rightarrow$ no oscillation possible,

2 (or more) amplitudes must come from different weak decay of B

B Meson is heavy $\Rightarrow$ many final states, multiple "paths."

2 classes of B decays come into play: "Tree" $\Rightarrow$ spectator decay like

"Penguin" $\Rightarrow$ FCNC loop diagrams with u,c,t

CP Violation is a quantum phenomenon

- CPV is due to Quantum interference between two or more amplitudes

- Phase of QM amplitudes is the key

- Need to consider two types of phases
  - *CP-conserving phases*: <u>don't change sign under CP</u>
    - Sometimes called *strong phases* since they can arise from strong, final-state interactions
  - *CP-violating phases*: <u>these do change sign under CP transformation</u>
    - originate in the Weak interaction sector

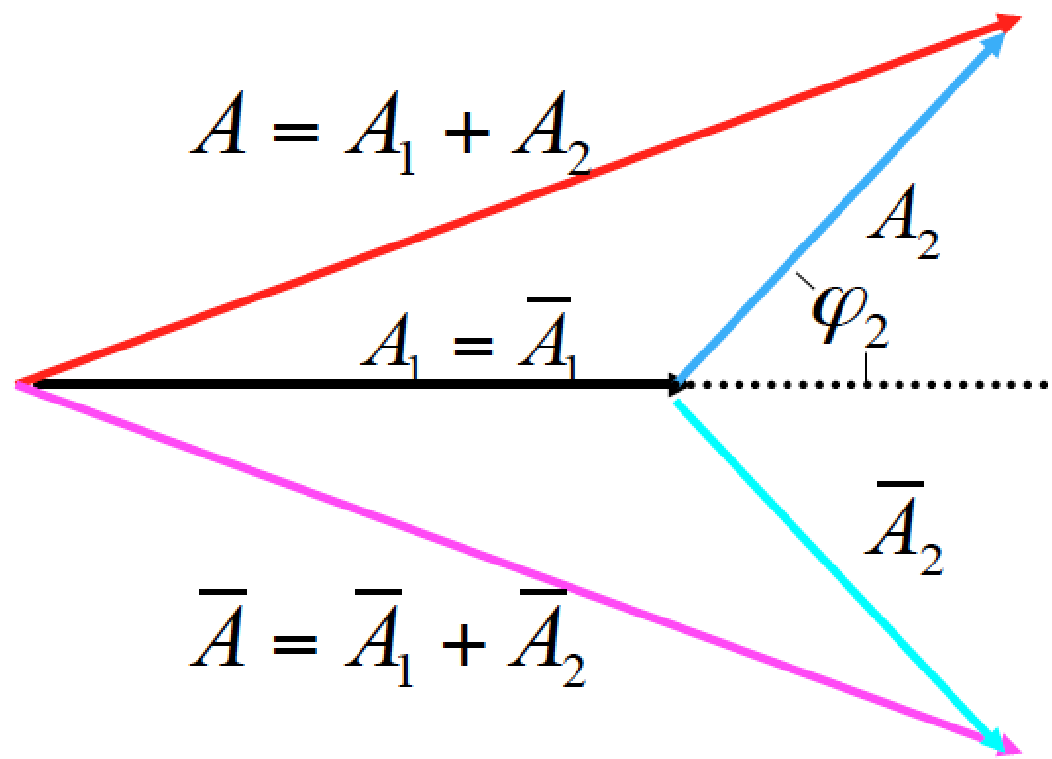
$$A = A\boxed{e^{i\varphi}}e^{i\delta}$$

$$\overline{A} = A\boxed{e^{-i\varphi}}e^{i\delta}$$

How can the CP asymmetries rise ?

- Suppose a decay can occur through two different processes, with amplitudes A_1 and A_2
- First, consider the case in which there is a (relative) CP-violating phase between A_1 and A_2 only

$$A = A_1 + a_2 e^{i\varphi_2}$$
$$\overline{A} = A_1 + a_2 e^{-i\varphi_2}$$



- Different decay rate for particle and anti-particle
  - Since new term added
- But no direct CP asymmetry

$$A = \bar{A}$$

How can the CP asymmetries rise ?

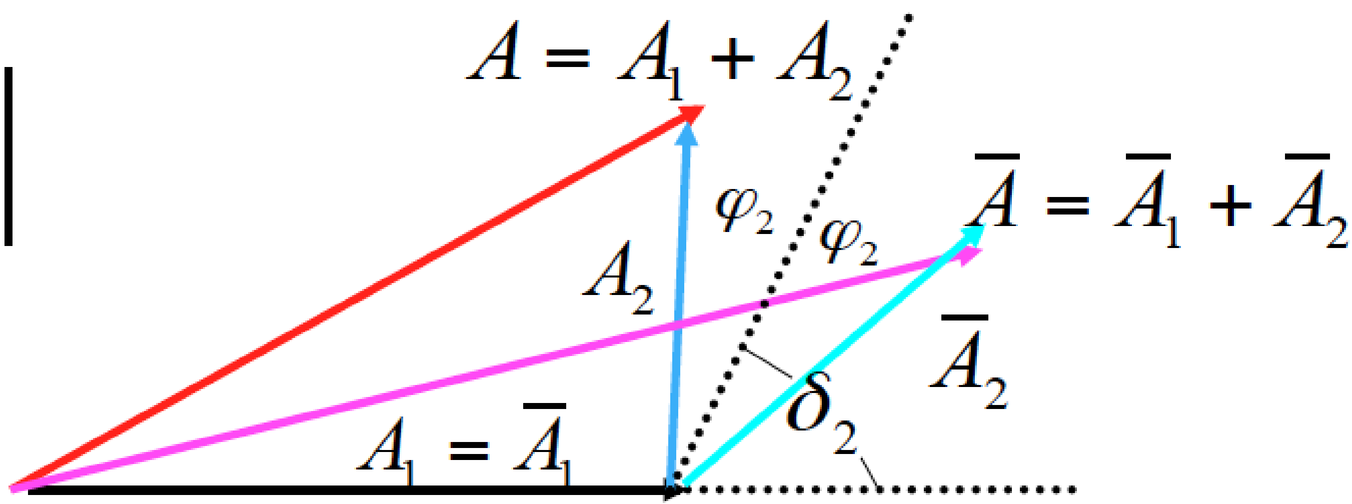
- Next, introduce a relative *CP-conserving* phase in addition to the relative *CP-violating* phase

$$A = A_1 + a_2 e^{i(\varphi_2 + \delta_2)}$$

$$\bar{A} = A_1 + a_2 e^{i(-\varphi_2 + \delta_2)}$$

- Now have a Direct CP Violation

$$|A| \neq |\bar{A}|$$



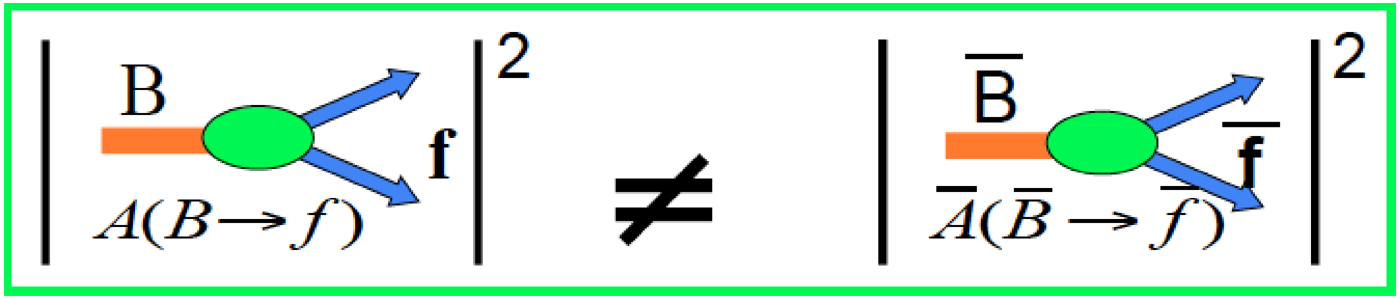
How can the CP asymmetries rise ?

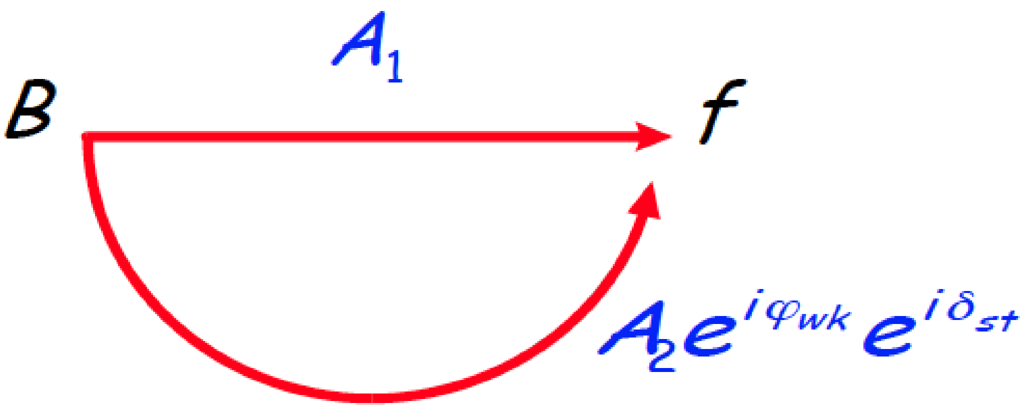
$$Asymmetry = \frac{|\bar{A}|^2 - |A|^2}{|\bar{A}|^2 + |A|^2} = \frac{2|A_1||A_2|\sin(\delta_1 - \delta_2)\sin(\phi_1 - \phi_2)}{|A_1|^2 + |A_2|^2 + |A_1||A_2|\cos(\delta_1 - \delta_2)\cos(\phi_1 - \phi_2)}$$

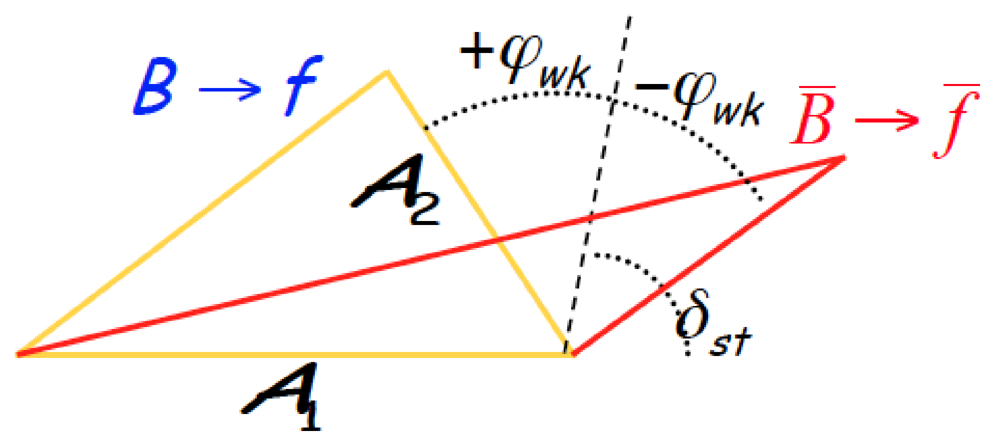
To extract the CP-violating phase from an observed CP asymmetry, we need to know the value of the CP-conserving phase difference

***B* system: extraordinary laboratory for quantum interference experiments: many final states, multiple "paths"→ Lots of channels for CP Violation**

