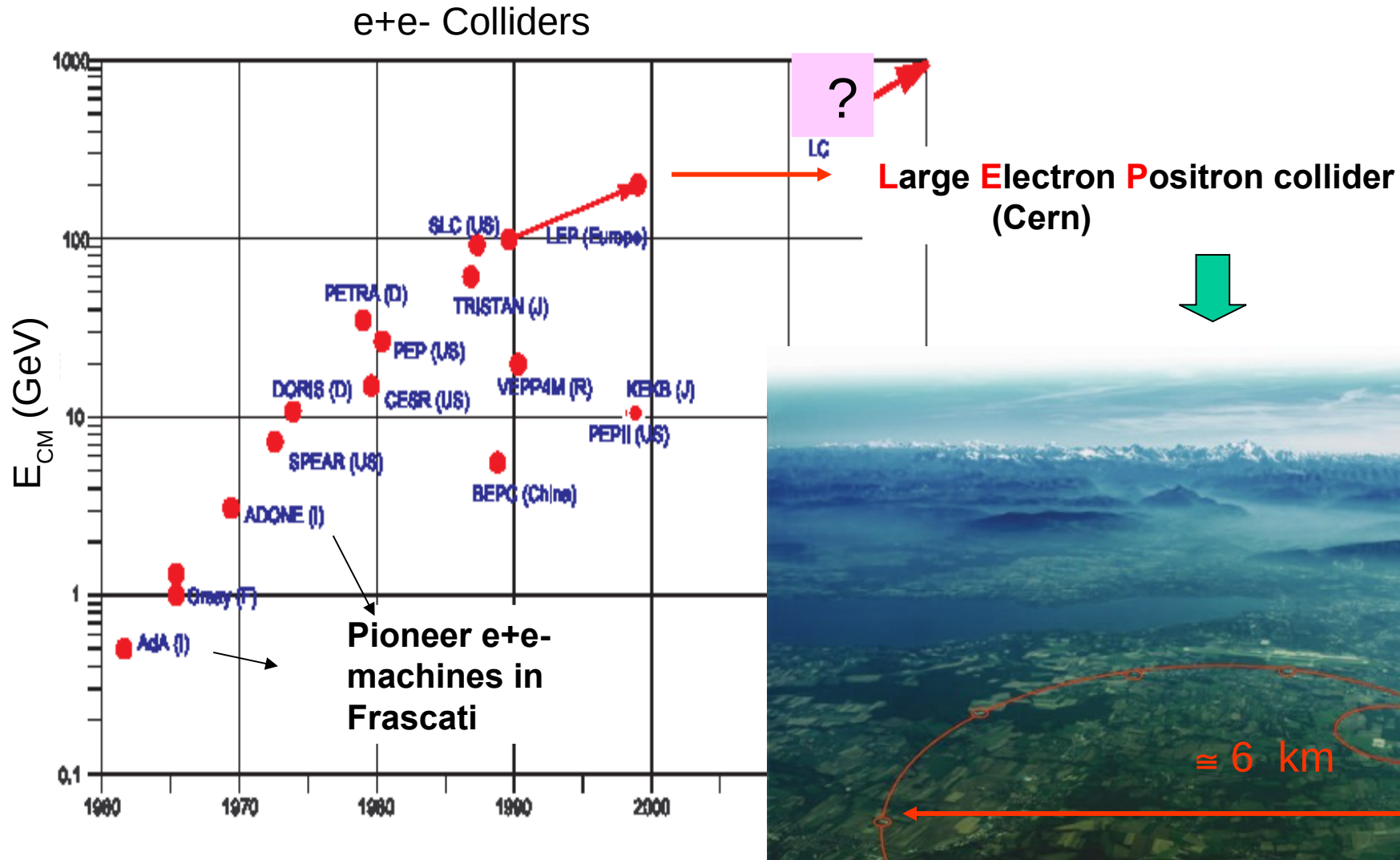


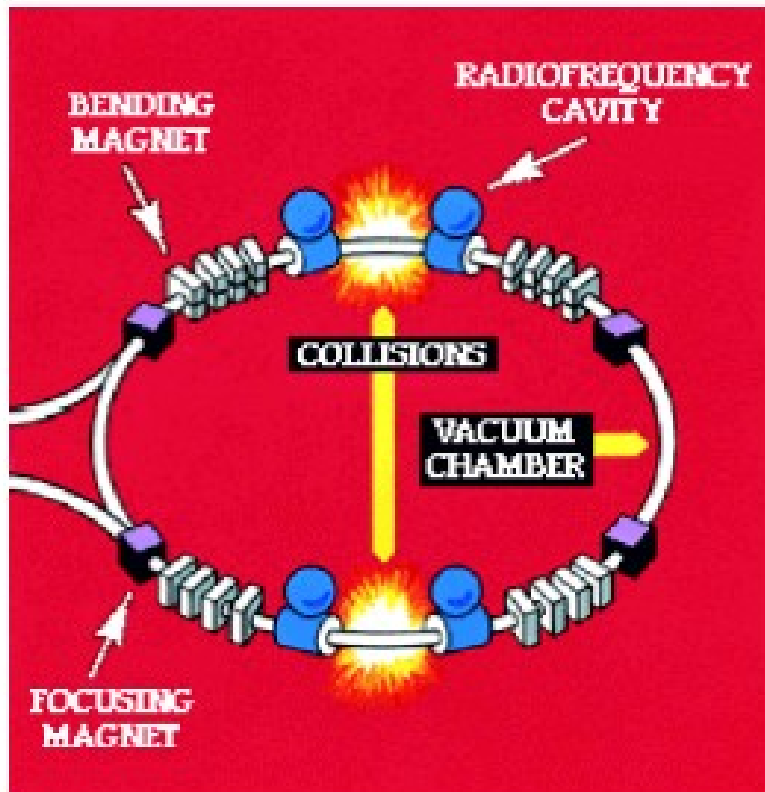
Experimental Tests of the Standard Model (2)

- Measurements of the Weinberg angle
- W^\pm and Z^0 discovery
- **Precision measurements of the Z^0**
- **Precision measurements of the W**
- Discovery/measurements of the top
- Discovery of the Higgs

$e^+ e^-$ Colliders



$e^+ e^-$ Colliders



$\sqrt{s}=200 \text{ MeV}$



Problem: Synchrotron radiation.

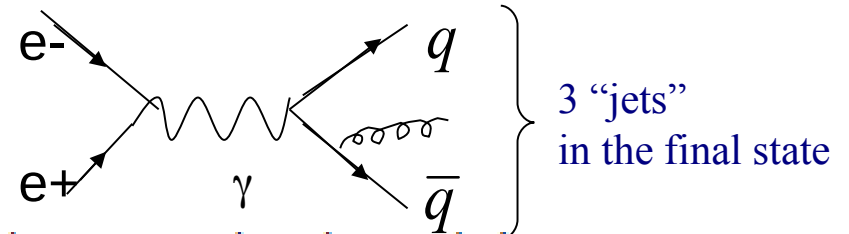
The energy loss $\Delta E \sim K(E/m)^4 \beta^3 r^{-1}$

- the mass is at the 4th power:

for the same radius, an $e^+ e^-$ collider irradiates $\sim 10^{12}$ times the power of a proton collider

1971 AdA demonstrated that $e^+ e^-$ can collide in the same tunnel

LEP (1989-2000)

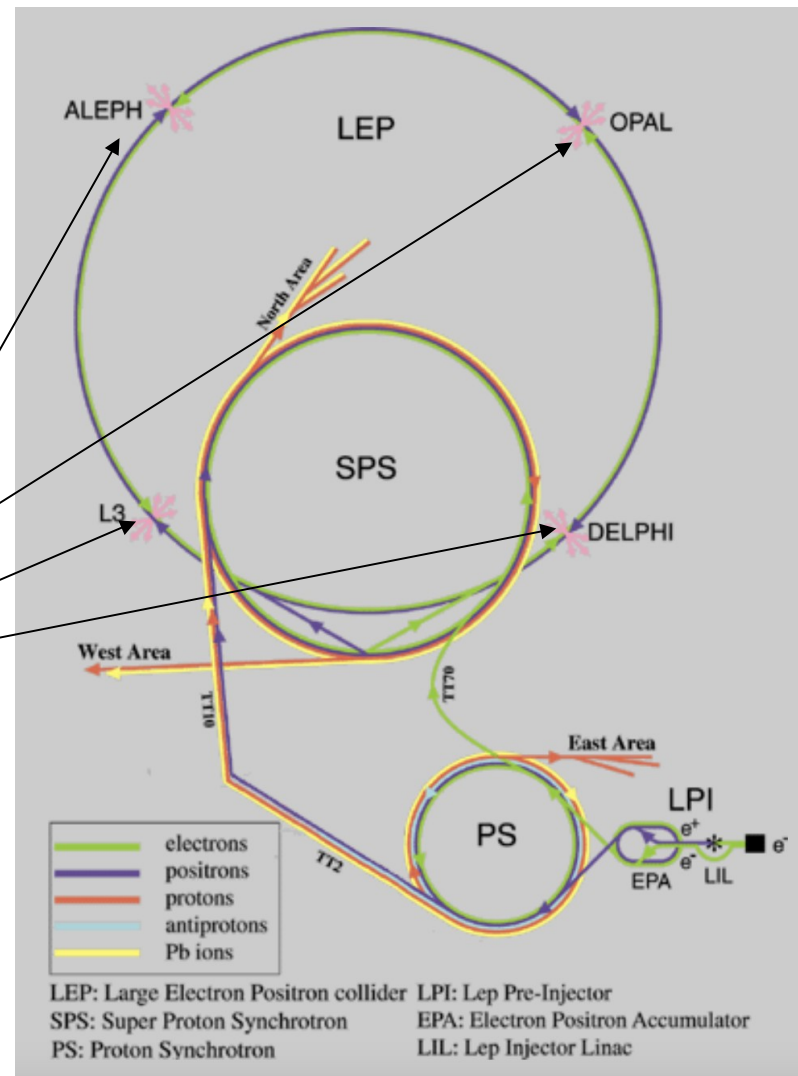


Petra at DESY discovered the gluon but the Standard Model was deeply verified at LEP

It ran at several \sqrt{s} up to 205 GeV

The large circumference, 27 Km, made it a "linear-like" collider to minimize synchrotron radiation losses

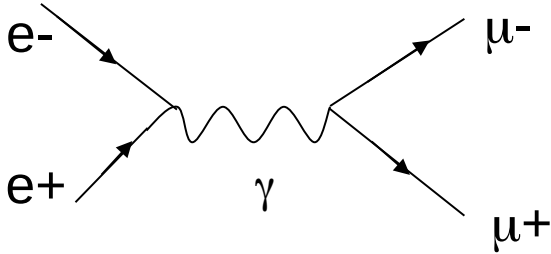
4 experiments:
Aleph, Delphi, L3, Opal



“Phase I” (1989-1995): $E_{\text{CM}} \sim M_Z$

“Phase II”(1996-2000): $E_{\text{CM}} \sim 2M_W \rightarrow 205 \text{ GeV}$

Scattering $e^+e^- \rightarrow f\bar{f}$ in QED



$$\left(\frac{d\sigma}{d\Omega} \right)_{e^+e^- \rightarrow \mu^+\mu^-}^{QED} = \frac{\alpha^2}{2s} \left[\frac{t^2 + u^2}{s^2} \right]$$

Reminding the definitions of the Mandelstam variables:

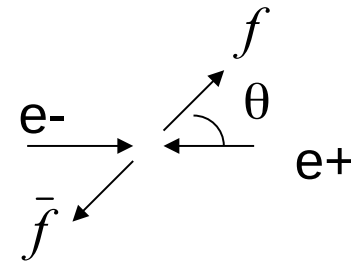
$$s \equiv (p_{e^-} + p_{e^+})^2 \simeq 4 p_e^2$$

$$t \equiv (p_{e^-} - p_f)^2 \simeq -2 p_e^2 (1 - \cos \vartheta)$$

$$u \equiv (p_{e^-} - p_{\bar{f}})^2 \simeq -2 p_e^2 (1 + \cos \vartheta)$$

→ $\frac{t^2 + u^2}{s^2} = \frac{1}{2} (1 + \cos^2 \vartheta)$

→ $\left(\frac{d\sigma}{d\Omega} \right)_{e^+e^- \rightarrow \mu^+\mu^-}^{QED} = \frac{\alpha^2}{4s} [1 + \cos^2 \vartheta]$



$$\begin{aligned} t &\equiv (p_e - p_f)^2 = p_e^2 + p_f^2 - 2p_e p_f \cos \vartheta \simeq \\ &= -2E_e E_f (1 - \cos \vartheta) = -2p_e^2 (1 - \cos \vartheta) \end{aligned}$$

QED foresees a symmetric distribution for the polar angle θ , where θ is the polar angle of the outgoing fermion respect to the direction of the incoming electron.

Scattering $e^+e^- \rightarrow f\bar{f}$ in QEWD

- The presence of the weak interaction mediated by the massive boson modifies largely, for energies near the boson mass, the QED prediction.
- The QED formula can be rewritten:

$$\left(\frac{d\sigma}{d\Omega}\right)_{e^+e^- \rightarrow \mu^+\mu^-}^{QED} = \frac{\alpha^2}{2s} \left[\frac{t^2 + u^2}{s^2} \right] = \frac{1}{64\pi^2 s} \overline{|M_{fi}^\gamma|^2}$$

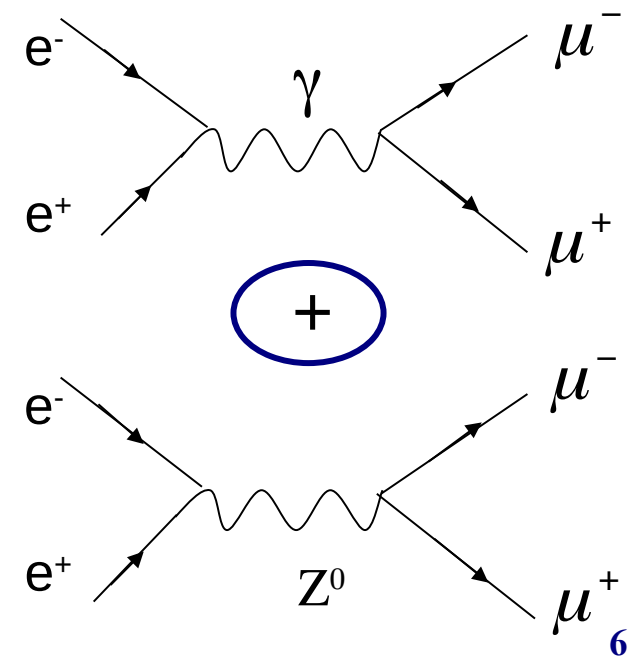
where the amplitude is:

$$\overline{|M_{fi}^\gamma|^2} \equiv 2e^4 \left[\frac{t^2 + u^2}{s^2} \right]$$

e.m. coupling (the electric charge e) has been absorbed.