Direct *CP* Violation: decays



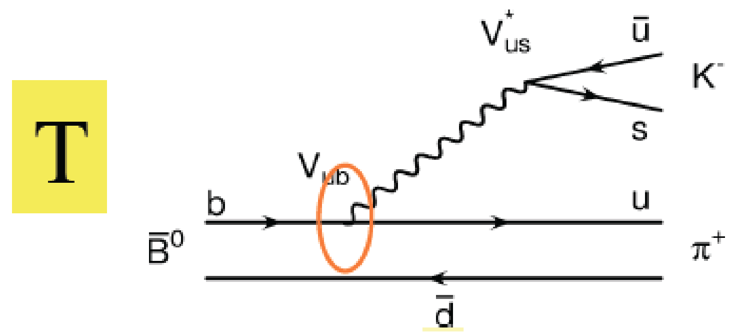


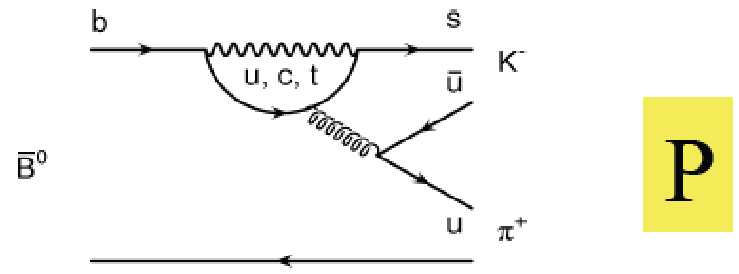


$$\Gamma(B \to f) = \left| A_1 + A_2 e^{i\varphi_{wk}} e^{i\delta_{st}} \right|^2 , \quad \Gamma(\bar{B} \to \bar{f}) = \left| A_1 + A_2 e^{-i\varphi_{wk}} e^{i\delta_{st}} \right|^2$$

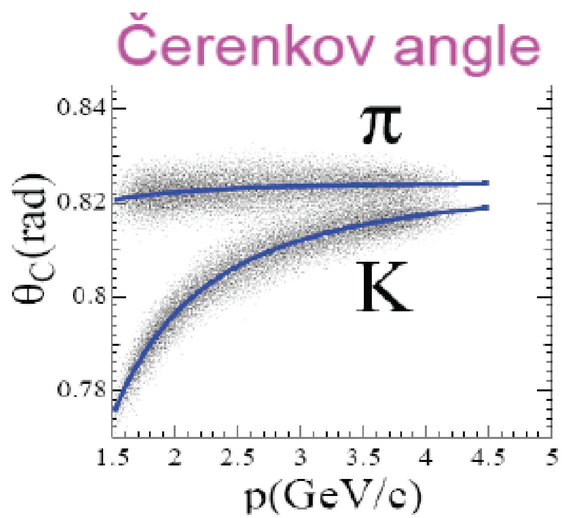
$$A_{CP} = \frac{Br\left(\bar{B} \to \bar{f}\right) - Br\left(B \to f\right)}{Br\left(\bar{B} \to \bar{f}\right) + Br\left(B \to f\right)} \equiv \frac{\left|\bar{A}_{\bar{f}}\right|^2 - \left|A_f\right|^2}{\left|\bar{A}_{\bar{f}}\right|^2 + \left|A_f\right|^2} \neq 0 \to \text{Direct } CPV$$

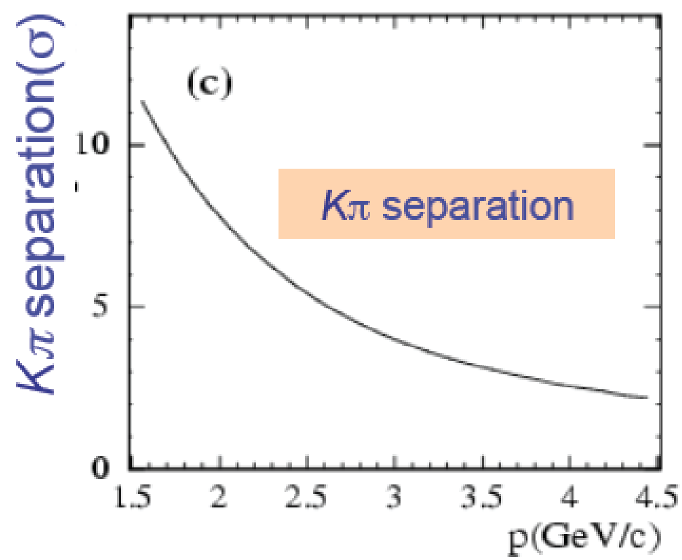
$B^0 \rightarrow K^-\pi^+$: *CP* direct violation

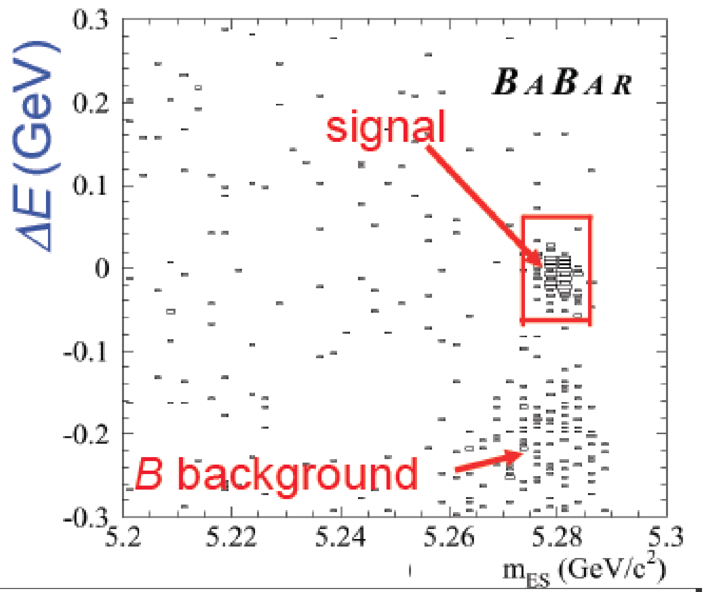




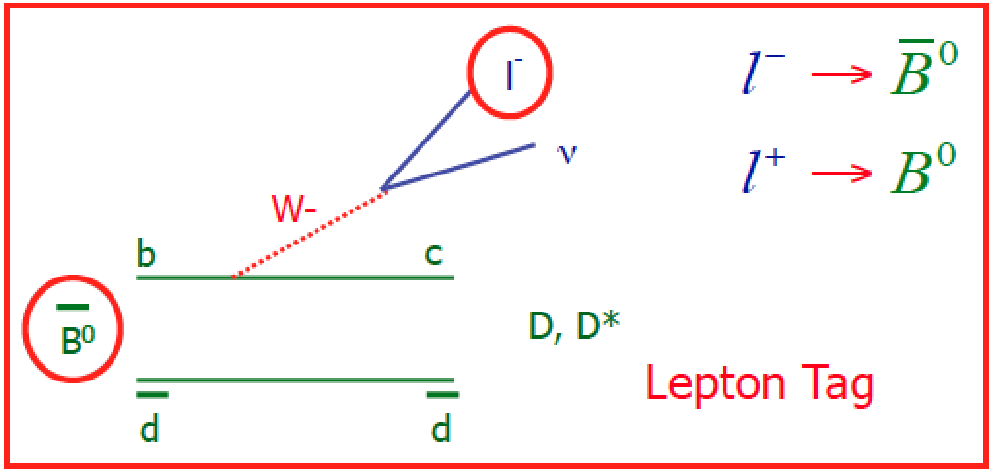
- Loop diagrams from New Physics (e.g. SUSY) can modify SM asymmetry via P
- Clean mode with "large" rate : $B^0 \rightarrow K^- \pi^+$
- Measure *charge* asymmetry, reject large $B \rightarrow \pi\pi$ background with Particle ID

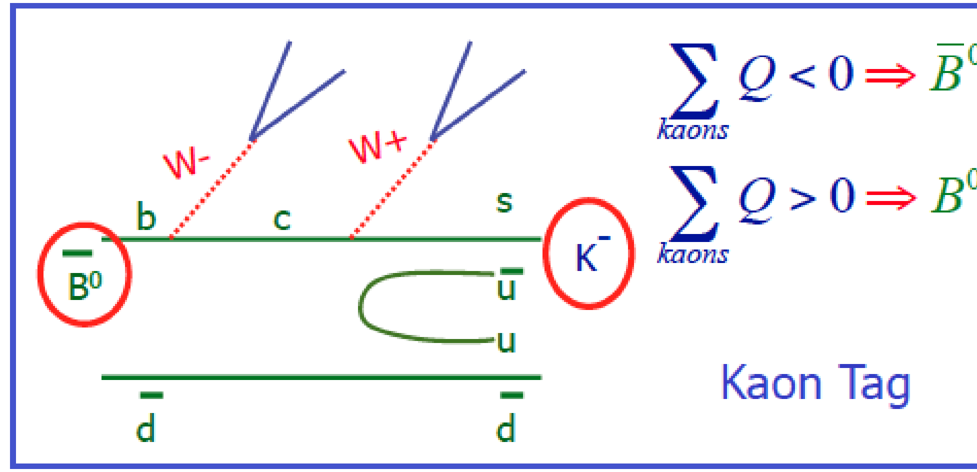


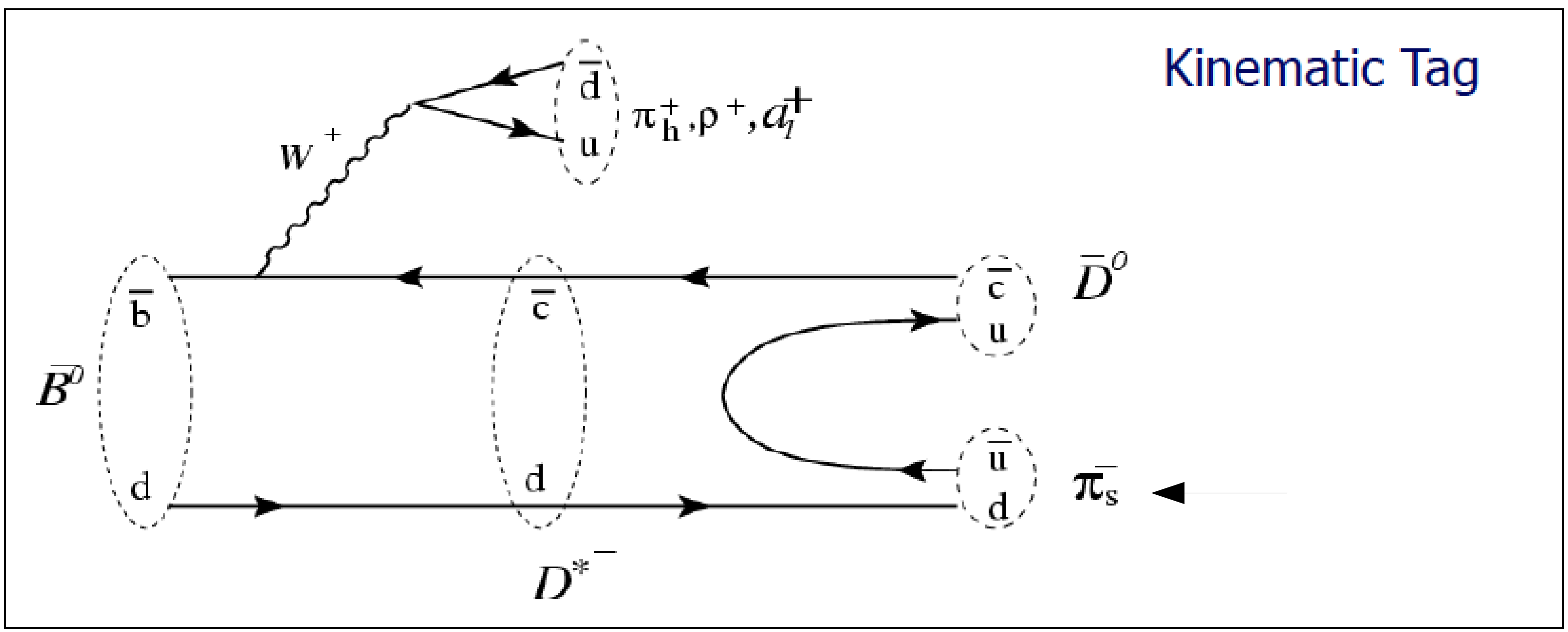




B^0 from $\overline{B}^0$ mesons separation







$B^0 \to K^- \pi^+$: *CP* direct violation

$$A_{K^-\pi^+} \equiv \frac{\Gamma(\bar{B}^0 \to K^-\pi^+) - \Gamma(B^0 \to K^+\pi^-)}{\Gamma(\bar{B}^0 \to K^-\pi^+) + \Gamma(B^0 \to K^+\pi^-)}$$

BaBar Collaboration

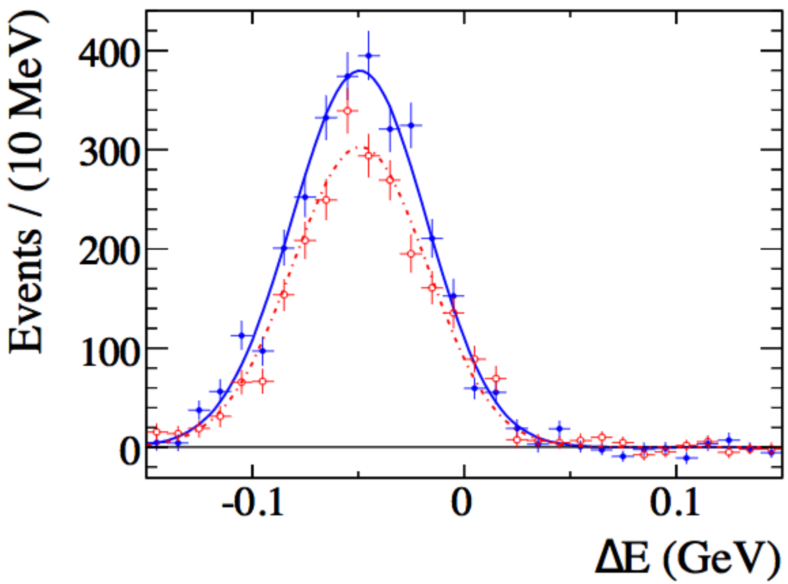


FIG. 3: ${}_s\mathcal{P}lots$ of the ΔE distribution for signal $K^\pm\pi^\mp$ events, comparing (blue solid lines, filled circles) B^0 and (red dashed lines, empty circles) $\overline{B}^0$ decays. The points with error bars show the data, and the lines represent the PDFs used in the fits and reflect the results of the fits.

Full data set
467 ± 5 millions of $B\bar{B}$ pairs

$$n_{K\pi} = 5410 \pm 90$$

$$\boxed{A_{K^-\pi^+} = -0.107 \pm 0.016^{+0.006}_{-0.004}}$$

$B^0 \rightarrow K^-\pi^+$: *CP* direct violation

Belle Collaboration

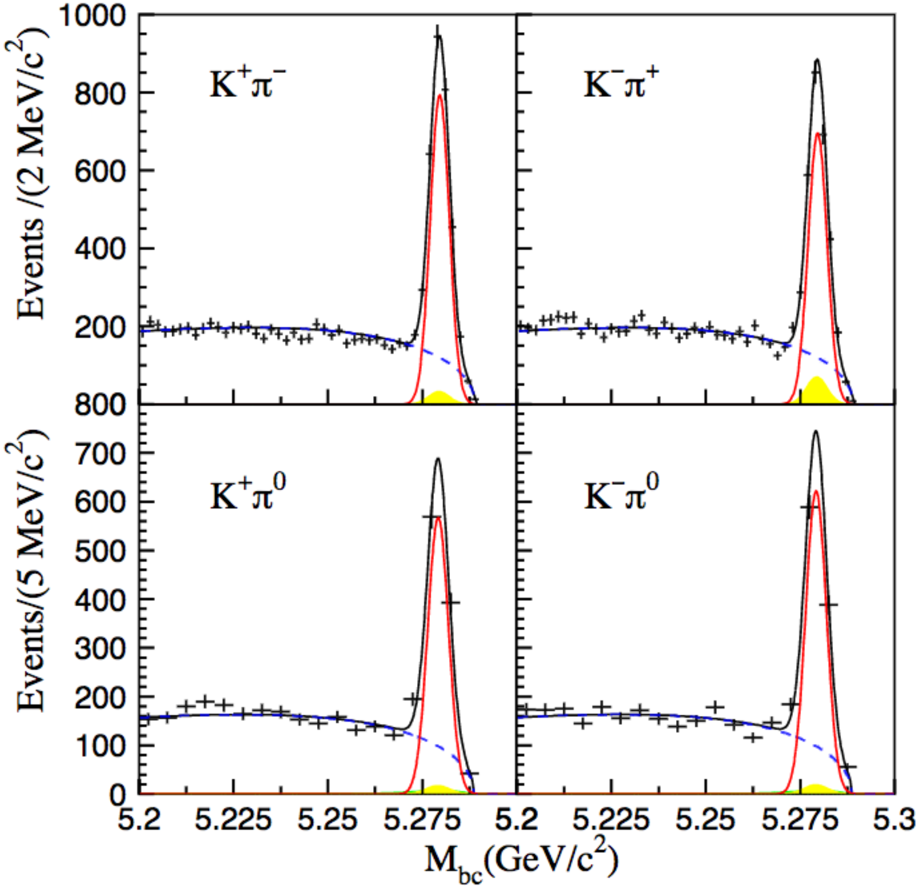


FIG. 3: The $M_{\rm bc}$ distributions for $B^0/\overline{B}^0 \to K^\pm\pi^\mp$ (top) and $B^\pm \to K^\pm\pi^0$ (bottom). The selections for fit projections and PDF component descriptions are identical to those described in Fig. 1.

Full data set
772 millions of $B\overline{B}$ pairs

$$n_{K\pi} = 7525 \pm 127$$

$$\boxed{A_{K^-\pi^+} = -0.069 \pm 0.014 \pm 0.007}$$

Non- perturbative QCD uncertainties large Standard Model CP Violation not precisely predictable

→insufficient to prove or rule out contribution from New Physics

Oscillation and *CP* Violation in the B^0 system

- Now we discuss two phenomena of the B^0 system.
- ***B* oscillations** and ***CP* violation *in the interference between decays with and without oscillation (between mixing and decay)***
- Both phenomena discovered in the "beauty factories", the first by the experiment ARGUS (DESY DORIS II), the second by the experiments BELLE and BABAR.
-

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{+i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12} s_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{+i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{+i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{+i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{+i\delta_{13}} & c_{23} c_{13} \end{pmatrix}$$

- 5 terms with phase factors
- 4 real terms
- phase factor always multiplied by s_{13} → the smallest angle → **effects of CP violation small**
- in 3 elements at least another sin multiplies the phase factor → terms almost real
- **with good approximation only V_{td} and V_{ub} are complex**

Oscillation and *CP* Violation in the B^0 system

- Measurement of the β phase of V_{td} defined as:

$$V_{td} \equiv |V_{td}| e^{i\beta}$$

- The precise definition of β is:

$$\beta \equiv arg\left(-\frac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}}\right)$$

where all factors are real or almost real except V_{td}

- The neutral *B* system behaves similarly as the neutral *K* system.

$$|B_L> \; = \; p|B^0> \; +q \; |\bar{B}^0>$$
$$|B_H> \; = \; p|B^0> \; -q \; |\bar{B}^0>$$

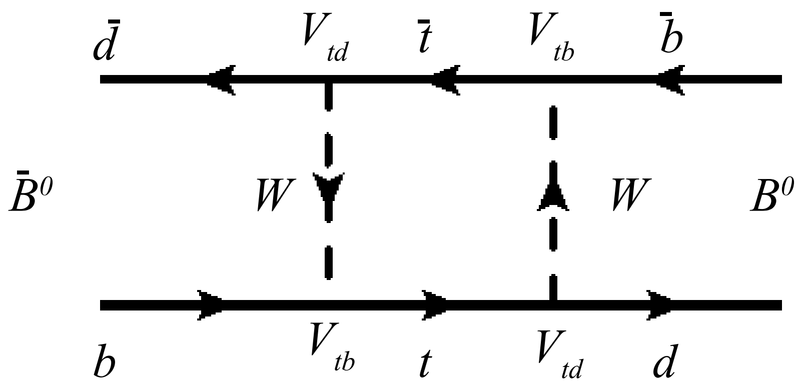
- But **two important differences**:
  - **The lifetime of the two *Bs* are equal within the uncertainties**: 1.530 ± 0.009 ps.
    Reasons: both *Q* values of the decays are large.
    Indicating the two eigenstates as B_H and B_L depending on their masses.
    Mass difference: $\Delta m_B = m_H - m_L \; > \; 0$

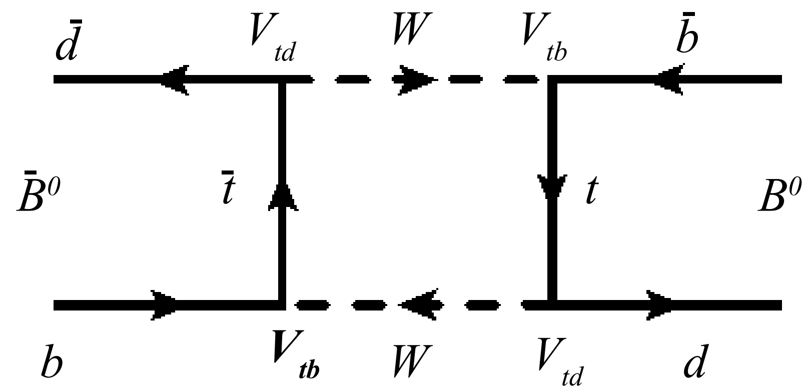
    Width: $\Gamma_B = \Gamma_{B_H} = \Gamma_{B_L} \simeq 0.43 \;\; \text{meV}$

Oscillation and *CP* Violation in the B^0 system

- **Suppression of the channels common to the decays of B^0 and $\overline{B}^0$ due to the smallness of the mixing elements.** Consequence $|p/q| \approx 1$ → CP violation in the mixing is small (first manner of CP violation)

  Probability amplitude for the transition between B^0 and $\overline{B}^0$ at the lowest level given by the following box diagrams:





  also the diagrams with u and c quarks replacing one or two t quarks should be taken into account. But the contribution of the internal lines is proportional to the squared quark masses. Contributions of u and c quarks are negligible.
  From the box diagrams one can calculate the mass difference, in particular:

$$|V_{td}|^2 |V_{tb}|^2 \propto \Delta m_B$$

Oscillation and *CP* Violation in the B^0 system

- From the expressions:

$$|B_L> \;=\; p|B^0> \;+q\;\;|\bar{B}^0>$$
$$|B_H> \;=\; p|B^0> \;-q\;\;|\bar{B}^0>$$

putting:

$$m \;\equiv\; \frac{(m_H + m_L)}{2}$$

the time evolution of the eigenstates *L* and *H* is:

$$|B_L(t)> \;=\; e^{-\frac{\Gamma_B}{2}t} e^{-imt} e^{+i\frac{\Delta m_B}{2}t} |B_L(0)>$$
$$|B_H(t)> \;=\; e^{-\frac{\Gamma_B}{2}t} e^{-imt} e^{-i\frac{\Delta m_B}{2}t} |B_H(0)>$$

with *t* being the proper time.