In QEWD, to the amplitude for the photon exchange the amplitude for the massive Z^0 exchange has to be added. In the amplitude there is the weak coupling that the Standard Model predict to be: $(g^2 + g'^2)^{1/2} = g/\cos\theta_w$

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{e^+e^- \rightarrow \mu^+\mu^-}^{QEWD} = \frac{1}{64\pi^2 s} \overline{|M_{fi}^\gamma + M_{fi}^{Z^0}|^2}}$$



Scattering $e^+e^- \rightarrow f\bar{f}$ in QEWD

- At the first perturbative order, known as “Born level” (only Feynman graphs at the lowest order in e^2 , g^2 are taken into account), averaging on initial polarization of the e^+, e^- beams

$$\left(\frac{d\sigma}{d\Omega} \right)_{e^+e^- \rightarrow \mu^+\mu^-}^{QEWD, Born} = \frac{\alpha^2}{4s} \left[F_1(s)(1 + \cos^2 \vartheta) + F_2(s) \cos \vartheta \right]$$

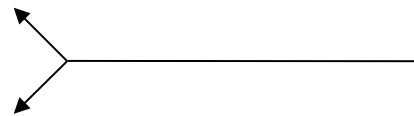
with:

forward- backward
term asymmetry

$$F_1(s) = 1 + 2 \operatorname{Re}(r(s)) g_V^2 + |r(s)|^2 (g_V^2 + g_A^2)^2$$

↓
QED

$$F_2(s) = 4 \operatorname{Re}(r(s)) g_A^2 + 8 |r(s)|^2 g_V^2 g_A^2$$



$$r(s) = \frac{1}{e^2} \frac{s(g/2 \cos \theta_W)^2}{s - M_Z^2 + i\Gamma_Z M_Z}$$

↓
resonant term:

M_Z boson mass

$\Gamma_Z = \Gamma_Z(g_A, g_V, M_Z)$: intrinsic width

Scattering $e^+e^- \rightarrow f\bar{f}$ in QEWD

● Integrating on the solid angle: $\sigma_{e^+e^- \rightarrow \mu^+\mu^-}^{Born}(s) = \frac{4\pi\alpha^2}{3s} F_1(s)$

● At $s = M_Z^2$ the resonant term is: $r(M_Z^2) = -i \frac{M_Z^2 g^2}{\Gamma_Z M_Z 4e^2 \cos^2 \theta_W} = -i \frac{M_Z g^2}{\Gamma_Z 4e^2 \cos^2 \theta_W}$

➡ $\text{Re}[r(M_Z^2)] = 0$, $|r(M_Z^2)|^2 = \frac{M_Z^2}{\Gamma_Z^2} \left(\frac{g^2}{4e^2 \cos^2 \theta_W} \right)^2$

● Then :

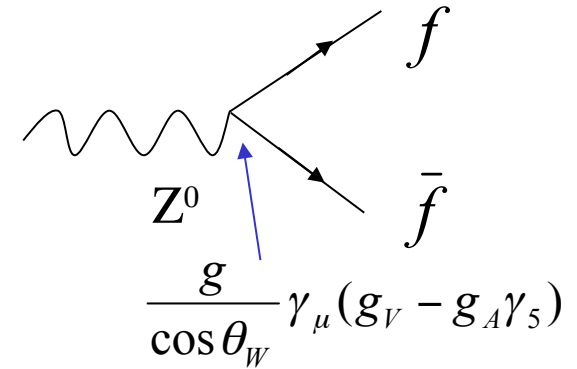
$$\begin{aligned} \sigma_{e^+e^- \rightarrow \mu^+\mu^-}^{Born} &= \frac{4\pi\alpha^2}{3M_Z^2} F_1(M_Z^2) = \frac{4\pi\alpha^2}{3M_Z^2} \left[1 + |r(M_Z^2)| (g_A^2 + g_V^2)^2 \right] = \frac{4\pi\alpha^2}{3M_Z^2} \left[1 + \frac{M_Z^2}{\Gamma_Z^2} \left(\frac{g^2}{4e^2 \cos^2 \theta_W} \right)^2 (g_A^2 + g_V^2)^2 \right] \\ &= \frac{4}{3} \frac{e^4}{16\pi \Gamma_Z^2} \frac{1}{16e^4} \left(\frac{g^2}{\cos^2 \theta_W} (g_A^2 + g_V^2) \right)^2 \quad \text{(neglecting 1 inside the parenthesis)} \end{aligned}$$

➡ $\sigma^{Born}(M_Z^2) = \frac{1}{12\pi\Gamma_Z^2} \frac{1}{16} \left(\frac{g^2}{\cos^2 \theta_W} (g_A^2 + g_V^2) \right)^2 = \frac{12\pi}{\Gamma_Z^2} \left(\frac{g^2}{48\pi \cos^2 \theta_W} (g_A^2 + g_V^2) \right)^2$

= $\Gamma_{e,\mu} / M_Z$

Scattering $e^+e^- \rightarrow f\bar{f}$ in QEWD

- The coupling of a fermion to vector boson is:



- The decay partial width into the $f\bar{f}$ state is given by:

$$\Gamma(Z \rightarrow f\bar{f}) = \left(\frac{g^2}{48\pi \cos^2 \theta_W} (g_A^2 + g_V^2) \right) M_Z$$

- Therefore, the **cross section at the resonance** can be expressed as:

$$\sigma_0^{Born} \equiv \sigma^{Born}(s = M_Z^2) = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$$

where the leptonic partial width are indicated by:

$$\Gamma_e \equiv \Gamma(Z \rightarrow e^+e^-) \quad , \quad \Gamma_\mu \equiv \Gamma(Z \rightarrow \mu^+\mu^-)$$

Scattering $e^+e^- \rightarrow f\bar{f}$ in QEWD

- Now we can calculate the value σ_0^{Born} predicted by Standard Model.

$$\Gamma_f = \left(\frac{g^2}{48 \pi \cos^2 \theta_W} (g_A^2 + g_V^2) \right) M_Z = \frac{g^2 M_Z^3}{48 \pi M_W^2} (g_A^2 + g_V^2)$$

$$\uparrow \\ M_W = M_Z \cos \theta_W$$

$$\frac{g^2}{8M_W^2} = \frac{G}{\sqrt{2}} \quad \rightarrow \quad \boxed{\Gamma_f = \frac{G}{\sqrt{2}} \frac{M_Z^3}{6\pi} (g_A^2 + g_V^2)}$$

- For neutrinos ($g_A = g_V = 1/2$), the partial width is:


$$\Gamma_\nu = \frac{G}{\sqrt{2}} \frac{M_Z^3}{12\pi} = 0.170 \text{ GeV}$$

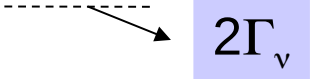
where: $G = 1.167 \cdot 10^{-5} \text{ GeV}^{-2}$ (from the μ decay) and $M_Z = 91.2 \text{ GeV}$ for the experimentally observed boson Z mass (in agreement with the Standard Model prediction)

Scattering $e^+e^- \rightarrow f\bar{f}$ in QEWD

- For charged fermions $f = e, \mu, \tau$: $g_A = -\frac{1}{2}$ $g_V = -\frac{1}{2} + 2\sin^2 \theta_W$

quindi:
$$g_A^2 + g_V^2 = \frac{1}{4} + (2\sin^2 \theta_W - 1/2)^2 = \frac{1}{2} + 4\sin^4 \theta_W - 2\sin^2 \theta_W$$


$$\Gamma_{e,\mu,\tau} = \frac{GM_Z^3}{6\pi\sqrt{2}} \left(\frac{1}{2} + 4\sin^4 \theta_W - 2\sin^2 \theta_W \right) = \Gamma_\nu (1 + 8\sin^4 \theta_W - 4\sin^2 \theta_W)$$



- For $\sin^2 \theta_W = 0.230$: $\Gamma_{e,\mu,\tau} = \Gamma_\nu (1 - 4\sin^2 \theta_W + 8\sin^4 \theta_W) = 0.085 GeV$

- in the same manner, for $Z \rightarrow u\bar{u}, d\bar{d}$ one has:

$$\Gamma_u = 3\Gamma_\nu [1 - (8/3)\sin^2 \theta_W + (32/9)\sin^4 \theta_W] = 0.28 GeV$$

$$\Gamma_d = 3\Gamma_\nu [1 - (4/3)\sin^2 \theta_W + (8/9)\sin^4 \theta_W] = 0.37 GeV$$

[the factor 3 is due to the quark colour]

Scattering $e^+e^- \rightarrow f\bar{f}$ in QEWD

- There are 3 quarks of “down” type: d, s, b with masses $m_q < M_Z/2$
- $Z \rightarrow d\bar{d}$ is kinematically possible, and 2 quark of “up” type: u, c (the top quark has a mass $m_t \cong 175 \text{ GeV} > M_Z$, discovered in 1994 at Tevatron (FNAL, Chicago)); therefore, the total width for Z is:

$$\Gamma_Z = 3\Gamma_\nu + 3\Gamma_e + 2\Gamma_u + 3\Gamma_d = 2.42 \text{ GeV}$$

[the factor 3 in front of $\Gamma_{\nu,e}$ takes into account the 3 leptonic families (e, μ, τ); together with the S.M. prediction and the experimental measurements of $\Gamma_e, \Gamma_{u,d}$ permits to establish that the number of neutrinos (with masses $< M_Z/2$) is 3]

- Inserting all these values one obtains at the end:
- $$\sigma_0^{Born} = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$$
- $$\sigma_0^{Born}(e^+e^- \rightarrow \mu^+\mu^-) = 1.9 \text{ nb}$$

Scattering $e^+e^- \rightarrow f\bar{f}$ in QEWD

- The "hadronic cross section" for the process $Z \rightarrow \text{hadrons}$ is:

$$\sigma_{\text{hadr}}^0 \equiv \sum_{q=u,d,c,s,b} \sigma_q = 2\sigma_u + 3\sigma_d \cong 40nb$$

being:

$$\sigma_{u,c}^0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_u}{\Gamma_Z^2} = \frac{\Gamma_u}{\Gamma_\mu} \sigma_\mu^0 = 3.5\sigma_\mu^0 = 6.7nb$$

$$\sigma_{d,s,b}^0 = \frac{\Gamma_d}{\Gamma_\mu} \sigma_\mu^0 = 4.6\sigma_\mu^0 = 8.8nb$$

Scattering $e^+e^- \rightarrow f\bar{f}$ in QEWD

- It is interesting to compare these cross sections with the QED cross section:

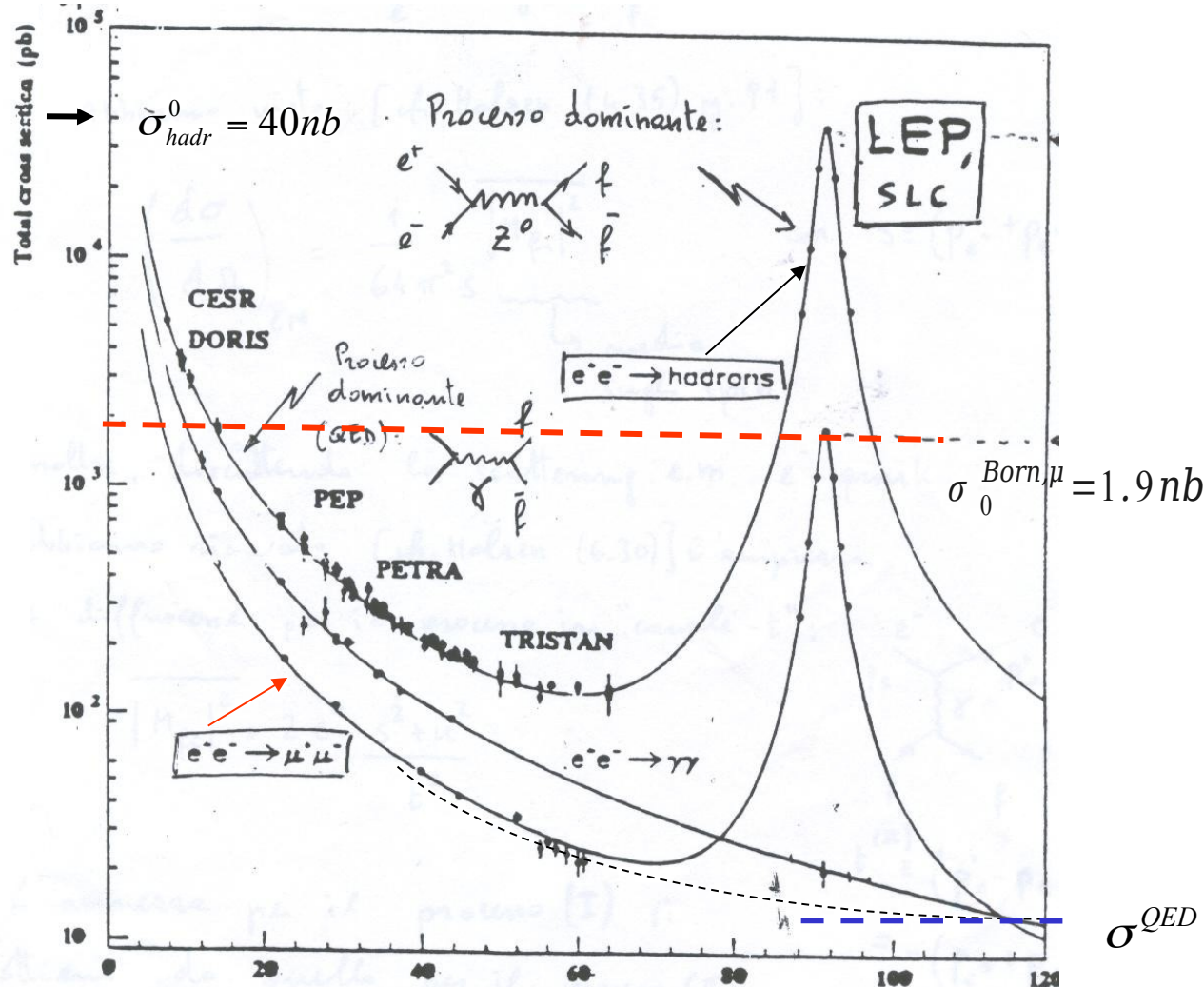
$$\sigma^{QED}(s=M_Z^2) = \frac{\sigma_{point-like}^{QED}}{M_Z^2}$$

$$\frac{87 \text{ nb GeV}^2}{(91.2)^2 \text{ GeV}^2} = 0.01 \text{ nb}$$



The cross section at the Z resonance is about 200 times higher than what is foreseen from QED

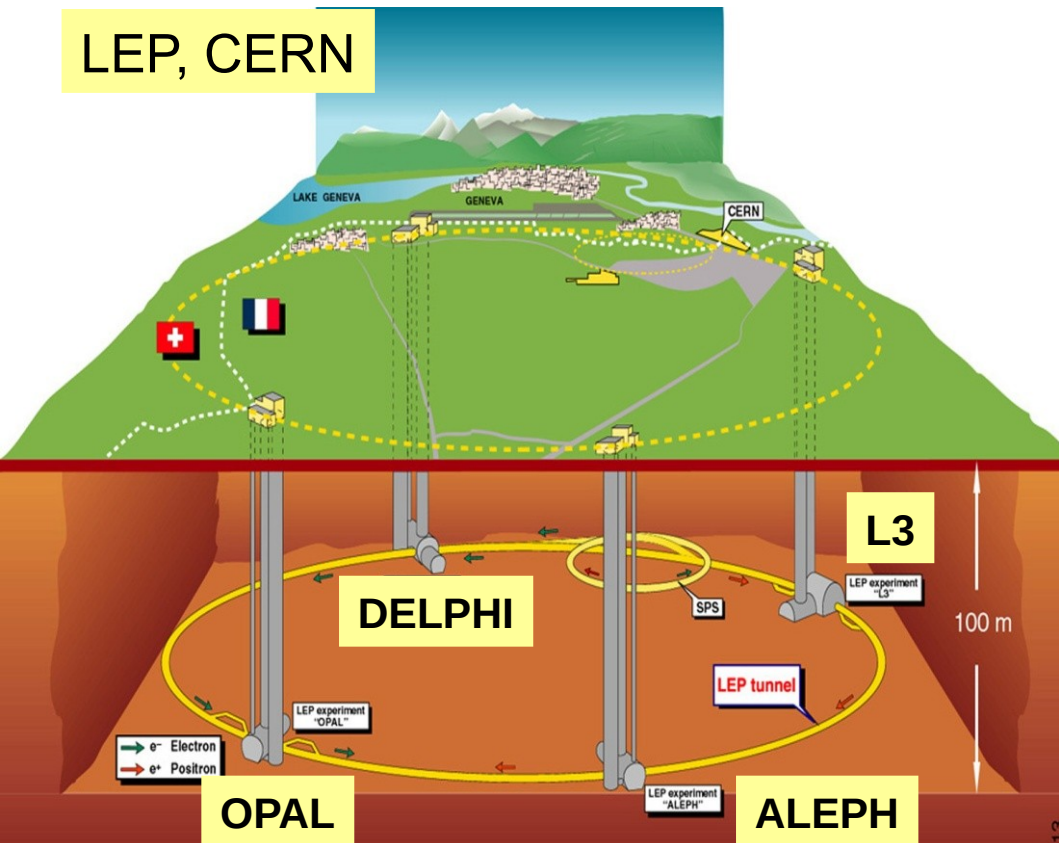
The cross section $Z \rightarrow \text{hadrons}$ is ~ 4000 higher (40 nb)