Oscillation and *CP* Violation in the B^0 system

- Supposing to start at the time $t = 0$ with a beam of pure B^0 and with another one of pure $\bar{B}^0$. Labeling them as $\Psi_0(t)$ and as $\Psi_{\bar{0}}(t)$:

$$\Psi_0(t) = h_+(t)B^0 + \frac{q}{p}h_-(t)\bar{B}^0$$

$$\Psi_{\bar{0}}(t) = \frac{p}{q}h_-(t)B^0 + h_+(t)\bar{B}^0$$

where:

$$h_+(t) = e^{-\frac{\Gamma_B}{2}t} e^{-imt} \cos\left(\frac{\Delta m_B t}{2}\right)$$

$$h_-(t) = i\, e^{-\frac{\Gamma_B}{2}t} e^{-imt} \sin\left(\frac{\Delta m_B t}{2}\right)$$

Oscillation and *CP* Violation in the B^0 system

- If at the time $t = 0$ one has a pure beam of B^0, the probability to find a B^0 at the general time t is:

$$\left|< B^0 | \Psi_0(t) >\right|^2 = \left|h_+(t)\right|^2 = e^{-\Gamma_B t}\cos^2\left(\frac{\Delta m_B}{2}t\right) = \frac{1}{2}e^{-\Gamma_B t}\left(1+\cos\Delta m_B t\right)$$

and the probability to find a $\bar{B}^0$ at the general time t is:

$$\left|< \bar{B}^0 | \Psi_0(t) >\right|^2 = \left|h_-(t)\right|^2 = \left|\frac{p}{q}\right|^2 e^{-\Gamma_B t}\sin^2\left(\frac{\Delta m_B}{2}t\right)$$

$$= e^{-\Gamma_B t}\sin^2\left(\frac{\Delta m_B}{2}t\right) = \frac{1}{2}e^{-\Gamma_B t}\left(1-\cos\Delta m_B t\right)$$

in the approximation $|p/q| = 1$

- Similar expression starting from a pure beam of $\bar{B}^0$

Oscillation and *CP* Violation in the B^0 system

- The difference between the probability to observe decays with opposite flavour and equal flavor , normalized to their sum, the so called **flavour asymmetry,** is:

$$\frac{|h_+(t)|^2 - |h_-(t)|^2}{|h_+(t)|^2 + |h_-(t)|^2} = \frac{P_{OF} - P_{SF}}{P_{OF} + P_{SF}} = \frac{|<B^0|\Psi_0(t)>|^2 - |<\bar{B}^0|\Psi_0(t)>|^2}{|<B^0|\Psi_0(t)>|^2 + |<\bar{B}^0|\Psi_0(t)>|^2} = \cos(\Delta m_B t)$$

This quantity determines Δm_B

Oscillation and *CP* Violation in the B^0 system

- To measure the phase of *p/q* it is necessary to have a second phase of reference (only phase differences have a physical meaning).
  One considers the *CP* eigenstate f of eigenvalue η_f in which both B^0 and $\bar{B}^0$ can decay. With A_f one indicates the amplitude for $B^0 \rightarrow f$ and with $\bar{A}_f$ the decay for $\bar{B}^0 \rightarrow f$
- If $A_f \neq \bar{A}_f$ *CP* is violated.
- If $|A_f| \neq |\bar{A}_f|$ *CP* is violated and is seen as a difference in the two decay rates.
- We discuss now the case in which $|A_f| = |\bar{A}_f|$ but the C*P* violation comes from a phase difference between the two amplitudes.
- The observable is (that is the relative phase between the complex numbers *p/q* and $A_f/\bar{A}_f$):

$$\boxed{\lambda_f \equiv \eta_f \frac{p}{q} \frac{A_f}{\bar{A}_f}} \qquad |\lambda_f| = 1$$

Oscillation and *CP* Violation in the B^0 system

- The amplitudes for the decay into the final state f are:

$$
\begin{aligned}
< f|\Psi_0(t)> &= A_f h_+(t) + \frac{q}{p}\bar{A}_f h_-(t) \\
&= A_f e^{-imt} e^{\frac{-\Gamma_B}{2}t} \cos\left(\frac{\Delta m_B}{2}t\right) + \frac{q}{p}\bar{A}_f i\, e^{-imt} e^{\frac{-\Gamma_B}{2}t} \sin\left(\frac{\Delta m_B}{2}t\right) \\
&= \frac{A_f}{\lambda_f} e^{-imt} e^{\frac{-\Gamma_B}{2}t}\left[\lambda_f \cos\left(\frac{\Delta m_B}{2}t\right) + i\sin\left(\frac{\Delta m_B}{2}t\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
< f|\Psi_{\bar{0}}(t)> &= \frac{p}{q} A_f h_-(t) + \bar{A}_f h_+(t) \\
&= \frac{p}{q} A_f e^{-imt} e^{\frac{-\Gamma_B}{2}t}\, i\sin\left(\frac{\Delta m_B}{2}t\right) + \bar{A}_f e^{-imt} e^{\frac{-\Gamma_B}{2}t} \cos\left(\frac{\Delta m_B}{2}t\right) \\
&= \bar{A}_f e^{-imt} e^{\frac{-\Gamma_B}{2}t}\left[i\lambda_f \sin\left(\frac{\Delta m_B}{2}t\right) + \cos\left(\frac{\Delta m_B}{2}t\right)\right]
\end{aligned}
$$

Oscillation and *CP* Violation in the B^0 system

- The observable violating *CP* is the ratio between the difference and the sum of the two probabilties. Remembering that:

$$|A_f| = |\bar{A}_f|$$

$$|\lambda_f| = 1$$

$$|<f|\Psi_0(t)>|^2 + |<f|\Psi_{\bar{0}}(t)>|^2 = 2|A_f|^2 e^{-\Gamma_B t}$$

$$|<f|\Psi_0(t)>|^2 - |<f|\Psi_{\bar{0}}(t)>|^2 = 2|A_f|^2 e^{-\Gamma_B t}\eta_f Im(\lambda_f)\sin(\Delta m_B t)$$

- and then:

$$\boxed{a_{fCP} = \frac{|<f|\Psi_0(t)>|^2 - |<f|\Psi_{\bar{0}}(t)>|^2}{|<f|\Psi_0(t)>|^2 + |<f|\Psi_{\bar{0}}(t)>|^2} = \eta_f Im(\lambda_f)\sin(\Delta m_B t)}$$

There is *CP* violation if $Im(\lambda_f) \neq 0$

Oscillation and *CP* Violation in the B^0 system

- **Measurement of Δm_B and of a_{fCP}**
- The beauty factories work at the resonance $\Upsilon(4^1S_3)$, only 20 MeV above the threshold $m_{B0} + m_{\overline{B0}}$
- The *B* mesons move slowly in the center of mass energy reference system, it is not possible to solve the secondary vertices in this reference system.
- asymmetric beauty factories:

  PEP2 $p(e-) = 9$ GeV and $p(e+) = 3.1$ GeV → $\langle\beta\gamma\rangle = 0.56$

  KEK $p(e-) = 8$ GeV and $p(e+) = 3.5$ GeV → $\langle\beta\gamma\rangle = 0.425$
- Mean distance between production vertex and decay vertex: $\Delta z \approx 200$ μm
- Measured by silicon vertex detectors with reconstruction accuracy of the vertex: 80-120 μm.

  About one and half flight length in a mean lifetime.
- The proper time of the preceding formula is the distance measured in the laboratory divided by $c\langle\beta\gamma\rangle$

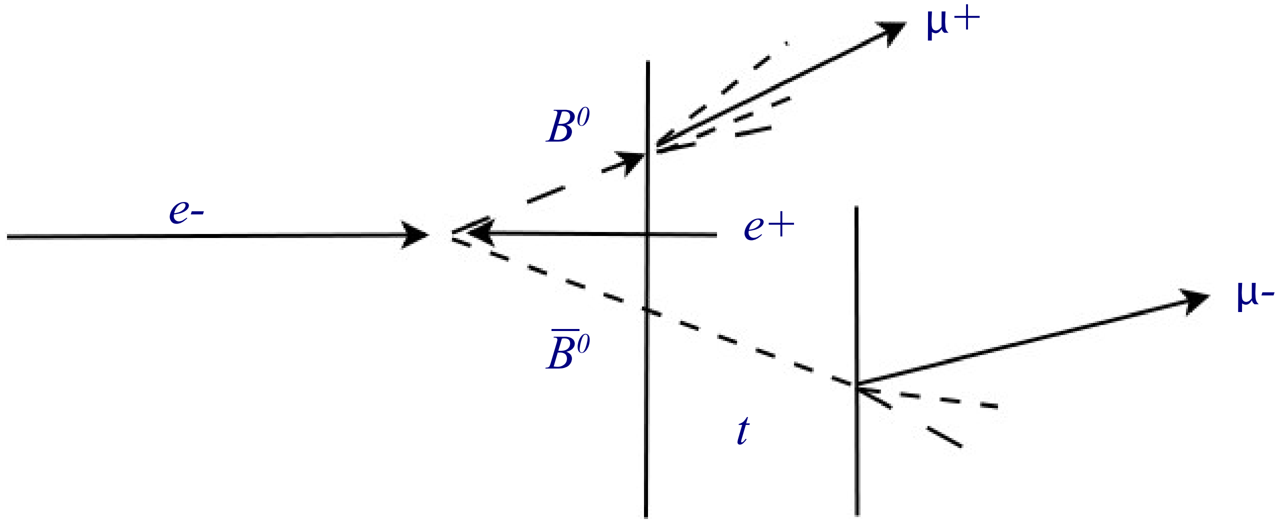
Oscillation and *CP* Violation in the B^0 system

- In an e^+e^- annihilation a B^0 and a $\overline{B}^0$ are produced, but one doesn't know which is one or the other.
- The time evolution of the two mesons is described by a single wave function. The phase difference between the two particles doesn't change with the time.
- However one of the two B can be identified as particle or antiparticle when it decays in a semileptonic mode:

$$
\begin{array}{llll}
B^0 = \bar{b}\,d & \bar{b} \to \bar{c}\, l^+ \nu_l & \Rightarrow & B^0 \to D^- l^+ \nu_l \\
\overline{B}^0 = b\,\bar{d} & b \to c\, l^- \bar{\nu}_l & \Rightarrow & \overline{B}^0 \to D^+ l^- \bar{\nu}_l
\end{array}
$$

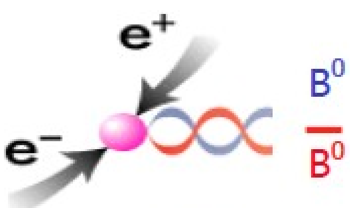
- So reconstructing the sign of the lepton or reconstructing the D meson one can identify (*tagging*) the neutral B as B^0 or $\overline{B}^0$.
- One estimates the time of this decay respect to the time of production measuring the distance between the production and decay vertices and the velocity of the particle from the momenta of its sons.
- The $t = 0$ is the decay time of the tagging
- If the tagged B is a $\overline{B}^0$, its partner is a B^0 with wave function evolving as $\Psi_0(t)$ and viceversa.
- Its evolution is $\Psi_0(t)$ even if $t < 0$.

Oscillation and *CP* Violation in the B^0 system



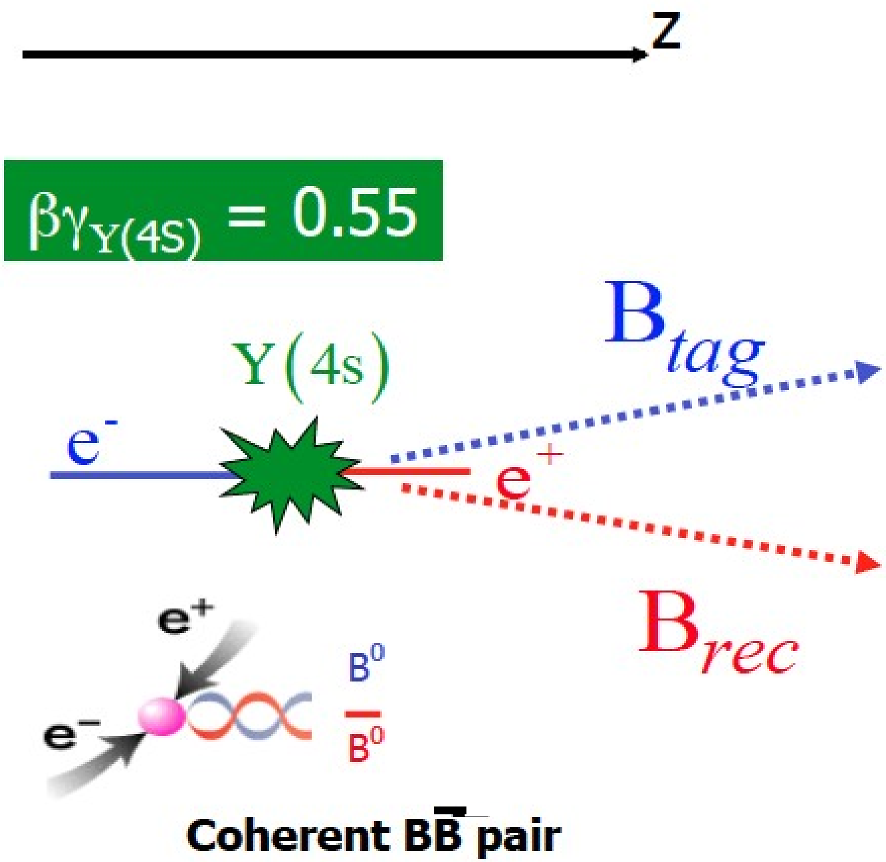
Oscillation and *CP* Violation in the B^0 system