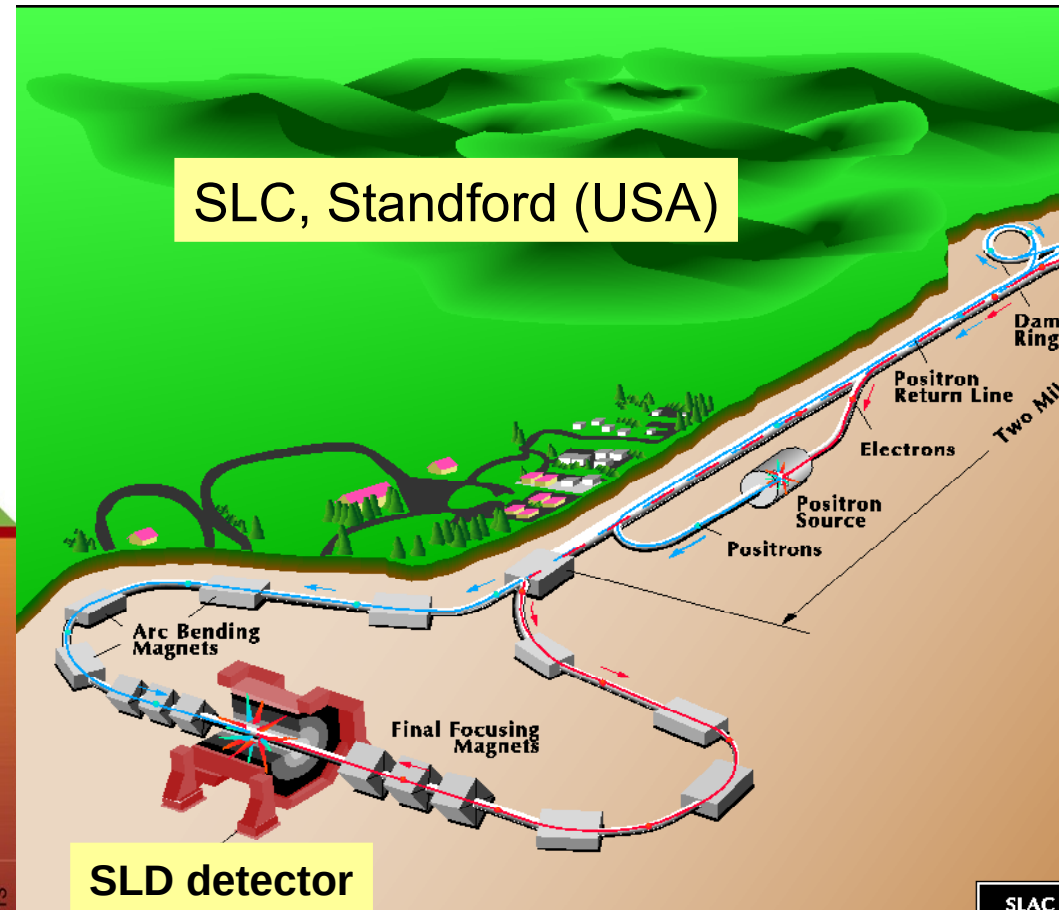
The resonance $e^+e^- \rightarrow Z$

- In the first half of the 90s the resonant production process: $e^+e^- \rightarrow Z \rightarrow f\bar{f}$ was studied in great detail by 4 dedicated experiments (ALEPH, DELPHI, L3, OPAL) at LEP (“Large Electron Positron collider”, CERN, Geneva) and by the SLD experiment at the linear accelerator SLC (“Stanford Linear Collider, with polarized beams) in USA

LEP, CERN



SLC, Stanford (USA)

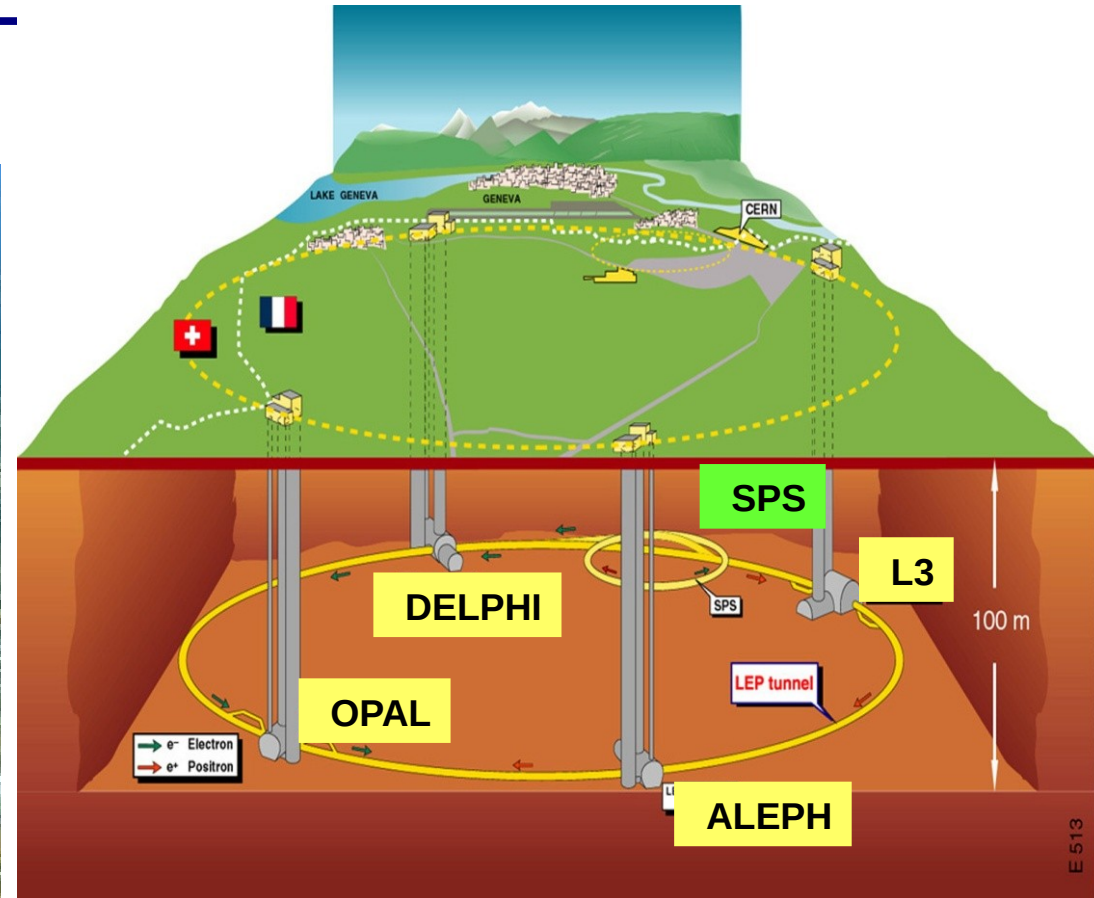


SLD detector

LEP: collider and detectors



Ring length: 27 km
Energy range: 20 – 104.5 GeV



4 interaction points
(=> experiments)

Initial beams energy:
22 GeV from SPS



LEP: collider

Parameter	Symbol	Value
Effective bending radius	ρ	3026.42 m
Revolution frequency	f_{rev}	11245.5 Hz
Length of circumference, $L = c/f_{\text{rev}}$	L	26658.9 m
Geometric radius ($L/2\pi$)	R	4242.9 m
Radio frequency harmonic number	h	31320
Radio frequency of the RF-system, $f_{\text{RF}} = h f_{\text{rev}}$	f_{RF}	352 209 188 Hz

Energy loss by synchrotron radiation
per turn :

$$U_0 \propto \frac{E^4}{\rho}$$



Example :

at $E_{\text{beam}} = 104 \text{ GeV} \sim 3\% \text{ of the beam energy}$

Large curvature radius.

However:

$V_{\text{rf}} \sim 3.6 \text{ GV}$ at 104 GeV.

the biggest RF system in the world

LEP: collider

1280 RF cavities

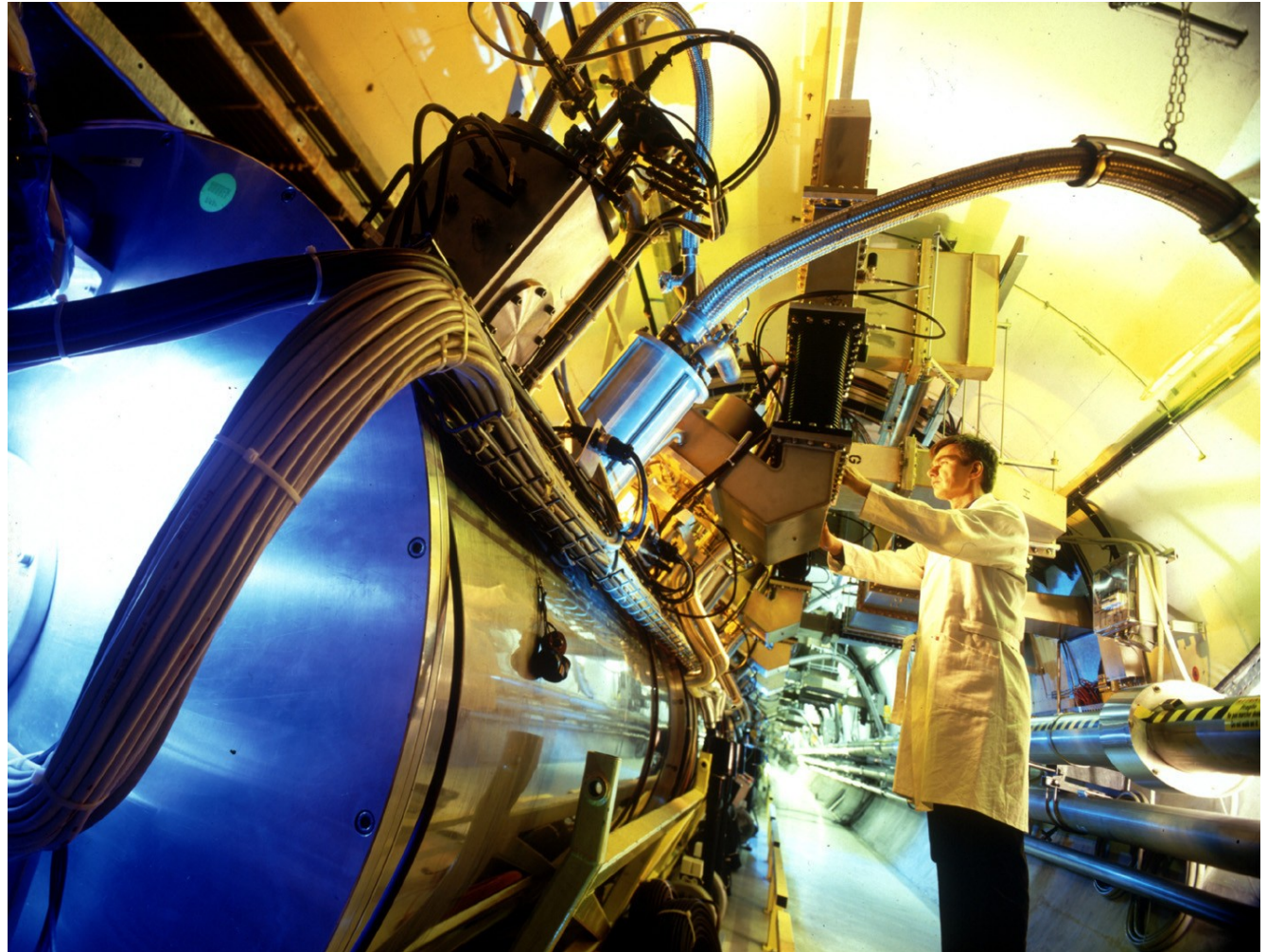
160 MWatt : delivered power
at the maximum energy
(104 GeV)

$$P_{sc} \propto I_{tot} U_0 \propto \frac{E_b^4}{E_0^4} \frac{I_{tot}}{\rho}$$

($E_0 = 0.511 \text{ MeV}$)

LEP1: copper cavity

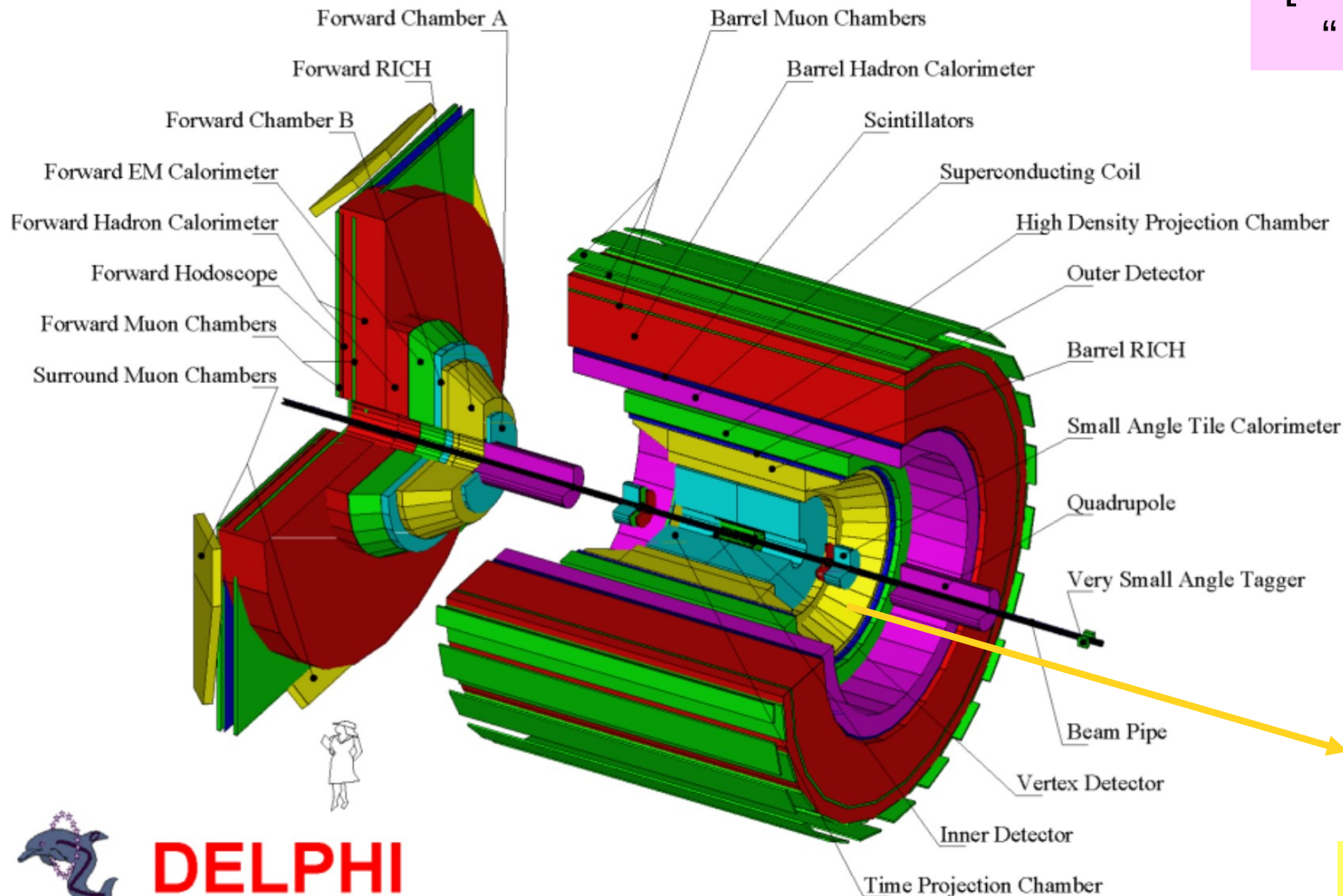
LEP2: superconductive cavity



LEP: detectors

DELPHI: DEtector with L Lepton P Photon H Hadron I Identification

[N.I.M. A303 (1991),233
“ A378(1996), 57]