$\beta\gamma_{\Upsilon(4S)} = 0.55$

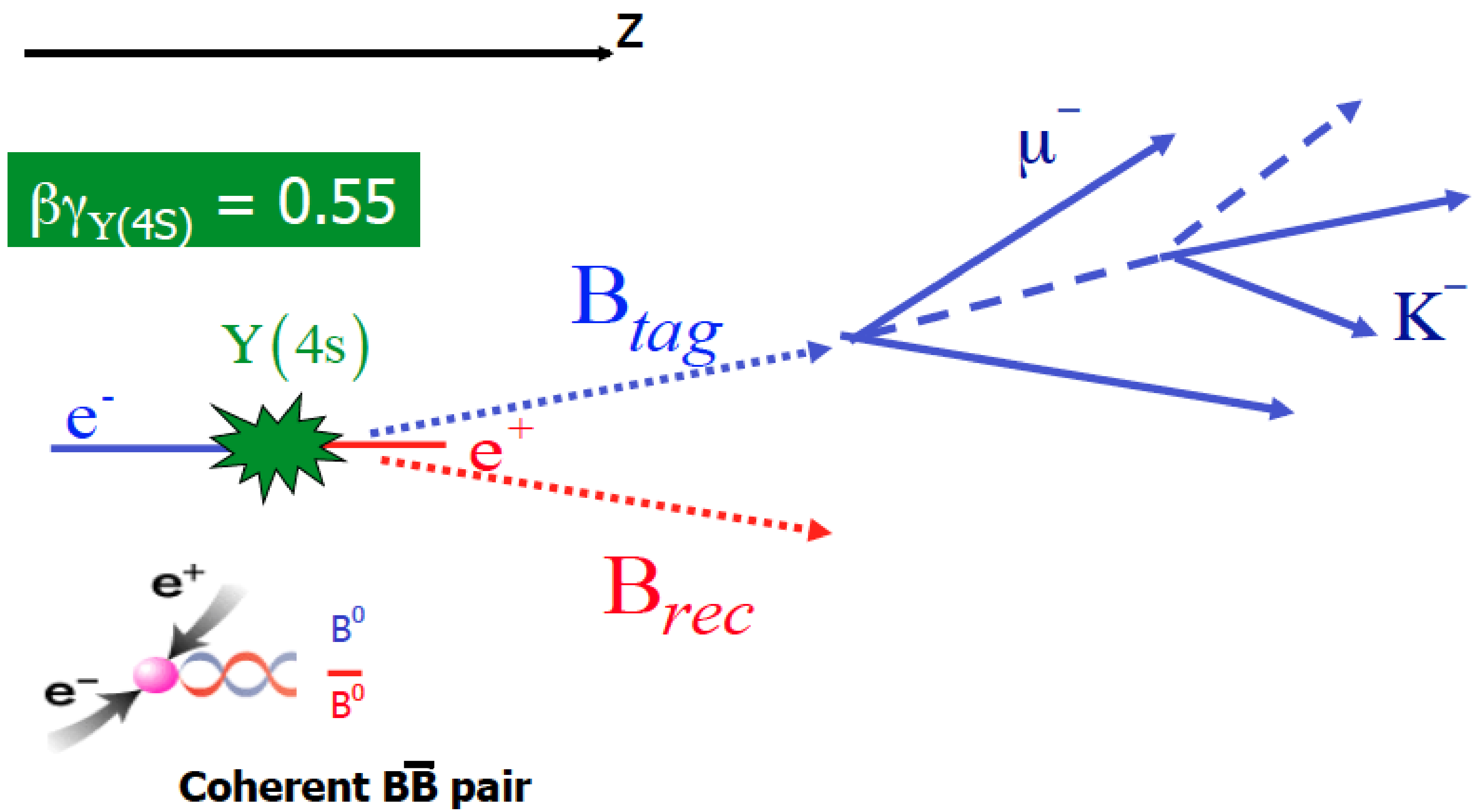


Coherent $B\bar{B}$ pair

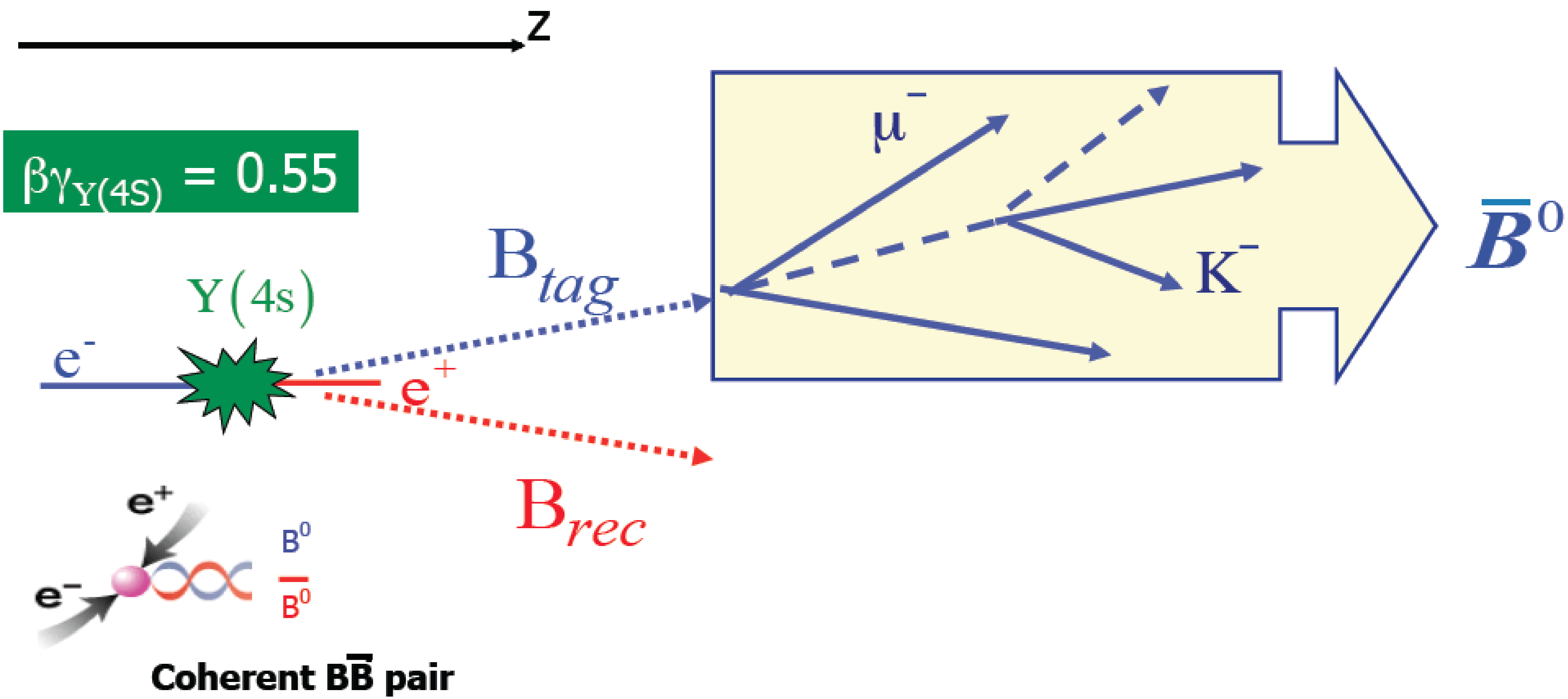
Oscillation and *CP* Violation in the B^0 system



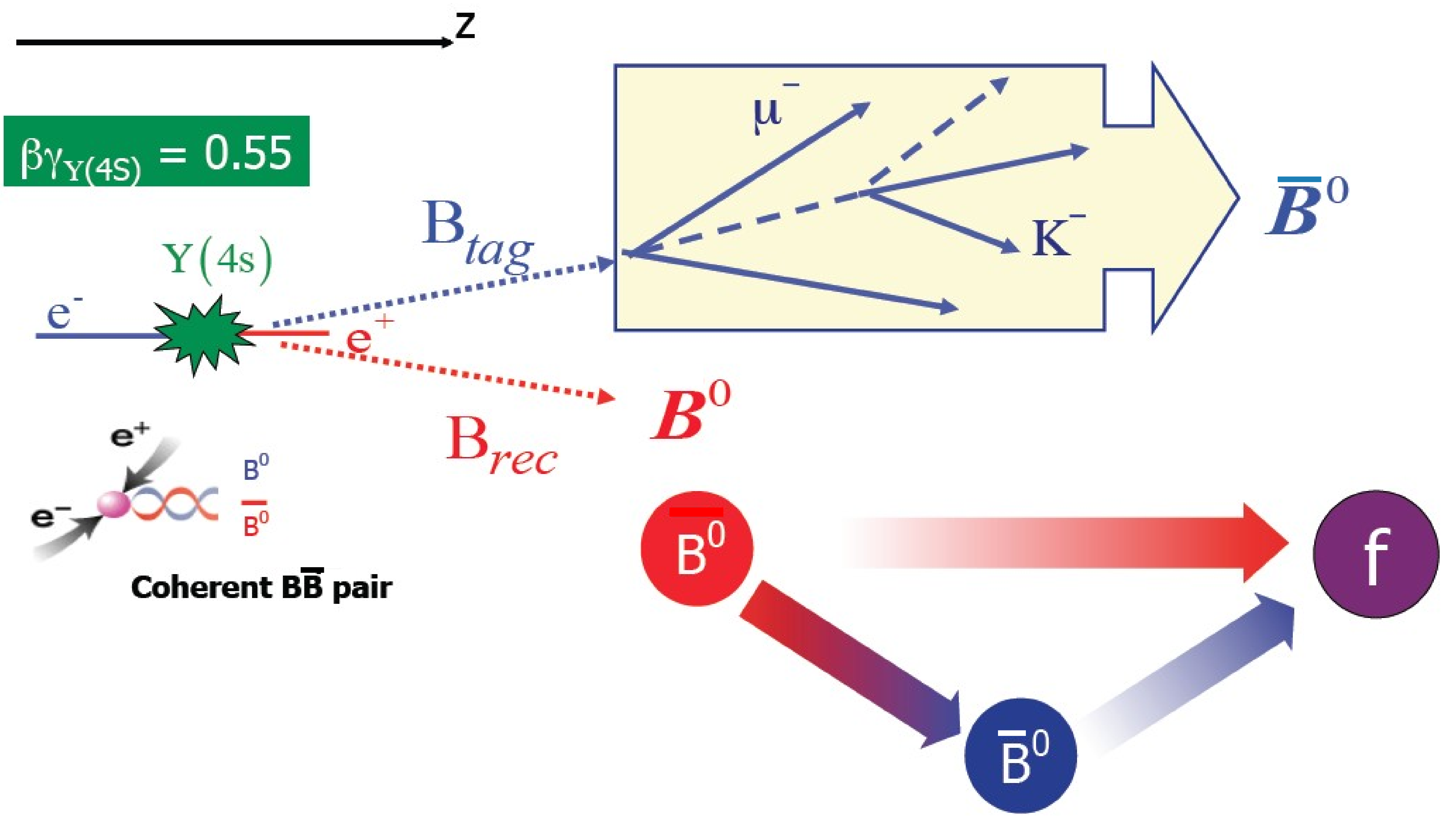
Oscillation and *CP* Violation in the B^0 system