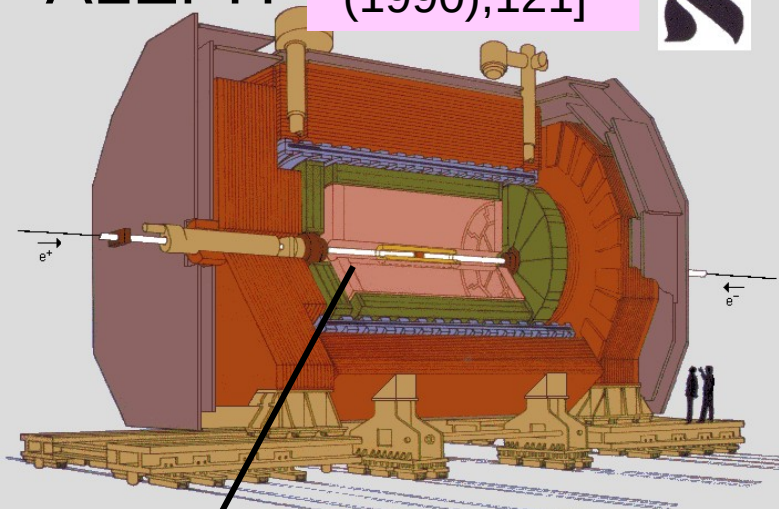


 **DELPHI**

weight to the particle
identification:
a dedicated detector:
Ring **I**maging**C**herenkov
[N.I.M. A323 (1992),351]

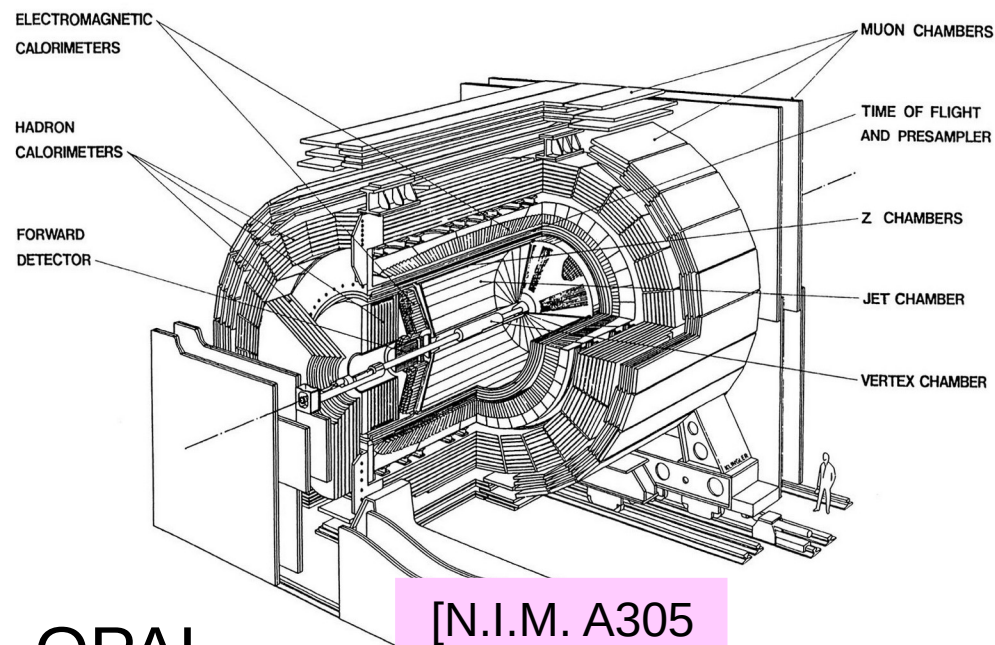
ALEPH

[N.I.M. A294
(1990),121]



- Vertex Detector
- Inner Tracking Chamber
- Time Projection Chamber
- Electromagnetic Calorimeter
- Superconducting Magnet Coil
- Hadron Calorimeter
- Muon Chambers
- Luminosity Monitors

the largest
Time Projection Chamber ever built



[N.I.M. A305
(1991),275]

OPAL

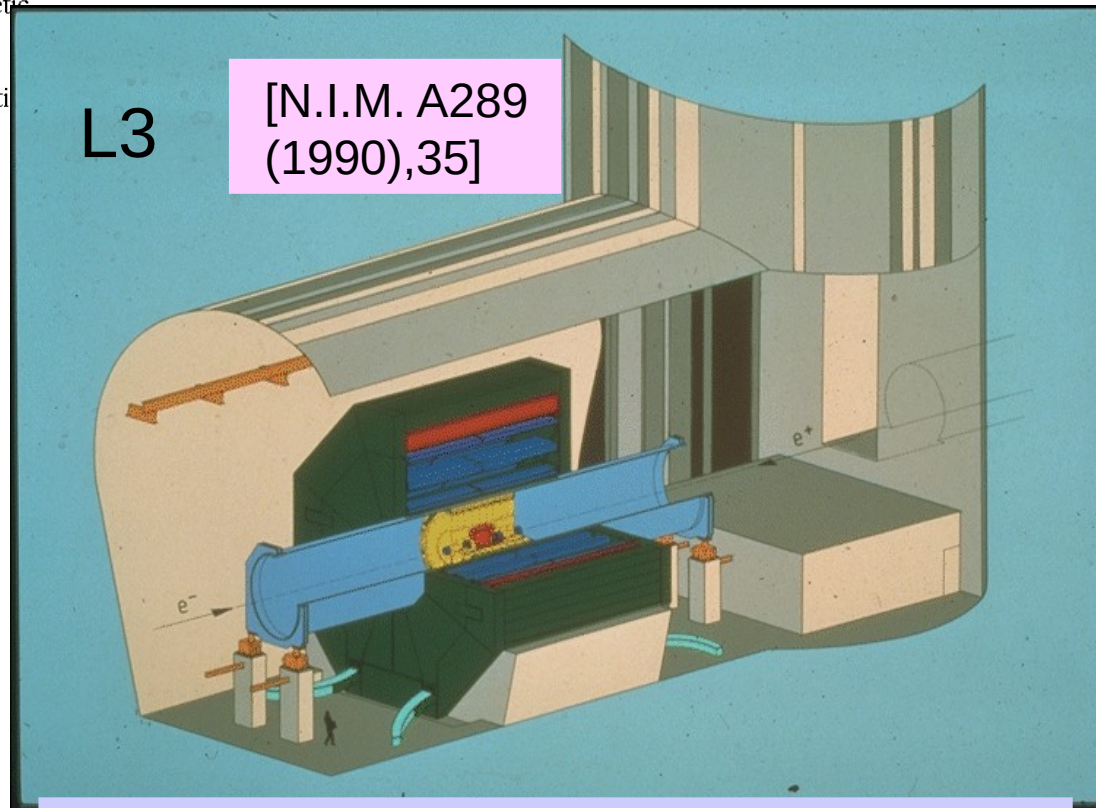
OPAL-ENSEMBLE EN PERSPECTIVE
JAN. 91 1991

OPAL: The OPAL Detector (an Omni Purpose Apparatus for Lep)

LEP: detectors

L3

[N.I.M. A289
(1990),35]

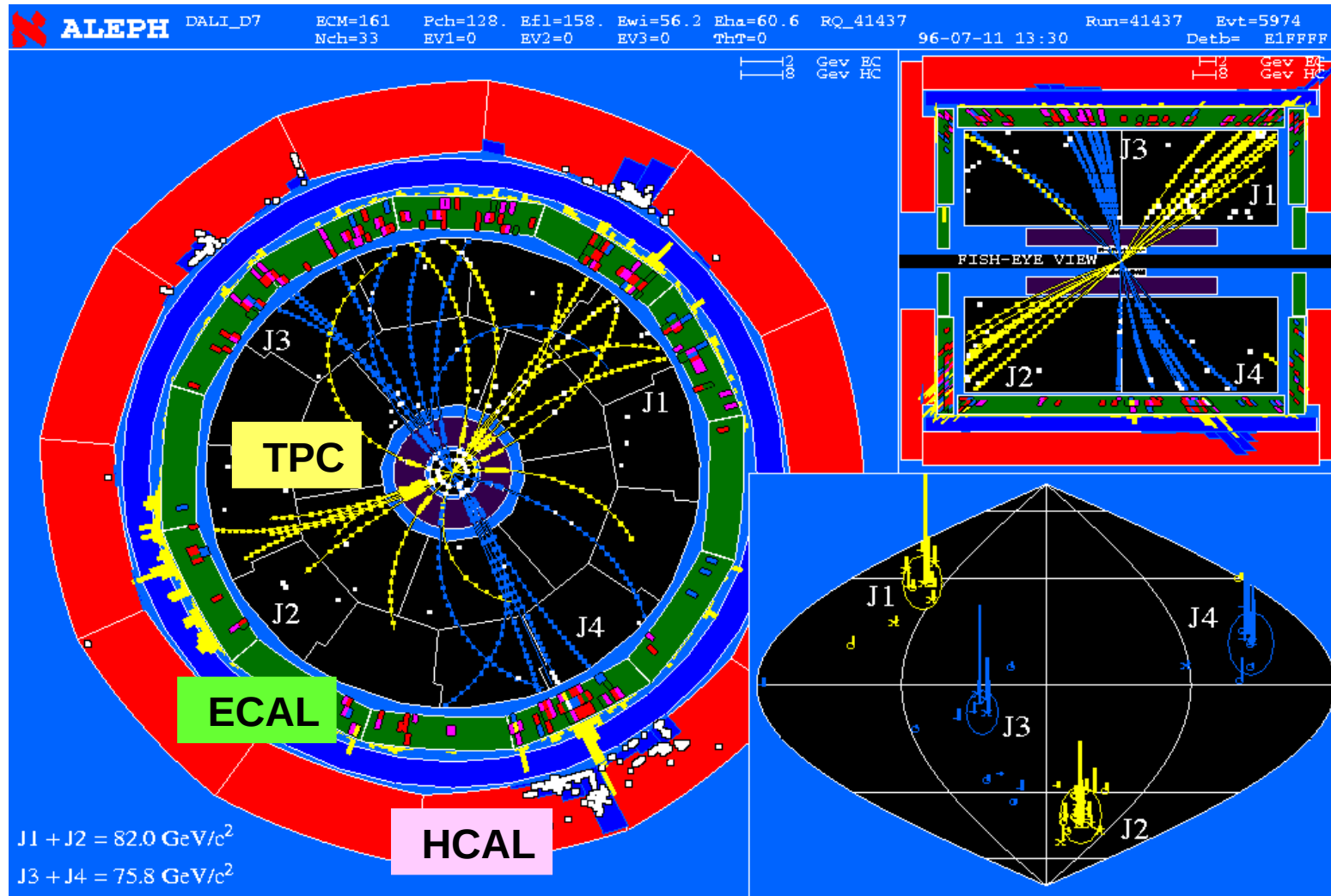


weight on the precise measurement of the leptons:
high resolution e.m. calorimeter
(BGO crystals),
open air muon spectrometer

'Subnuclear Physics

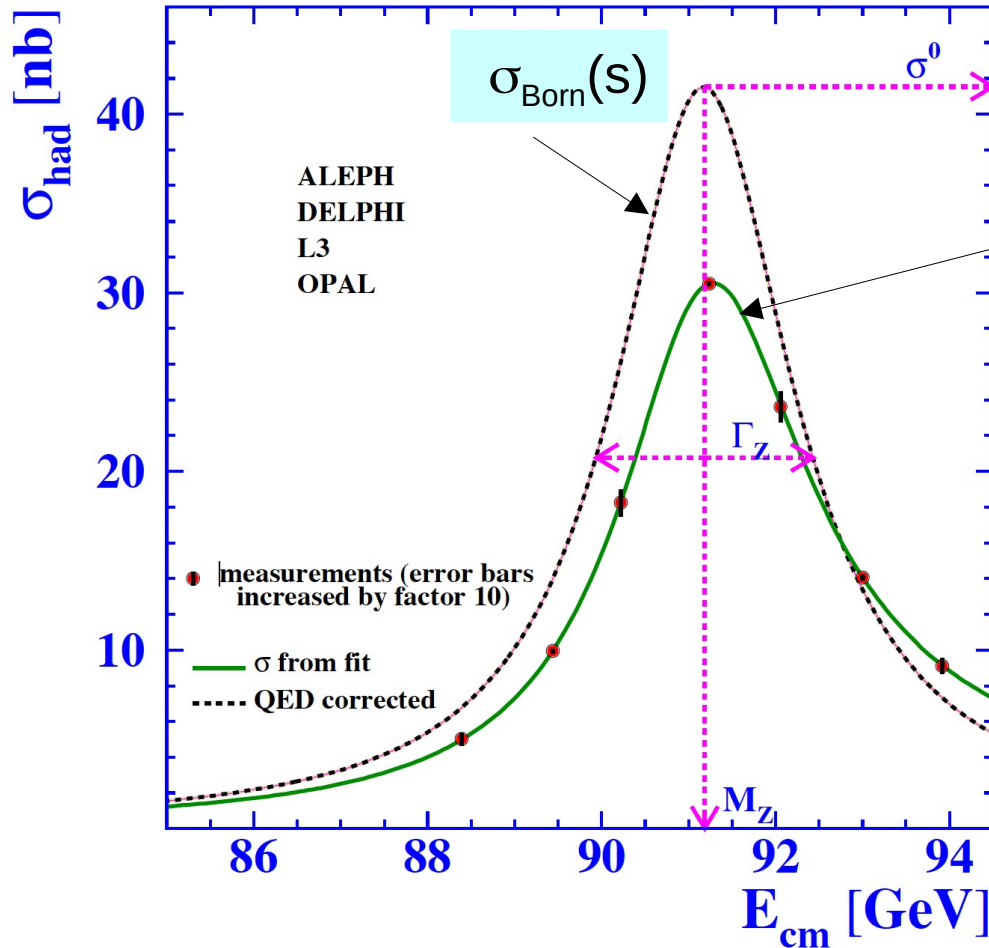
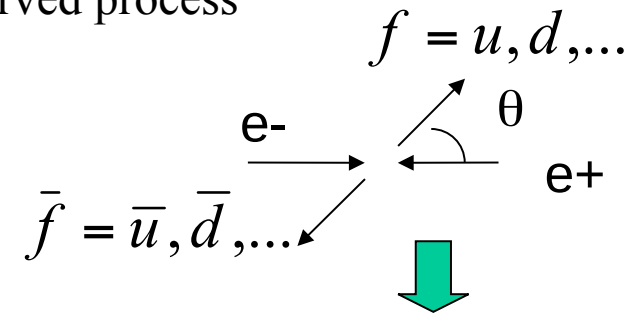
LEP: detectors

Event $ee \rightarrow WW \rightarrow 4\text{jets}$ in ALEPH ($\sqrt{s} = 161 \text{ GeV}$)



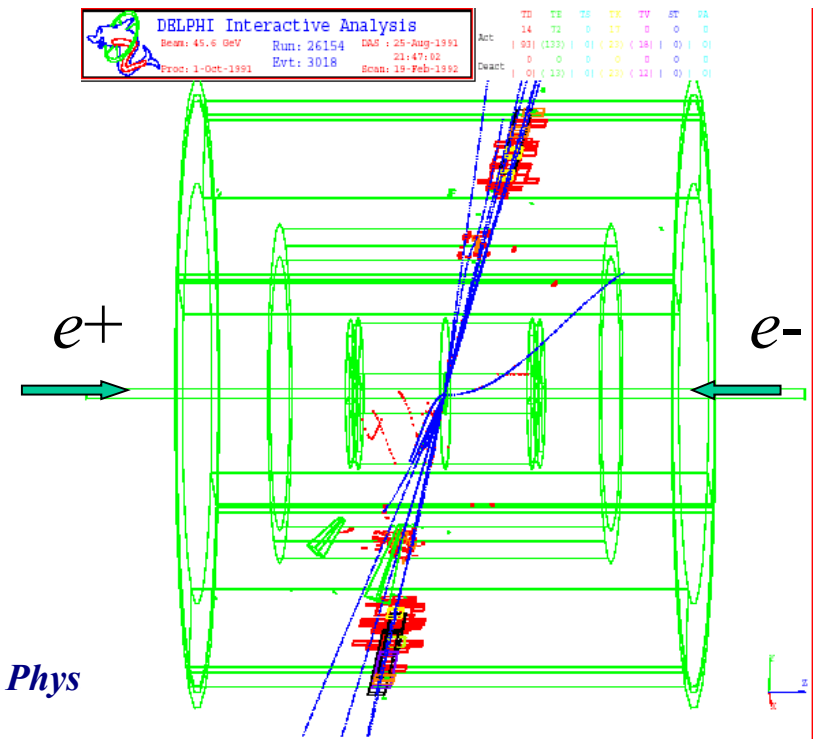
The Z resonance

When the final state is a quark-antiquark pair ($f=u,d,s,c,b$) the observed process is: $e^+e^- \rightarrow \text{hadrons}$ due to the hadronization process of the quarks



observed cross section for the process:

$e^+e^- \rightarrow \text{hadrons}$



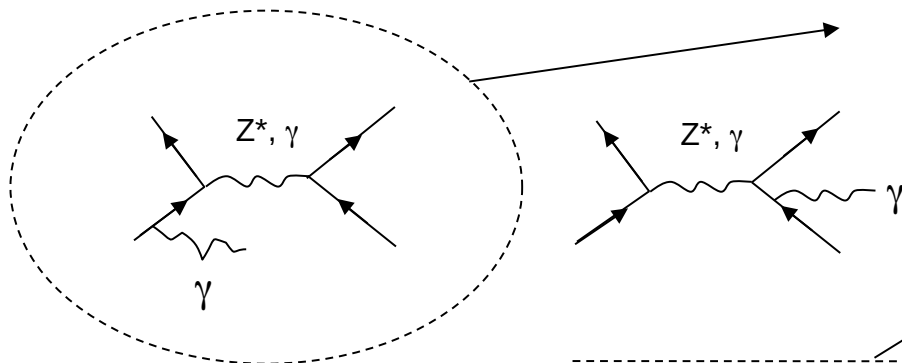
The Born cross section has to be highly modified to describe the experimental results

Experimental Subnuclear Phys

The Z resonance

- The radiative corrections **highly modify** the predictions at “tree level”:

- Photonic corrections (pure QED) :

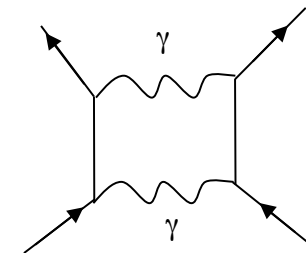


Initial State Radiation

(important effect: it lowers the total cross section of ~30% + peak shift ($O(100)$ MeV))

“vacuum polarization”: $\alpha \rightarrow \alpha(q^2)$

“vertex correction”



“box diagram”

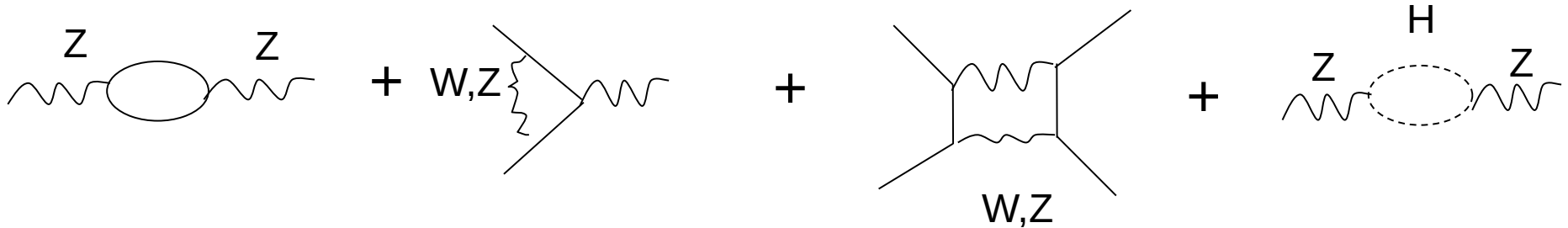
interference between initial state and final state radiation + “box” diagram of pure QED

$$\sigma_{Born}(s) \equiv \int_{\Omega} \left(\frac{d\sigma}{d\Omega} \right)_{Born} d\Omega \quad \Rightarrow \quad \sigma(s) = \int_0^1 \left[\sigma_{Born}(s'=sz) H(s,z) + \Delta(s,z) \right] dz$$

radiation function of initial state radiation (pure QED)

The Z resonance

● Non photonic corrections (theoretical model dependent):



➡ sensitive to new physics, and to the “unknown” parameters of the model, e.g. M_{top} , M_{Higgs}

➡ small effects (of the order of %)

● “IMPROVED BORN APPROXIMATION”:

The vertex photonic corrections and those of vacuum polarization + the NON photonic corrections are absorbed in $\sigma_{\text{Born}}(s, M_Z, \alpha, \Gamma_Z, \Gamma_f)$ with the substitutions:

$$\alpha \rightarrow \alpha(M_Z^2) = \alpha / (1 - \Delta\alpha) \approx 1.064 \alpha = 1/128$$

$$\Gamma \rightarrow \Gamma(s) = s\Gamma / M_Z^2$$

$$\Gamma_f(g_V, g_A) \rightarrow \Gamma_f = \frac{G}{\sqrt{2}} \frac{M_Z^3}{6\pi} (g_{Vf}^2 + g_{Af}^2) N_{\text{col}}$$

“effective” coupling constants, calculable inside a specific model

Standard Model: radiative corrections

In the Standard Model:

$$\Gamma_f = \frac{G_F m_Z^3}{6 \pi \sqrt{2}} (g_{Vf}^2 + g_{Af}^2) N_{col}$$

$\overline{g}_{Af} = \sqrt{\rho_f} I_{3f}$

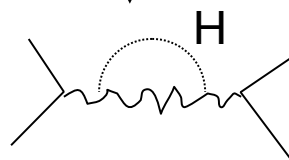
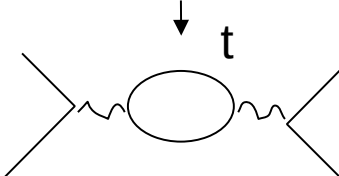
“effective Weinberg angle”

$$\overline{g}_{Vf} = \sqrt{\rho_f} (I_{3f} - 2Q_f \sin^2 \overline{\theta}_W)$$

$1 + \Delta\rho$

$(1 + \Delta\rho / \tan^2 \theta_W) \sin^2 \theta_W$

$$\frac{\alpha(M_Z)}{\pi} \frac{m_t^2}{m_Z^2} - \frac{\alpha(M_Z)}{4\pi} \log \left(\frac{m_H^2}{m_Z^2} \right) + \dots$$



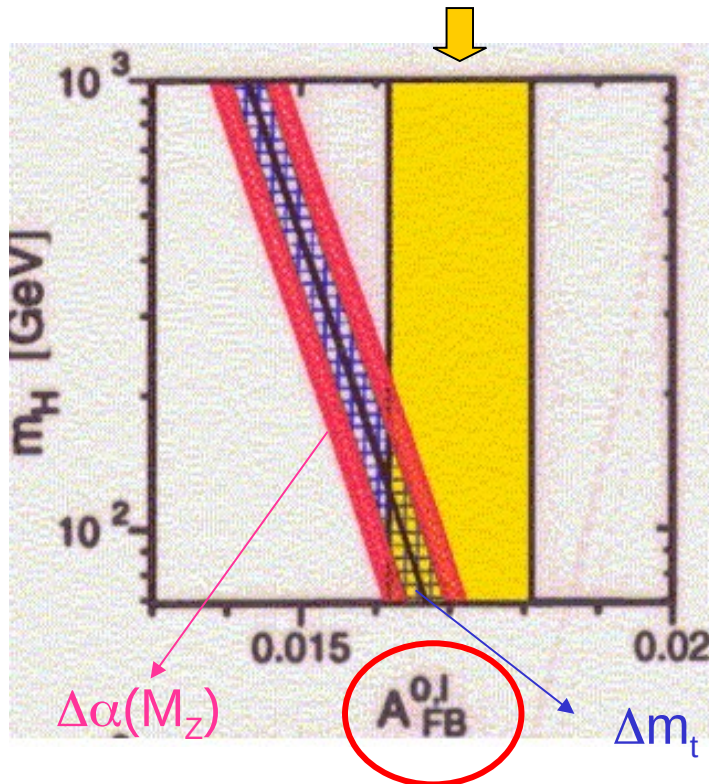
$$\equiv 1 - \frac{m_W^2}{m_Z^2}$$

Standard Model: radiative corrections

asymmetry at the peak: $s=M_Z^2$

$$A_{FB}^{0,f} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} A_e A_f = \frac{3}{4} \frac{g_{Ve} g_{Ae}}{g_{Ve}^2 + g_{Ae}^2} \frac{g_{Vf} g_{Af}}{g_{Vf}^2 + g_{Af}^2}$$

LEP : 0.01714 ± 0.00095



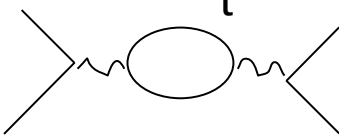
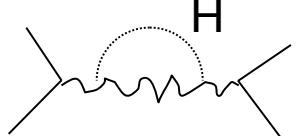
$$\overline{g_{Af}} = \sqrt{\rho_f} I_{3f}$$

$$\overline{g_{Vf}} = \sqrt{\rho_f} (I_{3f} - 2Q_f \sin^2 \overline{\theta_W})$$

“effective Weinberg angle”

e.g.

$$\frac{\alpha(M_Z)}{\pi} \frac{m_t^2}{m_Z^2} - \frac{\alpha(M_Z)}{4\pi} \log \left(\frac{m_H^2}{m_Z^2} \right) + \dots$$






$$\equiv 1 - \frac{m_W^2}{m_Z^2}$$


Precision Measurements at LEP

- The fits using the “**precise measurement**” at the Z resonance give a very accurate determination of:
 - M_Z ,
 - Γ_Z total width,
 - $\Gamma_{e,\mu,\tau,\nu,\text{quarks}}$ partial widths
 - asymmetries and consequently the theory coupling constants g_{Vf} , g_{Af} .

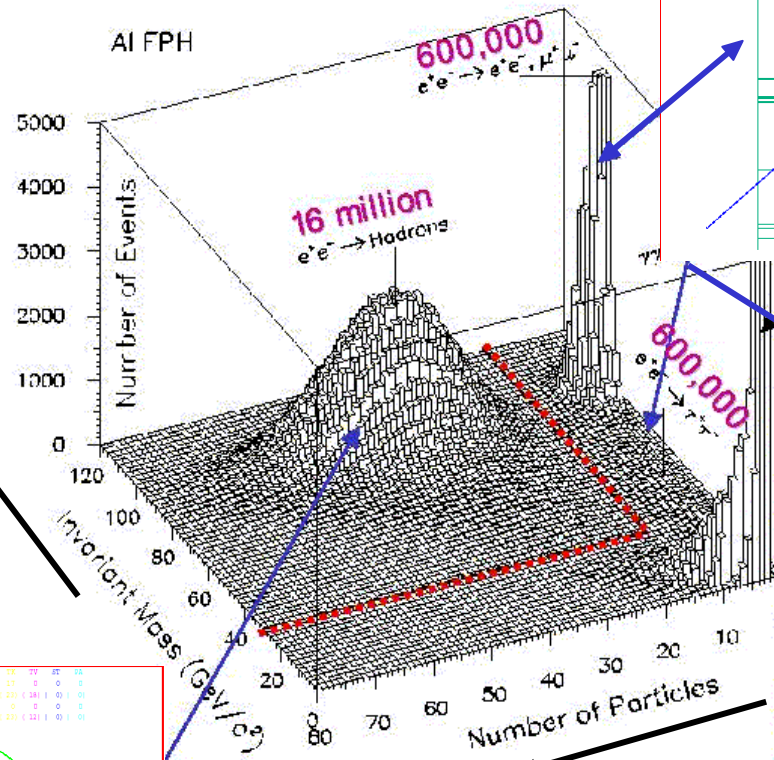
“Ingredients”:

- i) **counting** of the hadronic and leptonic events  high statistics
- ii) precise calculation of the **radiative effects**
(initial state, QED, final state, QCD)  theory
($\delta\sigma_{\text{peak}}=30\%$, $\delta M_Z \approx 200 \text{ MeV}$)
- iii) **relative luminosity** between different points
- iv) **beam energies**

 very good “luminometers”: $\Delta\mathcal{L}/\mathcal{L} \cong 0.1\%$

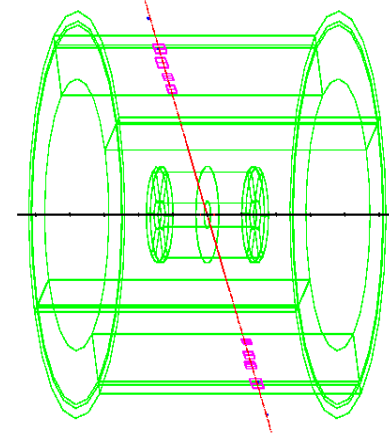
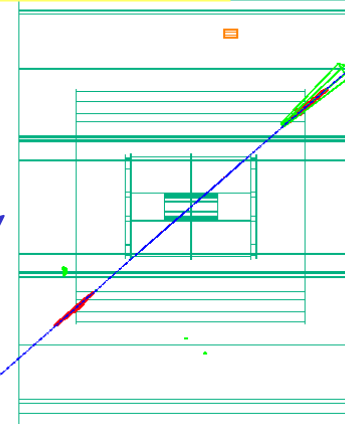
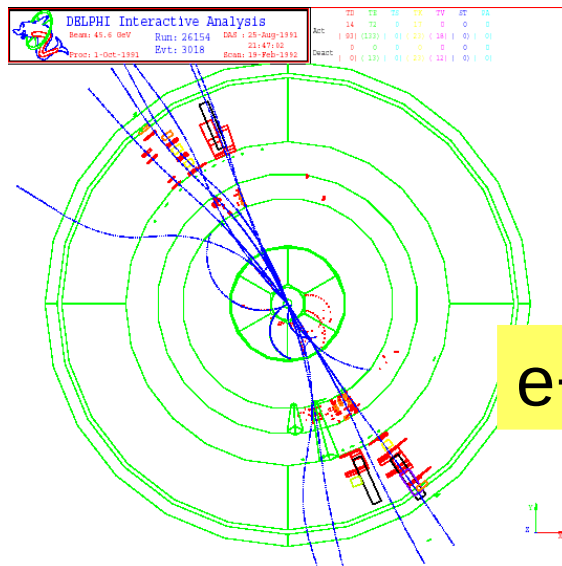
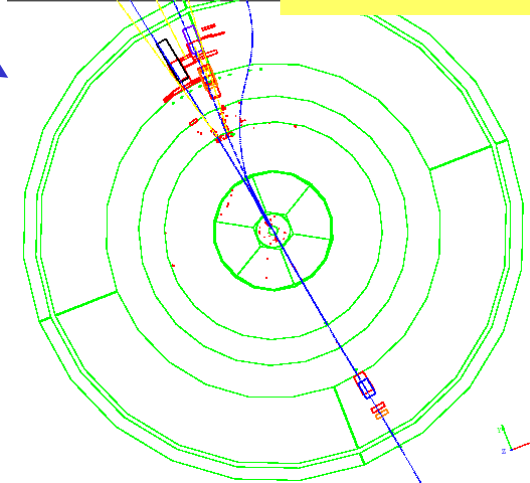
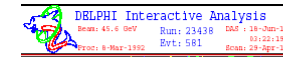
 Precise measurement with the method of the
“resonant depolarization”: $\Delta E_{\text{int.point}} \cong 2 \text{ MeV}$