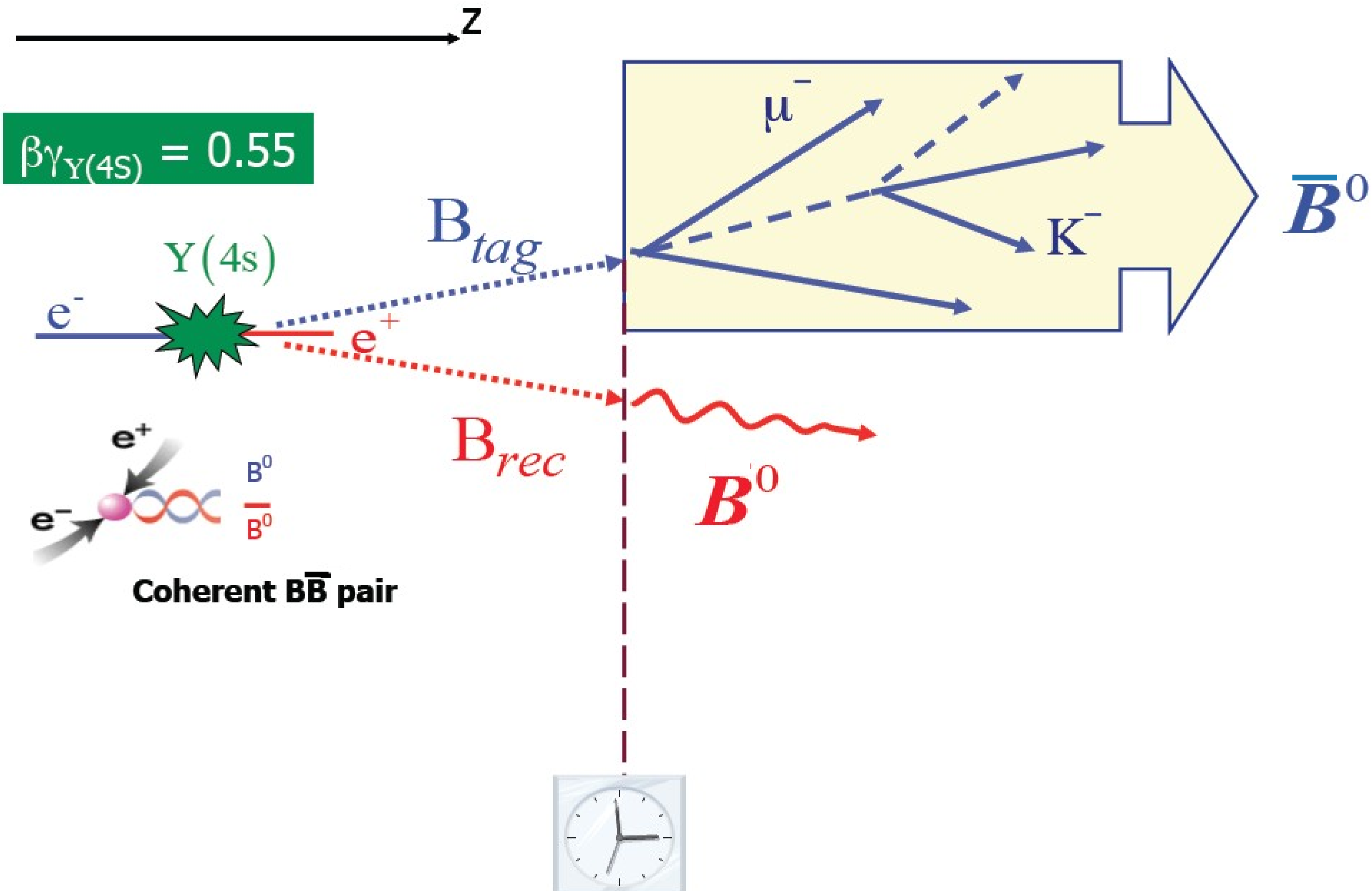
Oscillation and *CP* Violation in the B^0 system



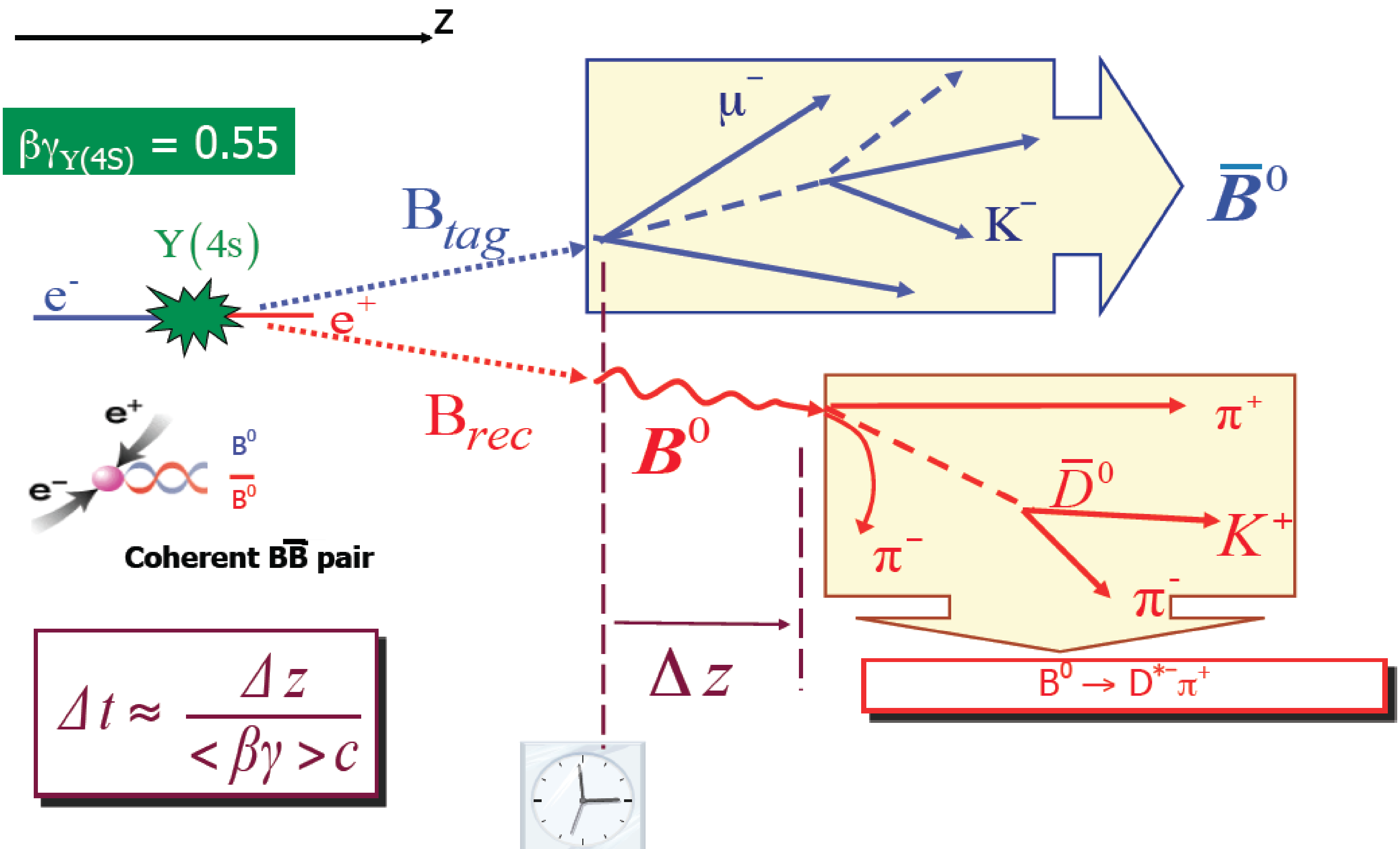
Oscillation and *CP* Violation in the B^0 system



Oscillation and *CP* Violation in the B^0 system



Oscillation and *CP* Violation in the B^0 system



$$\Delta t \approx \frac{\Delta z}{<\beta\gamma>c}$$

Oscillations in Hadronic B^0 decays

- **Beauty oscillation**
- The flavours of both *B* can be identified from the final states of their decays.

The large sample of fully reconstructed events provides the precise determination of the tagging parameters required in the CP fit

factor of merit

neural net tags

Tagging category	Fraction of tagged events ε (%)	Wrong tag fraction w (%)	Q = ε (1-2w)² (%)
Lepton	11.1 ± 0.2	**8.6 ± 0.9**	7.6 ± 0.4
Kaon	**34.7 ± 0.4**	18.1 ± 0.7	14.1 ± 0.6
NT1	7.7 ± 0.2	22.0 ± 1.5	2.4 ± 0.3
NT2	14.0 ± 0.3	37.3 ± 1.3	0.9 ± 0.2
ALL	67.5 ± 0.5		**25.1 ± 0.8**

Highest "efficiency"

Smallest mistag fraction

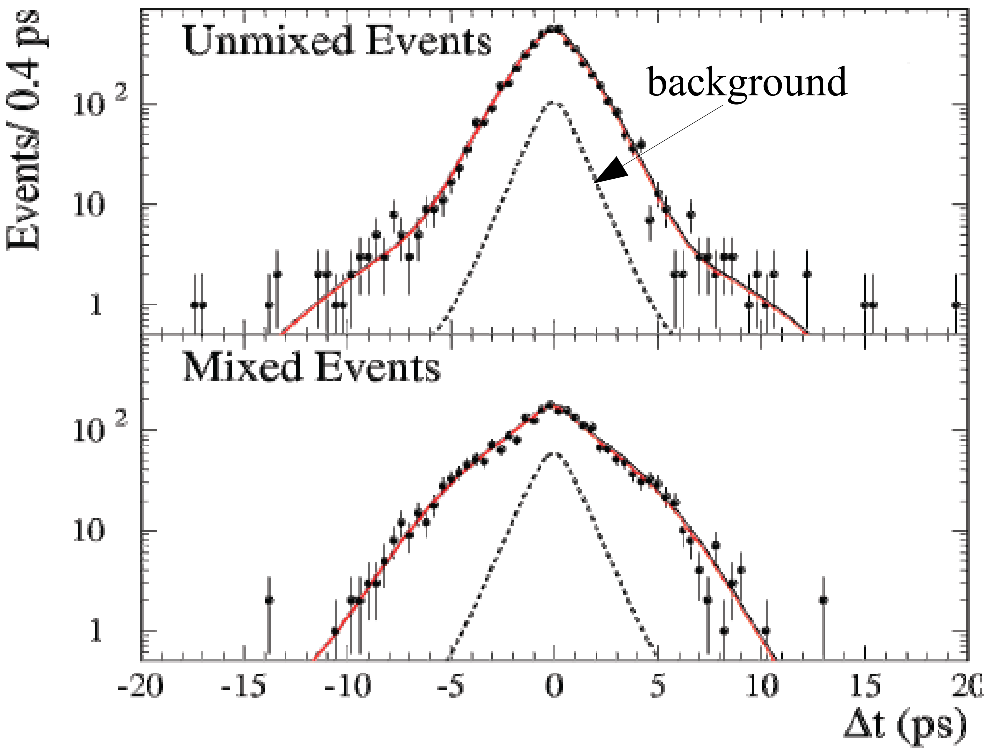
- The times between the production of the $B\bar{B}$ pair and the time of each of the two decays are measured obtaining the time t between the two decays.
- Because the time of one or of the other decay can be taken as $t = 0$ indifferently, t is known only in absolute value.

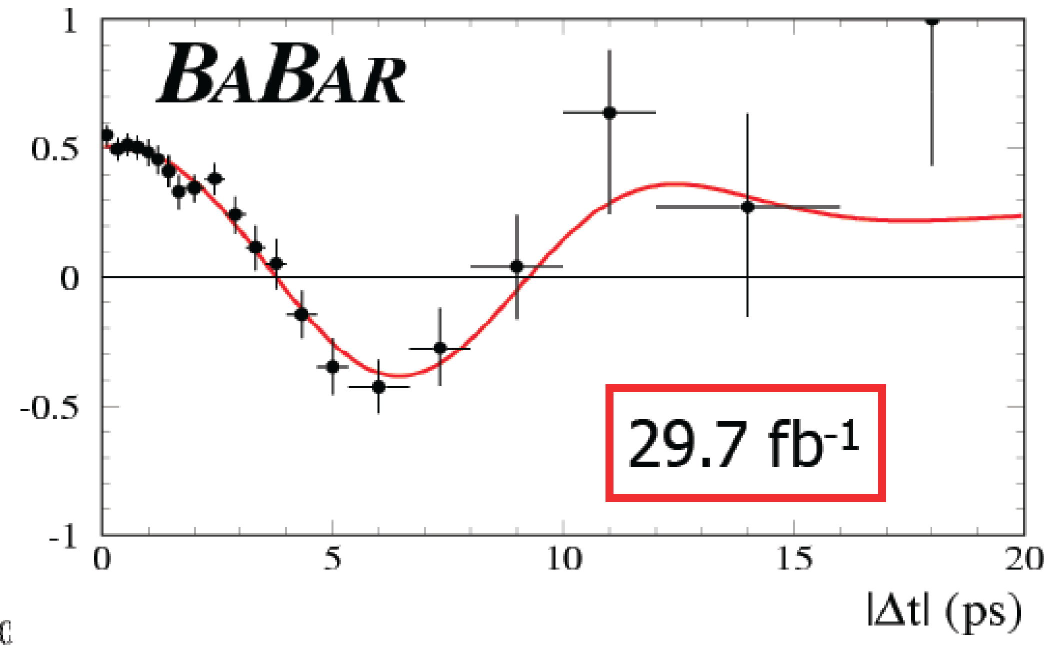
Oscillations in Hadronic B^0 decays

$$Asym\,(\Delta t) = \frac{N(unmixed) - N(mixed)}{N(unmixed) + N(mixed)} \sim (1 - 2\langle w \rangle) \times \cos\,(\Delta m_d \Delta t)$$