Processes $e^+e^- \rightarrow ff$ at LEP

$$e^+e^- \rightarrow e^+e^-$$
$$e^+e^- \rightarrow \mu^+\mu^-$$


Invariant Mass of the system $f\bar{f}$

Numbers of particles

$$e^+e^- \rightarrow \text{hadrons}$$

$$e^+e^- \rightarrow \tau^+\tau^-$$


Hadronic and leptonic cross sections at LEP

E. Phys. J. C16(2000)371

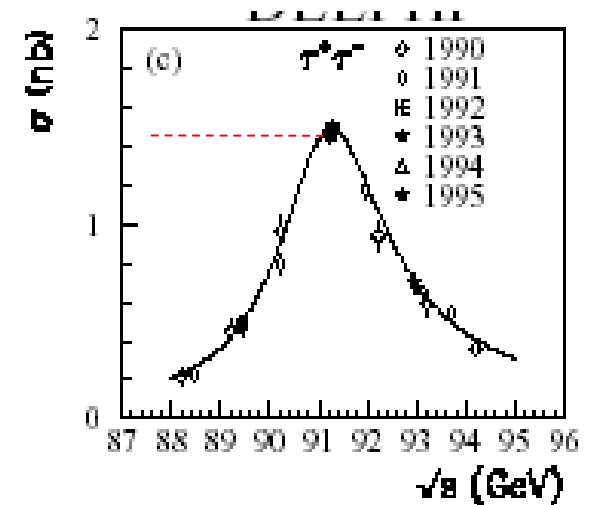
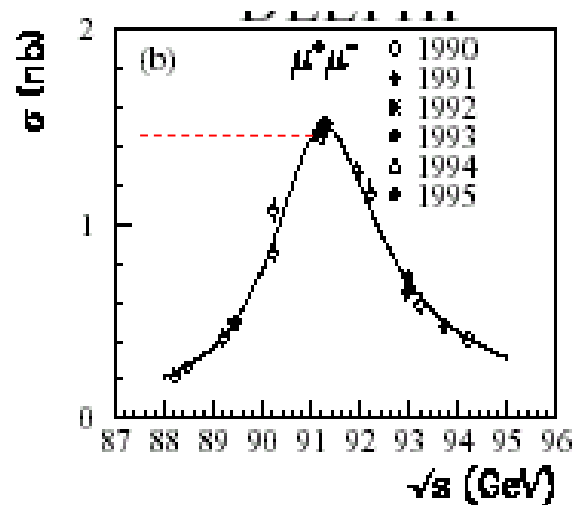
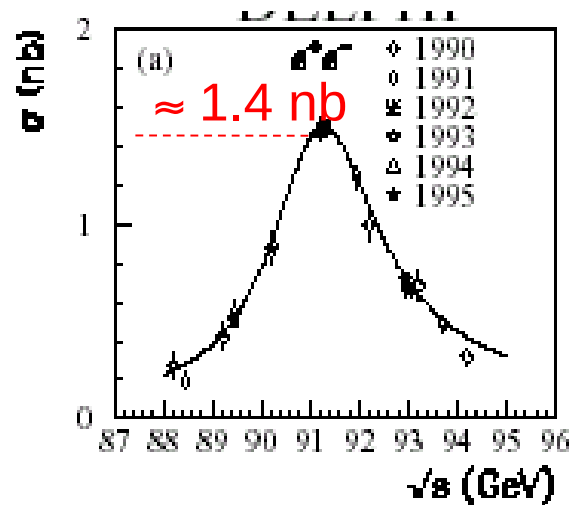
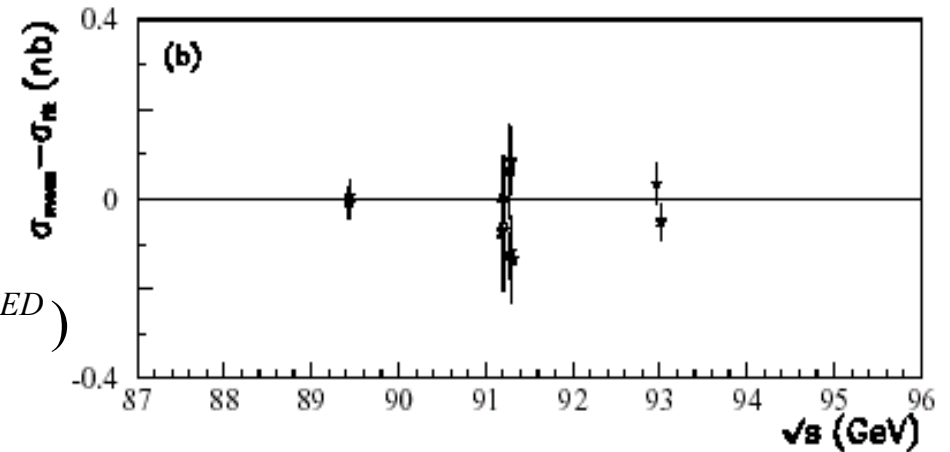
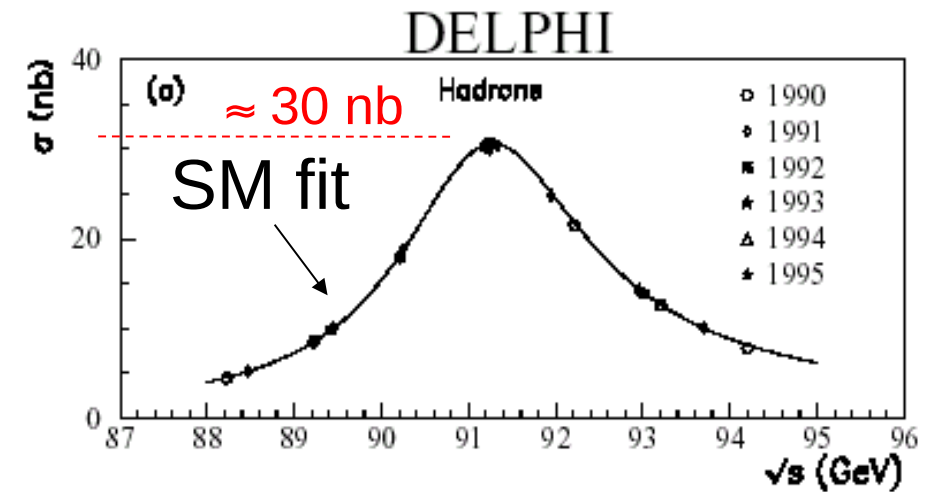
interf. between initial and final state radiation (QED)

$$\sigma(s) = \int_0^1 \left[\sigma_{\text{Born}}(s'=sz) H(s,z) + \Delta(s,z) \right] dz$$

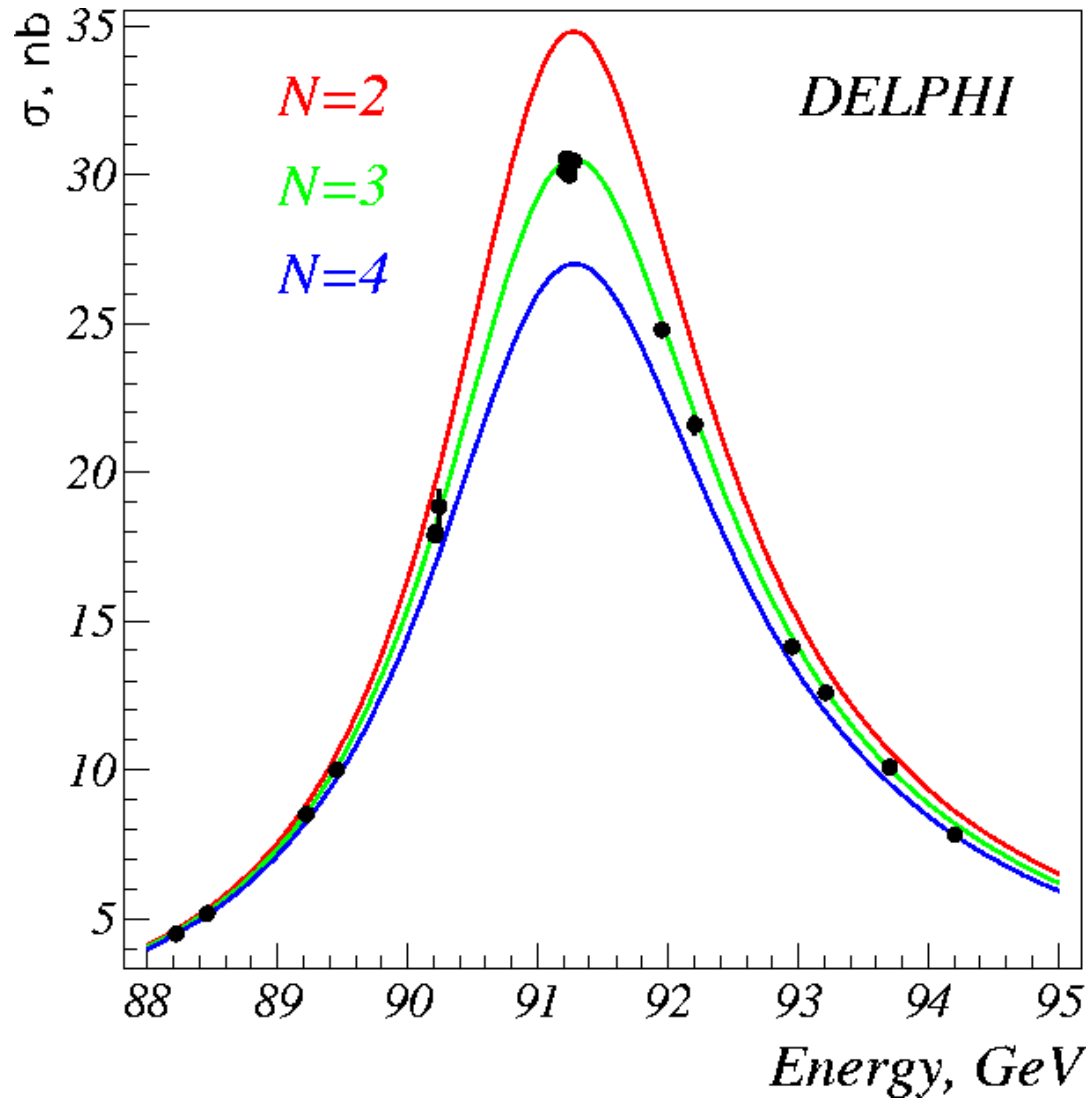
$$= \sigma_0 \frac{s' \Gamma_Z^2}{(s' - M_Z^2)^2 + (s'^2 / M_Z^2) \Gamma_Z^2} + \sigma_{\gamma Z} + \sigma_{\gamma}$$

interf. γZ pure QED

$$\sigma_0 = \frac{12\pi \Gamma_e \Gamma_{\text{hadr}}}{M_Z^2 \Gamma_Z^2} \quad \Gamma_f = \frac{G_F M_Z^3}{6\pi \sqrt{2}} (g_{Vf}^2 + g_{Af}^2) (1 + \delta_f^{\text{QED}})$$



Determination of the number of neutrinos



From the measurement of the total and partial width of the Z:

➡ $N_\nu = 2.9841 \pm 0.0083$

$\Gamma_{inv}^x = -2.7_{-1.5}^{+1.7} \text{ MeV}$



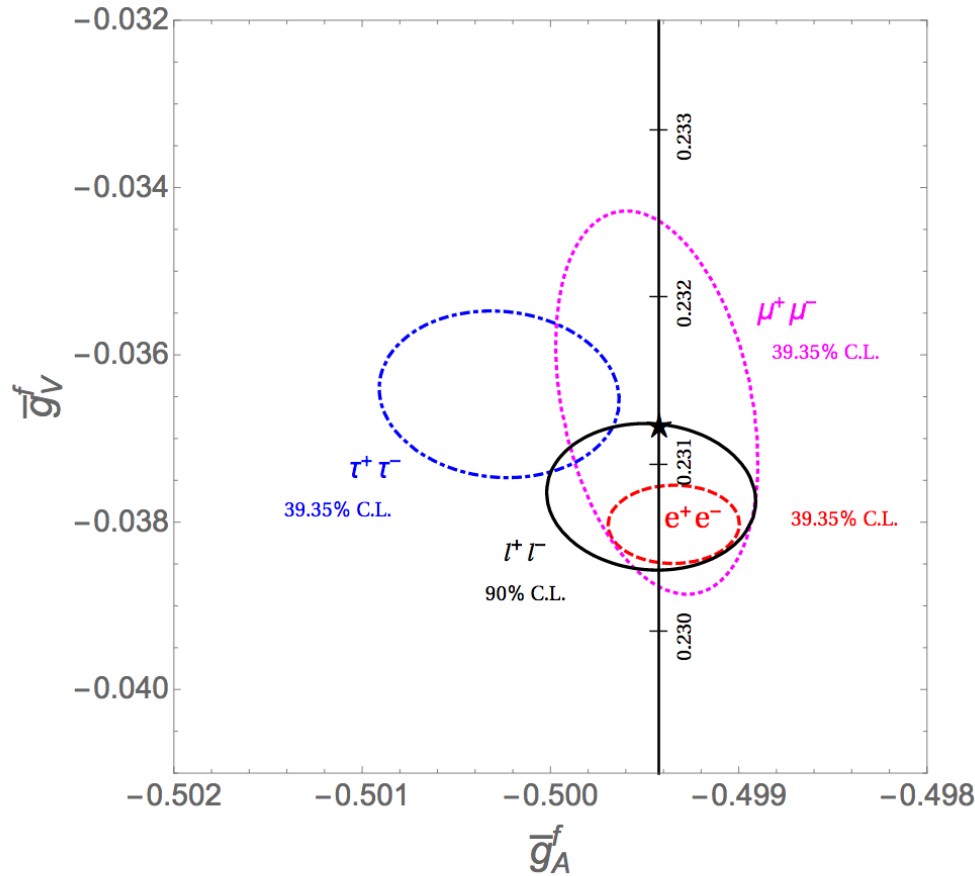
$$\Gamma_{inv} = \Gamma_Z - \Gamma_{had} - 3\Gamma_{lept} - 3\Gamma_\nu$$

(assuming, from the SM:

$$\frac{\Gamma_\nu}{\Gamma_\ell} = 1.990 \quad)$$

Leptonic Universality

- In the fit of the data the equality of the coupling constants of the Z to the fermions is not assumed; the universality, foreseen in the SM, is verified using the fit results:



effective \bar{g}_{VI} and \bar{g}_{AI} come from $\Gamma(Z \rightarrow ll)$ and from the asymmetries.

$$\bar{g}_{Af} = \sqrt{\rho_f} I_{3f}$$

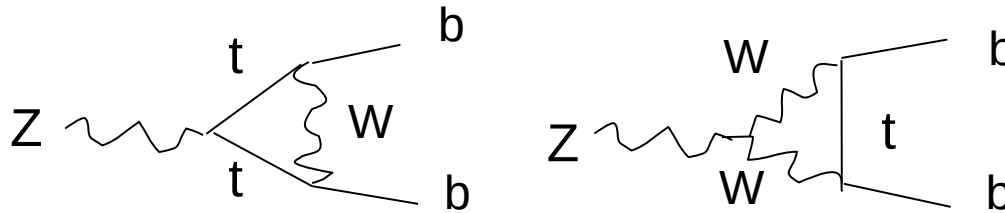
$$\bar{g}_{Vf} = \sqrt{\rho_f} (I_{3f} - 2Q_f \sin^2 \theta_W)$$

Figure 10.3: 1σ (39.35% CL) contours of the effective couplings \bar{g}_A^f and \bar{g}_V^f for $f = e, \mu$ and τ from LEP and SLC, compared to the SM expectation as a function of \hat{s}_W^2 . (The SM best fit value $\hat{s}_W^2 = 0.23121$ is also indicated.) Also shown is the 90% CL allowed region in $\bar{g}_{A,V}^e$ obtained assuming lepton universality.