$B^0\,\bar{B}^0$ $\quad$ $B^0\,B^0, \bar{B}^0\,\bar{B}^0$

diluition factor





$\boxed{\Delta m_d = 0.516 \pm 0.016\ (\mathrm{stat}) \pm\ 0.010\ (\mathrm{syst})\ \mathrm{ps}^{-1}}$

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= 0.3354 ± 0.0104 ± 0.0065 meV

Oscillations in Hadronic B^0 decays

- From this measurement one can extract:

$$|V_{td}||V_{tb}| = (7.4 \pm 0.8) \times 10^{-3}$$

- Pay attention that the neutral *B* mesons are reconstructed in a sample of multihadron events in the flavor eigenstate decay modes:

$$D^{(*)-}\pi^+; D^{(*)-}\rho^+, D^{(*)-}a_1^+, J/\psi K^{*0}$$

- The $\overline{D}^0$ candidates are reconstructed through the channels:

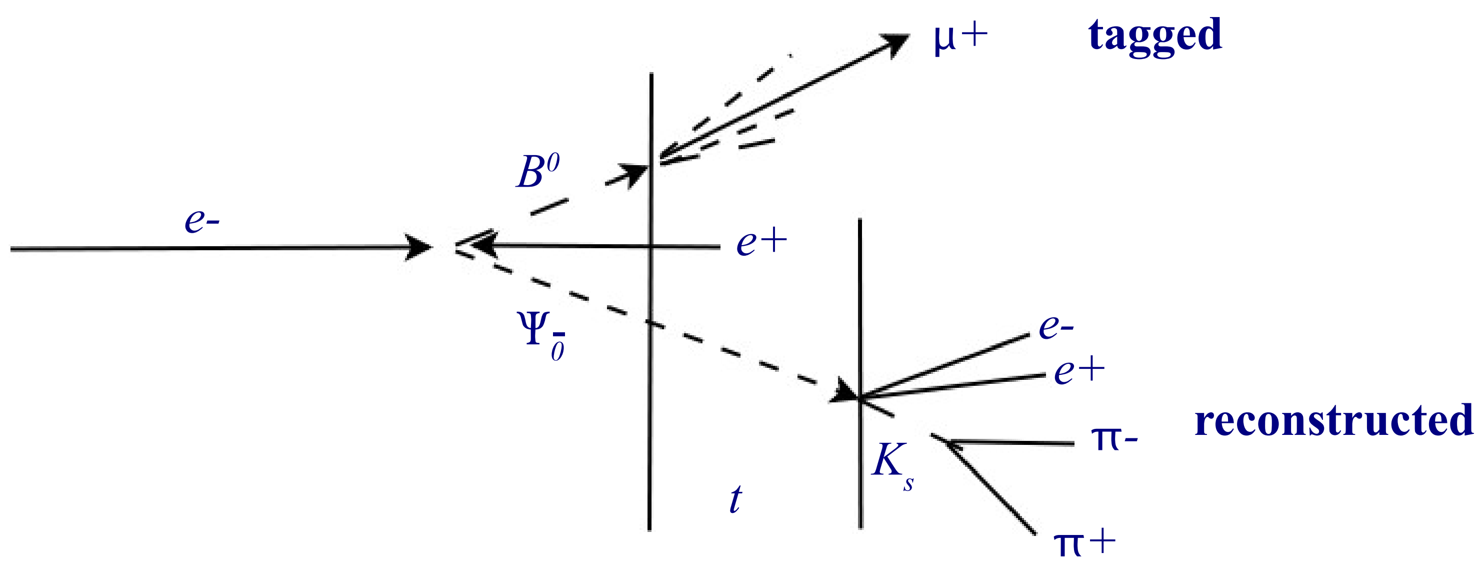
$$K^+\pi^-, K^+\pi^-\pi^0, K^+\pi^+\pi^-\pi^-, K_S^0\pi^+\pi^-$$

- The D^- candidates are reconstructed through the channels:

$$K^+\pi^-\pi^-, K_S^0\pi^-$$

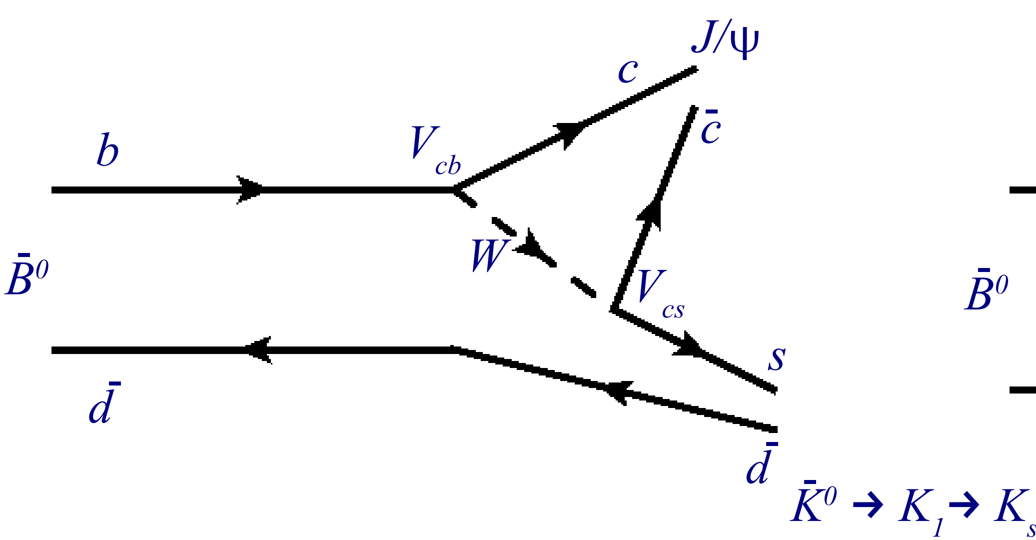
Oscillation and *CP* Violation in the B^0 system

- a_{fCP} **asymmetry** observed in more than one channel
- We consider the final eigenstates of *CP*: $f = J/\psi + K_s$ and $f = J/\psi + K_L$
- The final orbital angular momentum is $L = 1$ due to the angular momentum and parity conservation
- Eigenstates of *CP*: $\eta_{J/\psi + KL} = +1$, $\eta_{J/\psi + Ks} = -1$
- Br ~ 0.9×10^{-3}, but the peak luminosity is $> 10^{34}$ cm^{-2} s^{-1} → production of 10^6 $B\overline{B}$ pairs/day → collected 5×10^8 eventi.
- One tags the events as described before and selects the events in which the reconstructed *B* (the second *B*) decays into an eigenstate of *CP*.

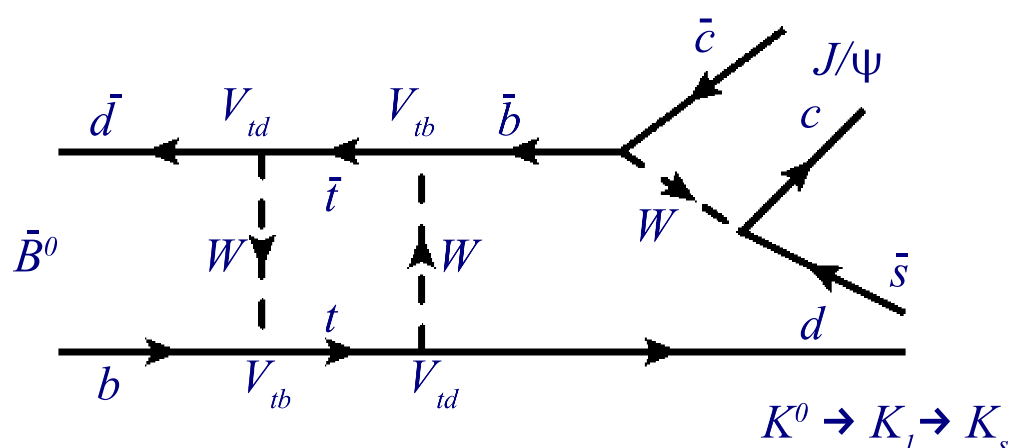


Oscillation and *CP* Violation in the B^0 system

- The decay of the reconstructed state ($\Psi_{\bar{0}}(t)$) can happen directly (a) or oscillates into a state B^0 and then decays (b). The two amplitudes do not interfere because the final states are different.
- But if the K decays as a CP eigenstate, that is as a $K^0{}_1$ (or $K^0{}_2$), the final states are equal and the two amplitudes interfere. (The differences because $K^0{}_1$ and K_S and between $K^0{}_2$ and K_L can be neglected).
- In the Feynman graphs the important elements of the mixing matrix are reported. They are all real (also those not shown) except for V_{td}
- V_{td} appears two times, then squared, in the amplitude.
- $V_{tb} \approx 1$



(a)



(b)

Oscillation and *CP* Violation in the B^0 system

- Taking everything together:

$$\lambda_{J/\psi+K_S} = \eta_f \frac{p}{q}\frac{A_{J/\psi+K}}{\bar{A}_{J/\psi+K}} = \eta_f \left(\frac{V_{td}V^*_{tb}}{V^*_{td}V_{tb}}\right)\left(\frac{V^*_{cb}V_{cs}}{V^*_{cs}V_{cb}}\frac{V^*_{cs}V_{cd}}{V^*_{cd}V_{cs}}\right) = -e^{-2i\beta}$$

$$Im(\lambda_{J/\psi+K_S}) = 2Im(V_{td}) = \sin(2\beta)$$

$$Im(\lambda_{J/\psi+K_L}) = -\sin(2\beta)$$

- Concluding the two observables $a_{CPJ/\psi+KS}(t)$ e $a_{CPJ/\psi+KL}(t)$ are two sinusoidal function of the time with the same period, amplitude and opposite phases.

$$a_{fCP} = \frac{\left|<f|\Psi_0(t)>\right|^2 - \left|<f|\Psi_{\bar{0}}(t)>\right|^2}{\left|<f|\Psi_0(t)>\right|^2 + \left|<f|\Psi_{\bar{0}}(t)>\right|^2} = \eta_f Im(\lambda_f)\sin(\Delta m_B t) = -\eta_f \sin(2\beta)\sin(\Delta m_B t)$$