$Z \rightarrow bb, cc$

- The study of the final states with heavy quarks are of the outmost importance. In particular the partial width Γ_b for the decay $Z \rightarrow bb$ has a dependence on M_{top} different from the other hadronic widths.

This is due to the fact that the diagrams:



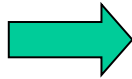
(negligible for all the other quarks due to the smallness of the elements of the CKM mixing matrix: $V_{qt} \ll V_{tb}$) they are no more negligible: $V_{tb} \sim 1$.

- ➡ The ratio $R_b = \Gamma_b / \Gamma_{\text{hadr}}$ is sensible to the top mass (and independent from α_s and vacuum polarization corrections);
if one knows with precision M_{top} , possible differences of R_b from the predicted value can be due to the presence of diagrams connected to new physics processes

b-tagging

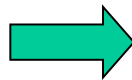
“tagging” of the events based on the characteristics of the *b* quarks:

$$m_B \cong 5 \text{ GeV}$$



- high p_T of the decay products (leptons in particular)
- higher p_L in the jets
- events with higher sphericity

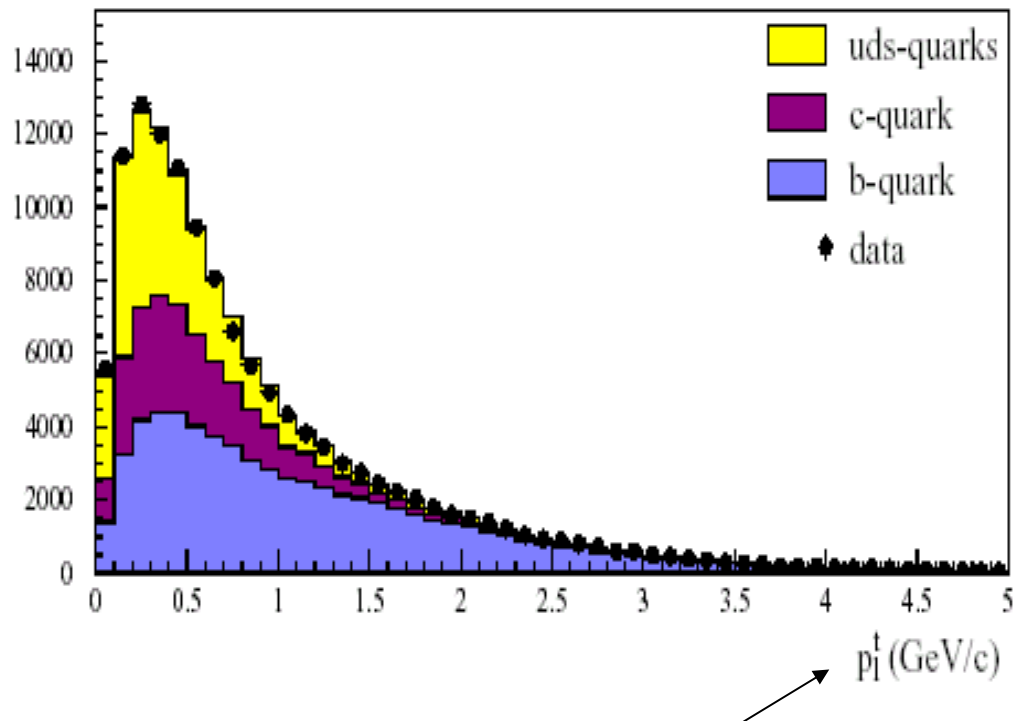
“long” mean life
($\tau_B \cong 10^{-12} \text{ s}$)



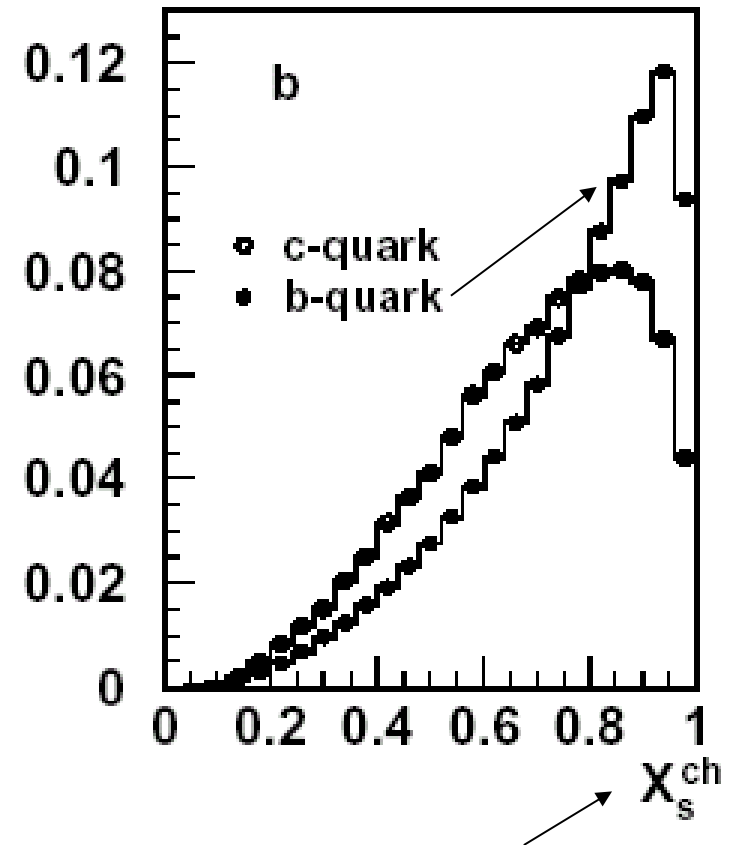
- secondary vertices measurable (at LEP : Lorentz boost $\cong 7$)
- tracks with high impact parameter compared to the primary vertex

Note: the *b*-tagging techniques will be very important for the search of the Higgs boson (which has a high coupling with the heavy quarks) and for the discovery of the top quark.

b-tagging: kinematic variables



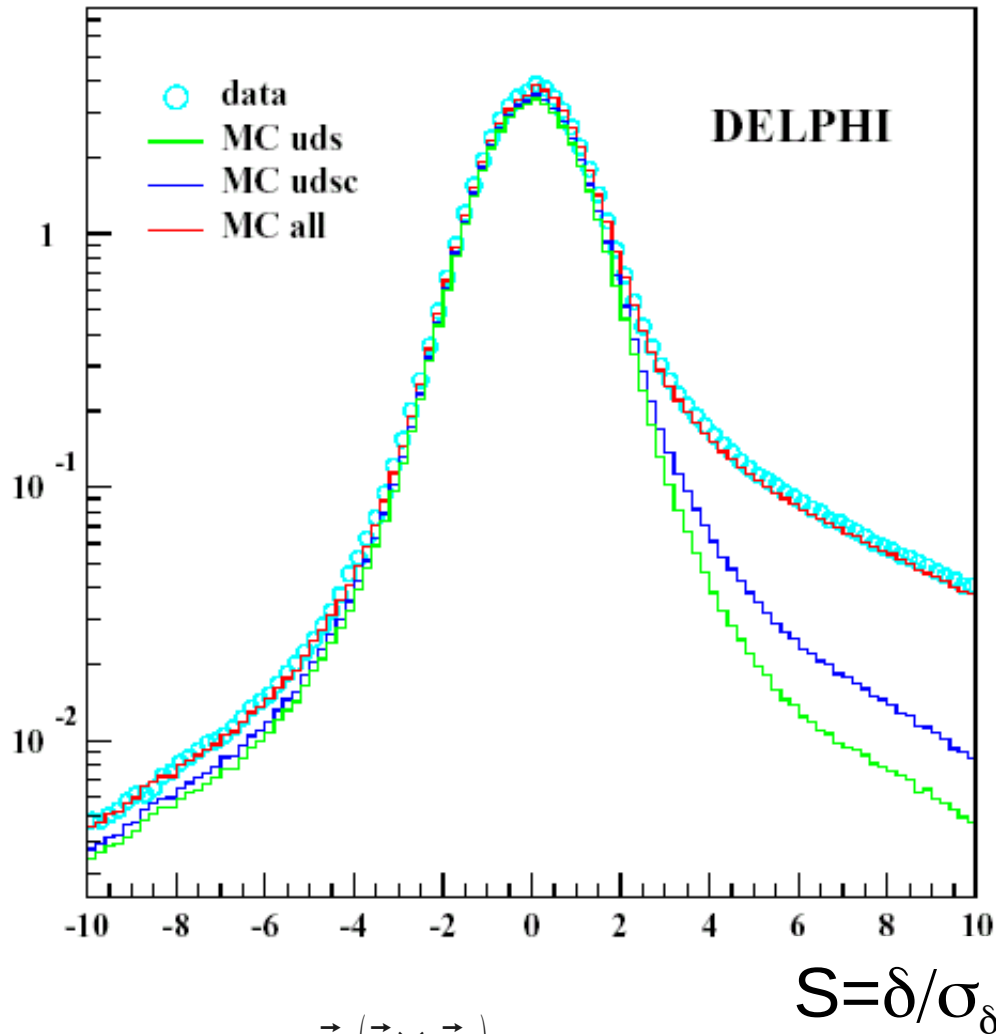
p_T of the lepton wth respect to the jet axis



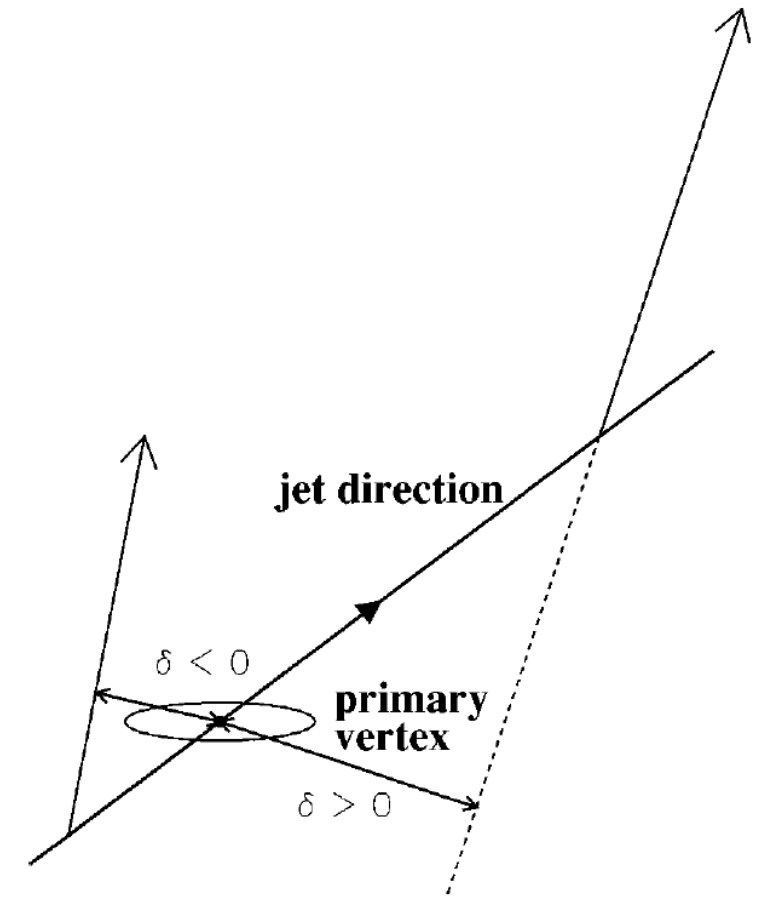
energy fraction of the jet
associated to the secondary vertices

b-tagging: impact parameter and its sign

Impact parameter significance

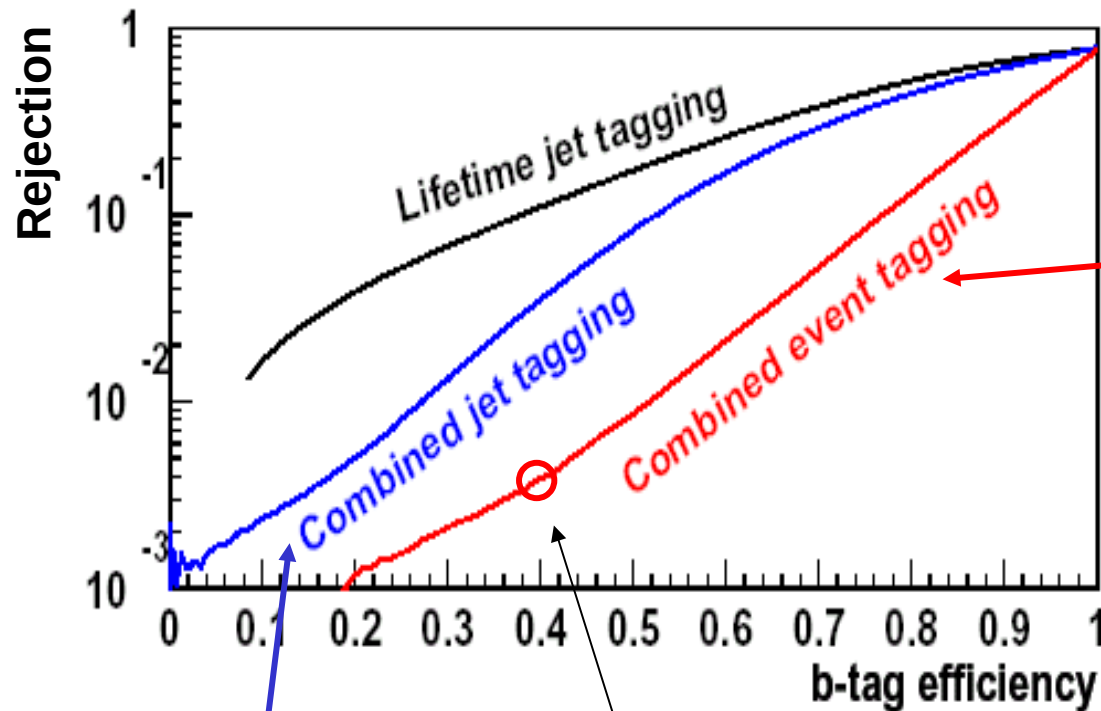


definition:
$$\delta = \frac{\vec{z} \cdot (\vec{r} \times \vec{p}_T)}{|\vec{p}_T|}$$