Oscillation and *CP* Violation in the B^0 system

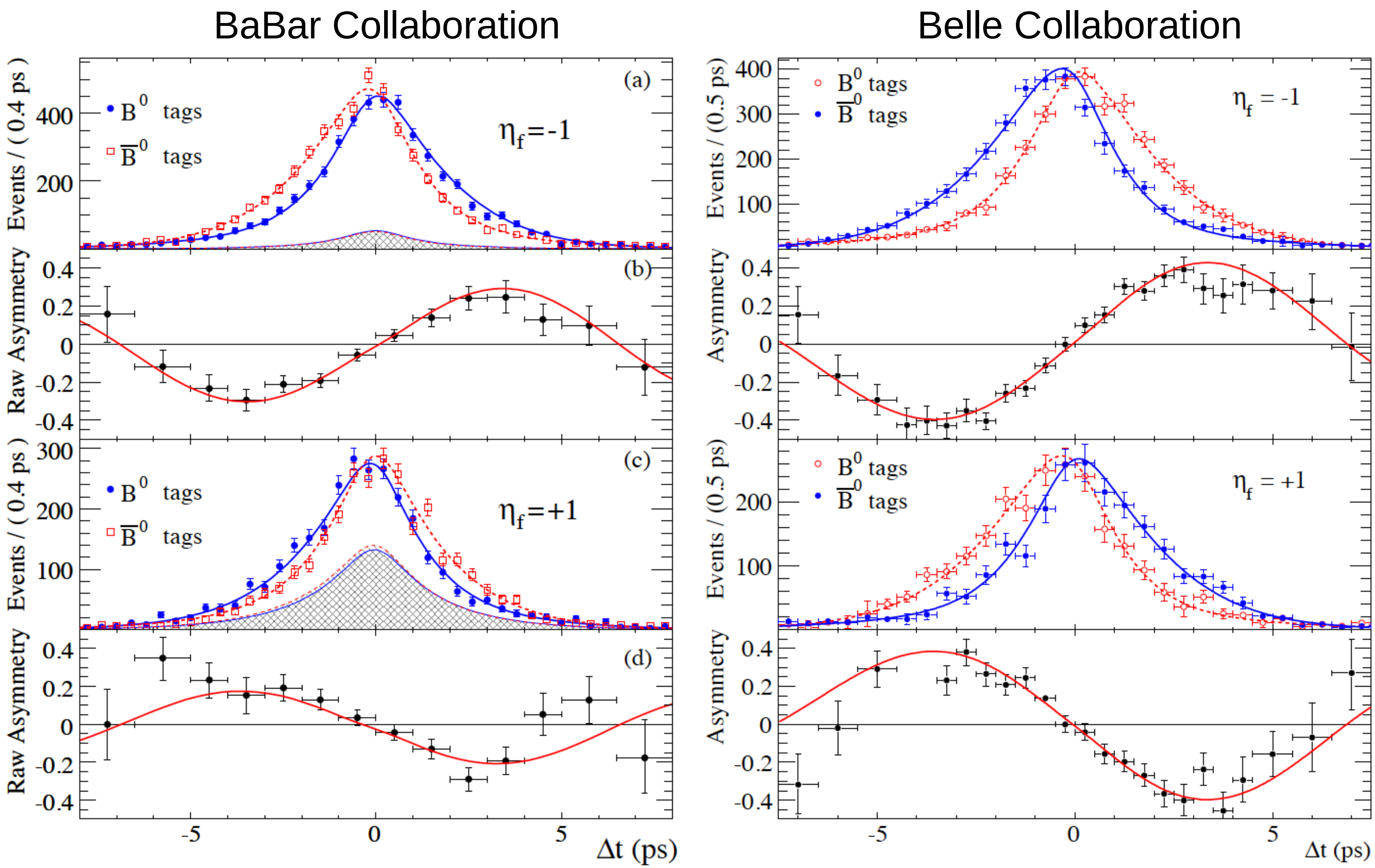


Figure 17.6.7. Flavor-tagged Δt distributions (a,c) and raw *CP* asymmetries (b,d) for the BABAR (left, (Aubert, 2009z)) and Belle (right, (Adachi, 2012c)) measurements of $\sin 2\phi_1$. The top two plots show the $B \to (c\bar{c})K_S^0$ ($\eta_f = -1$) samples, and the bottom two show the $B \to J/\psi K_L^0$ ($\eta_f = +1$) sample. The shaded regions for BABAR represent the fitted background, while the Belle distributions are background subtracted. The two experiments adopt the opposite color code in Δt distribution plots.

Oscillation and *CP* Violation in the B^0 system

- One talks of raw asymmetry because:
  - there is the presence of background (larger background in the case of K_L because such particle doesn't decay in the detector due to its long average life time. It is seen in the hadronic calorimeter as an hadronic shower.);
  - the experimental resolution in the time measurement is limited (1-1.5 ps);
  - presence of mis-tag, B^0 classified and $\overline{B}^0$ and viceversa.
- These effects reduce the oscillation amplitude.
- The mean value of the two experiments (corrected for all the aforementioned effects) is:

$$\boxed{\sin(2\beta) = 0.677 \pm 0.020}$$

Oscillation and *CP* Violation in the B^0 system

- *CKM* matrix should be unitary for *SM:* $V^{\dagger}V = 1$
- This gives 9 relationships between the individual elements:

$$\sum_{i=1,3} \left|V_{i,j}\right|^2 = 1$$

$$\sum_{i=1,3} V_{ji} V^*_{ki} = \sum_{i=1,3} V_{ij} V^*_{ik} = 0 \quad (\mathrm{j, k = 1, 2, 3; j \neq k})$$

- Unitarity tests:

$$\left|V_{ud}\right|^2 + \left|V_{us}\right|^2 + \left|V_{ub}\right|^2 = 0.9999 \pm 0.0006 \ \textit{(first row)}$$

$$\left|V_{cd}\right|^2 + \left|V_{cs}\right|^2 + \left|V_{cb}\right|^2 = 1.067 \pm 0.047 \ \textit{(second row)}$$

$$\left|V_{ud}\right|^2 + \left|V_{cd}\right|^2 + \left|V_{td}\right|^2 = 1.002 \pm 0.005 \ \textit{(first column)}$$

$$\left|V_{us}\right|^2 + \left|V_{cs}\right|^2 + \left|V_{ts}\right|^2 = 1.065 \pm 0.046 \ \textit{(second column)}$$

Oscillation and *CP* Violation in the B^0 system

- The last relations, each of which is a sum of 3 complex numbers, form a **unitarity triangle (UT)** in the complex plane.
- It can be shown that

$$\left|\Im\left(V_{km}^{*}V_{lm}V_{kn}V_{\ln}^{*}\right)\right| = \left|\Im\left(V_{mk}^{*}V_{ml}V_{nk}V_{nl}^{*}\right)\right| = J$$

irrispective of *k,l,m,n* and all six triangles have the same area: ***A* = ½ *J*** independent of any phase convention, *J* is known as the *Jarlskog invariant.*

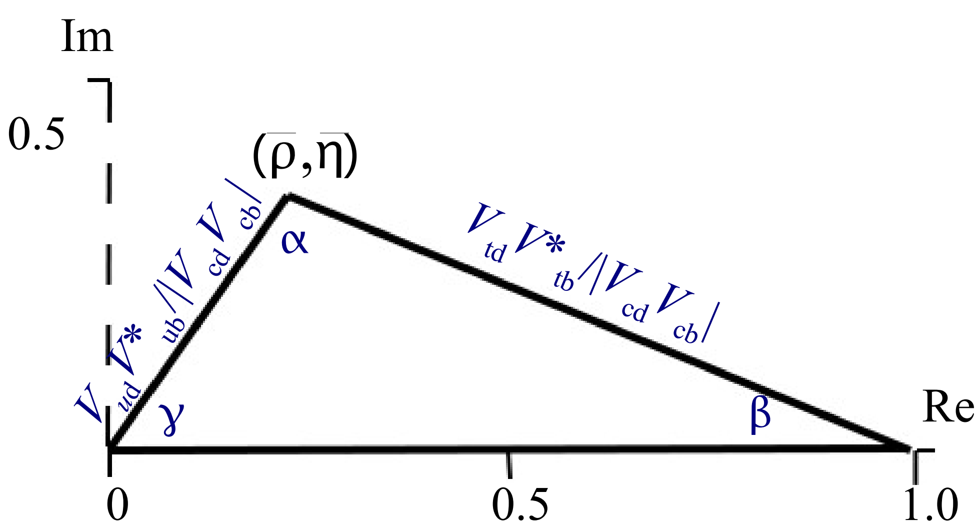
- A particular interesting triangle is the triangle involving the *B* decays (coming from the product of the first with the third column):

$$V_{ud}V_{ub}^{*}+V_{cd}V_{cb}^{*}+V_{td}V_{tb}^{*} = 0$$

- This triangle contains the matrix elements: V_{ub} and V_{tb} which are really complex numbers.

Oscillation and *CP* Violation in the B^0 system

- One knows $|V_{cd}|$, $|V_{cb}|$, $|V_{ud}|$ and $|V_{ub}|$ and $|V_{td}||V_{tb}|$ (from the Δm_B measurement). One knows the lengths of the three sides.
- One can neglect the imaginary part of $V_{cd}V^*_{cb}$ and writes: $V_{cd}V^*_{cb} = |V_{cd}||V_{cb}|$.
- One can divide the three sides by this quantity



$$\phi_1 = \beta \equiv arg\left[-V_{cd}V^*_{cb}/V_{td}V^*_{tb}\right]$$
$$\phi_2 = \alpha \equiv arg\left[-V_{td}V^*_{tb}/V_{ud}V^*_{ub}\right]$$
$$\phi_3 = \gamma \equiv arg\left[-V_{ud}V^*_{ub}/V_{cd}V^*_{cb}\right]$$

β measured through $B^0 \rightarrow J/\psi\, K^0{}_s$
α measured through $B^0 \rightarrow \pi^+\pi^-$; $\rho^+\rho^-$
γ measured through $B^- \rightarrow D\, K^-$

Oscillation and *CP* Violation in the B^0 system

- Knowing the length of the three sides one can verify if the triangle is closed.
- A further constraint can from sin2β which fixes β within a fourfold ambiguity: β, β+π, ±π/2 - β
- There are other constraints given by other measurements.

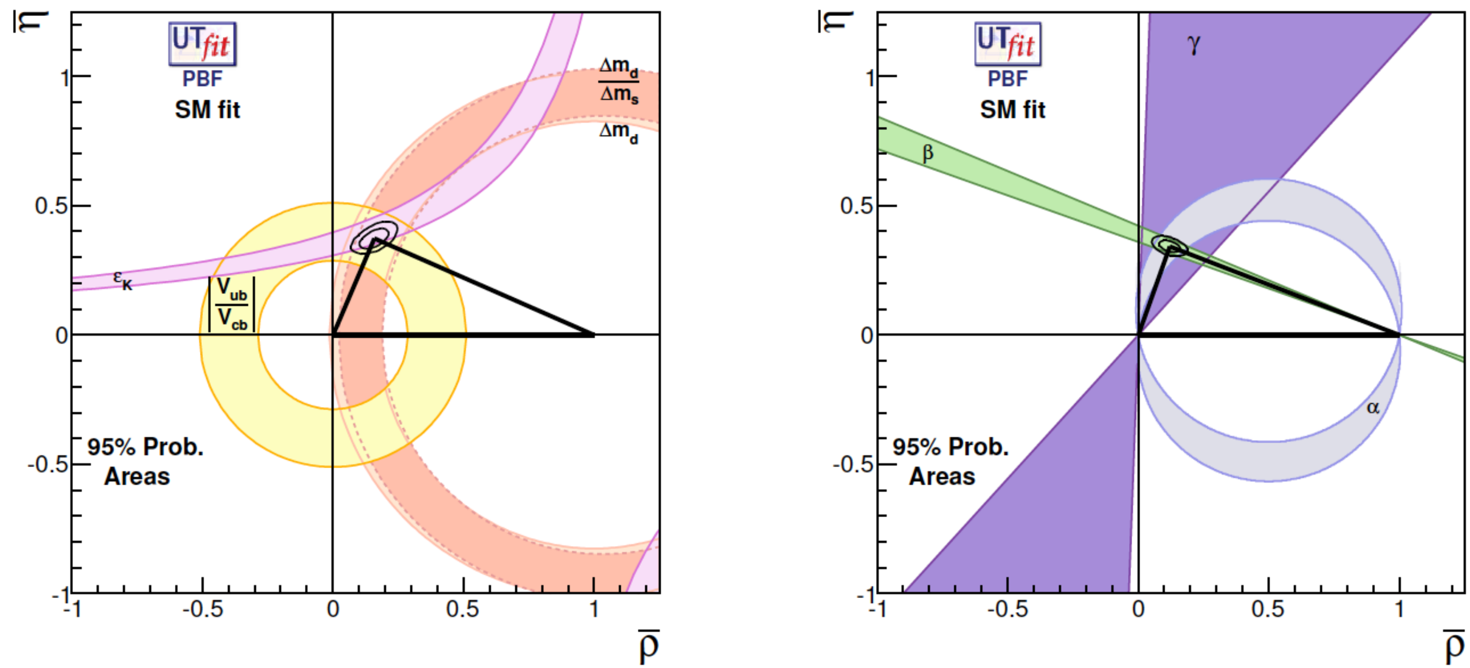


Figure 25.1.3. The consistency between the fit to CP-conserving observables and from K measurements (on the left) and the angles (B Factory-dominated on the right) from the UTfit group. The constraints used in the left plot are the $B^0 - \overline{B}^0$ and $B^0_s - \overline{B}^0_s$ mass differences, Δm_d and Δm_s, respectively, the measurements of $|V_{ub}|$ and $|V_{cb}|$ from semileptonic B decays, and the CP-violation parameter ε_K. The constraints used in the right plot are the "angles" observables, *i.e.* measurements of ϕ_1 $(= \beta)$, ϕ_2 $(= \alpha)$, and ϕ_3 $(= \gamma)$.