“impact parameter with sign”: δ

b-tagging: combined methods



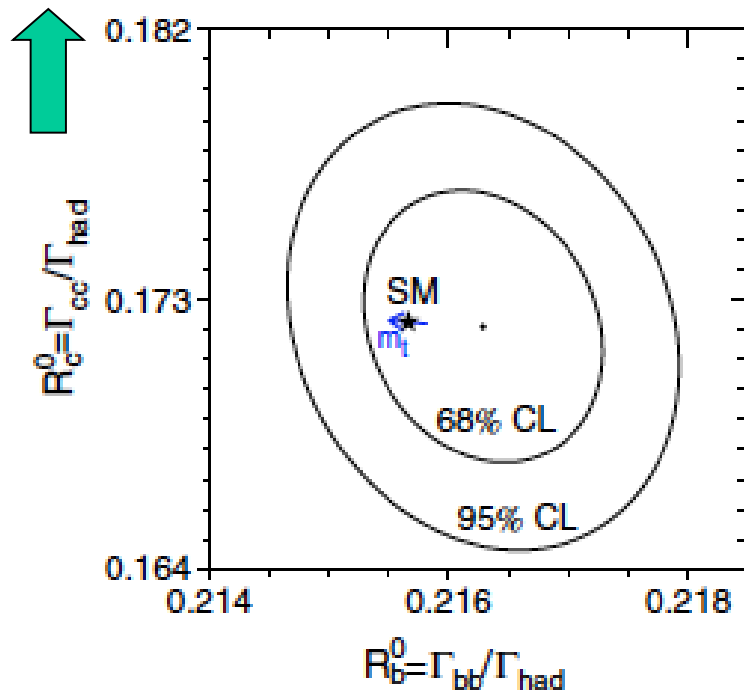
lifetime +
kinematic variable of the event

lifetime +
kinematic variables of the jet

Typical working point:
efficiency \approx 40%, rejection \approx 500

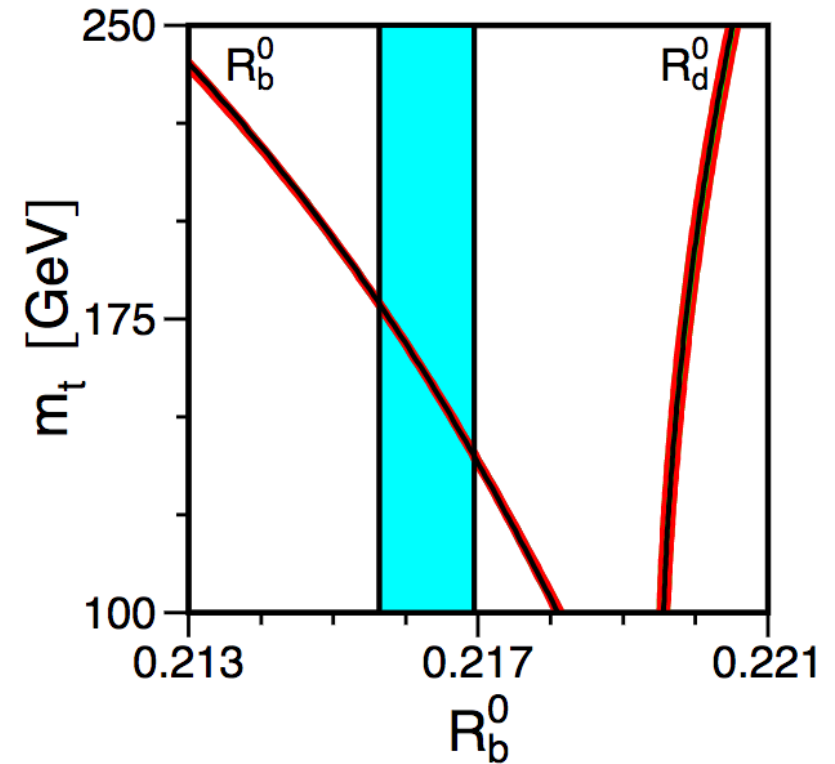
$Z \rightarrow bb, cc$

$$R_c = \frac{\Gamma_c}{\Gamma_h} = 0.1719 \pm 0.0031$$



$$\longrightarrow R_b = \frac{\Gamma_b}{\Gamma_h} = 0.21646 \pm 0.00065$$

M_{top} sensitivity:



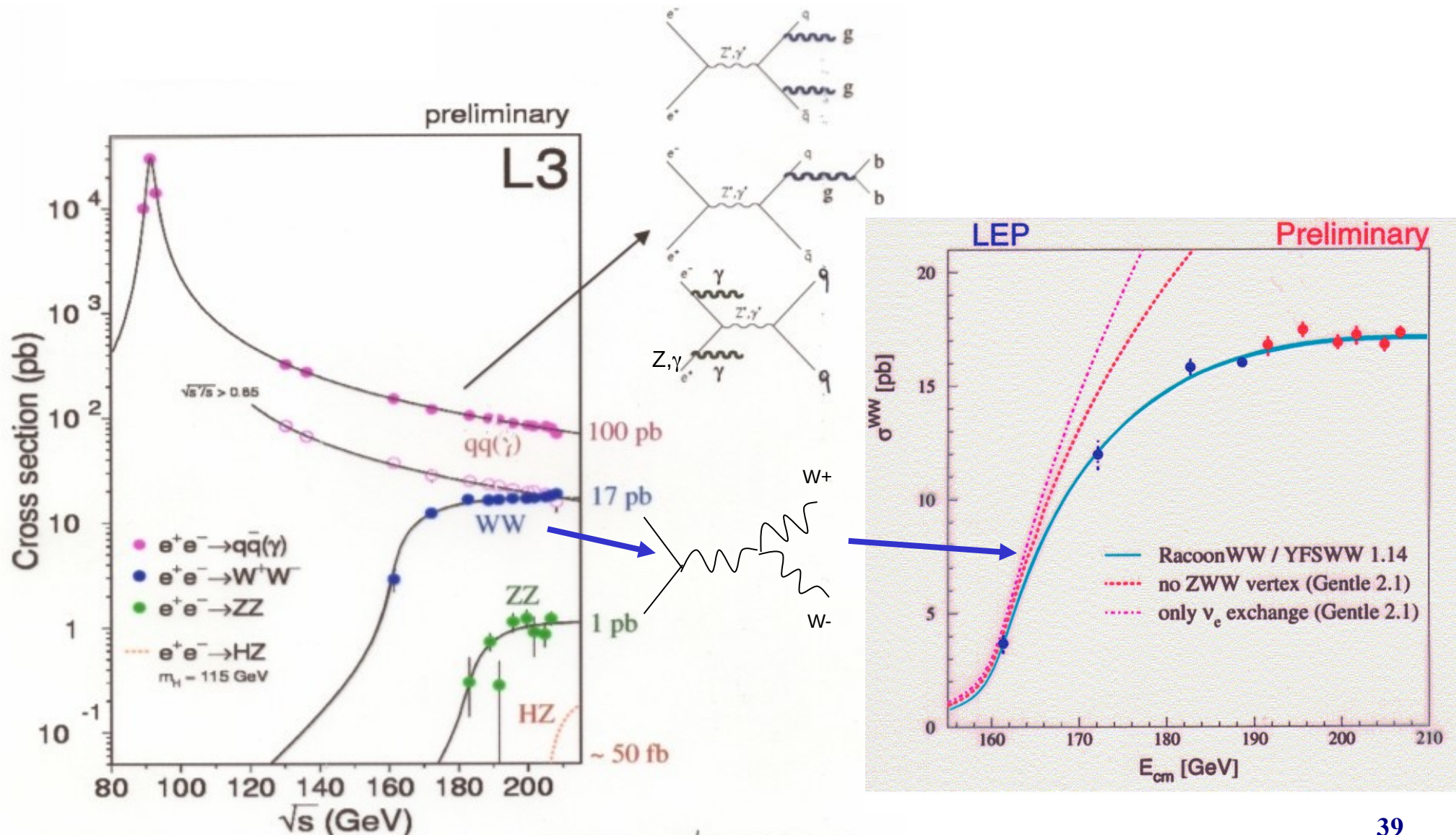
Results in agreement with the Standard Model; the predicted top mass (1993) is in agreement with the top mass discovered at Tevatron ($m=175 \pm 5$ GeV) few years after.

Beyond the Z: the physics of “LEP2”

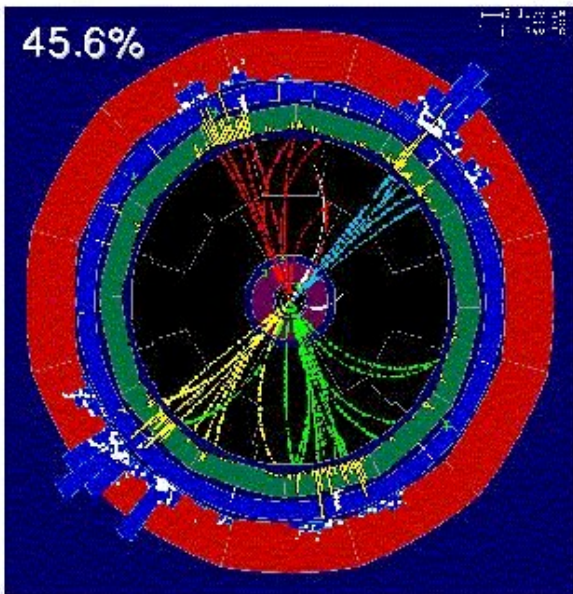
- In its second phase (“LEP2”: [1996-2000]), LEP was heavily modified, the beam energy was gradually increased up to a factor 2 (with also a slight increase of the luminosity)
- The goals were the following:
 - to go above the **threshold of the 2 W bosons production**: $2M_W \cong 160 \text{ GeV}$
and to study in detail the auto-interactions of the bosons, which is a characteristic aspect of the non-abelian structure of the electroweak gauge theory
 - to push to the maximum possible energies **to search for the Higgs boson**

Production of WW at LEP2

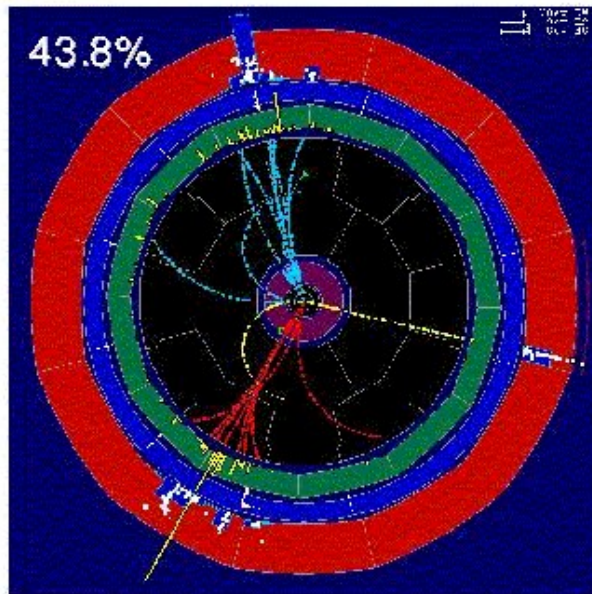
- Important test of the **auto-interaction of the bosons** as foreseen by the structure of the non-abelian gauge theory:



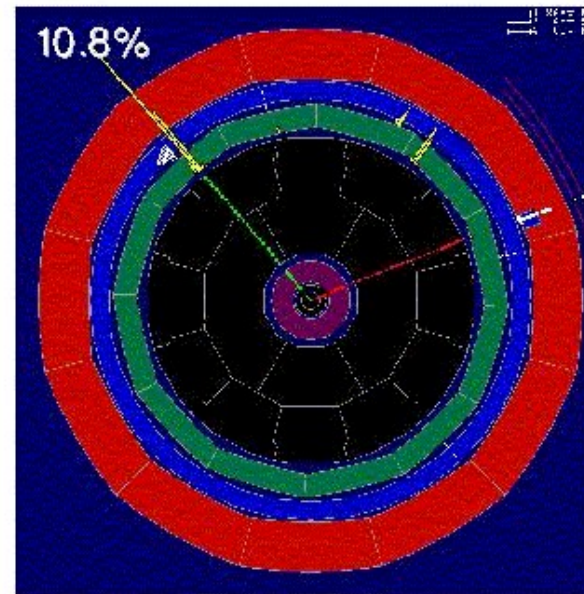
M_W at LEP2



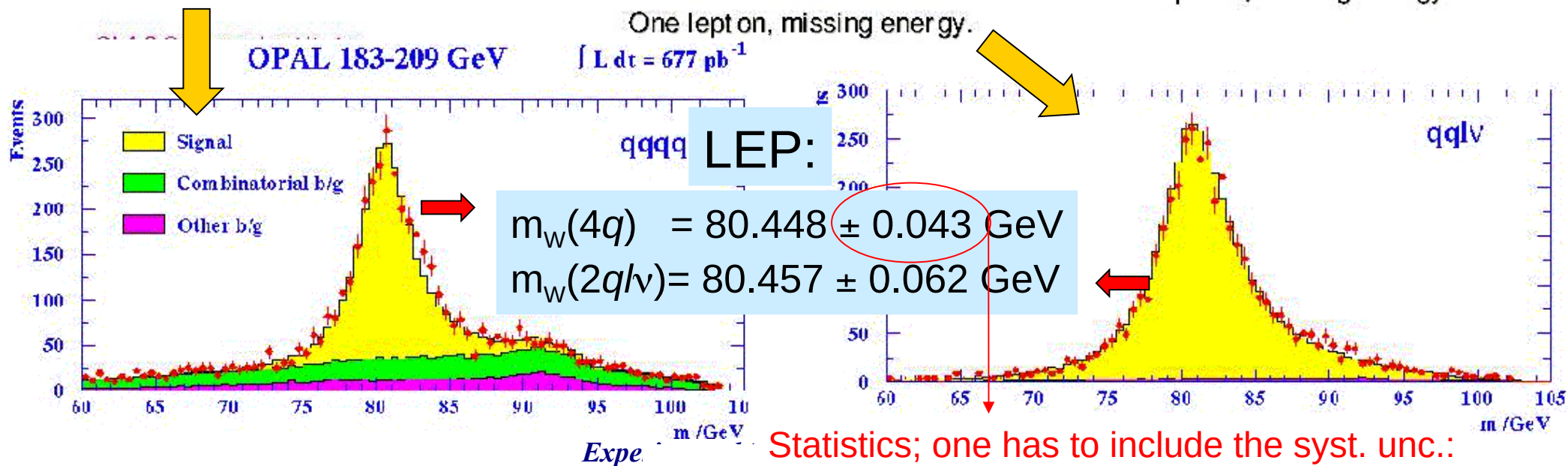
$W^+W^- \rightarrow q_1\bar{q}_2q_3\bar{q}_4$
Four well separated jets.



$W^+W^- \rightarrow q_1\bar{q}_2l\bar{\nu}$
Two hadronic jets,
One lepton, missing energy.

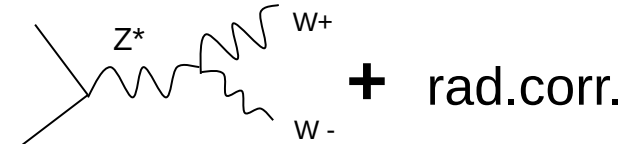
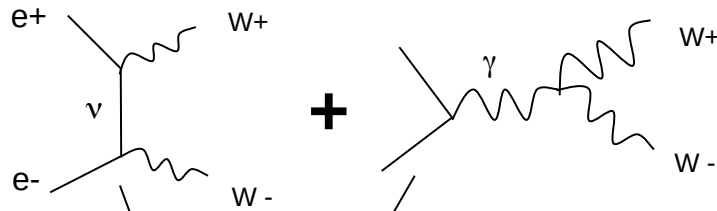


$W^+W^- \rightarrow l_1\nu_1l_2\bar{\nu}_2$
Two leptons, missing energy



Production of WW at LEP2

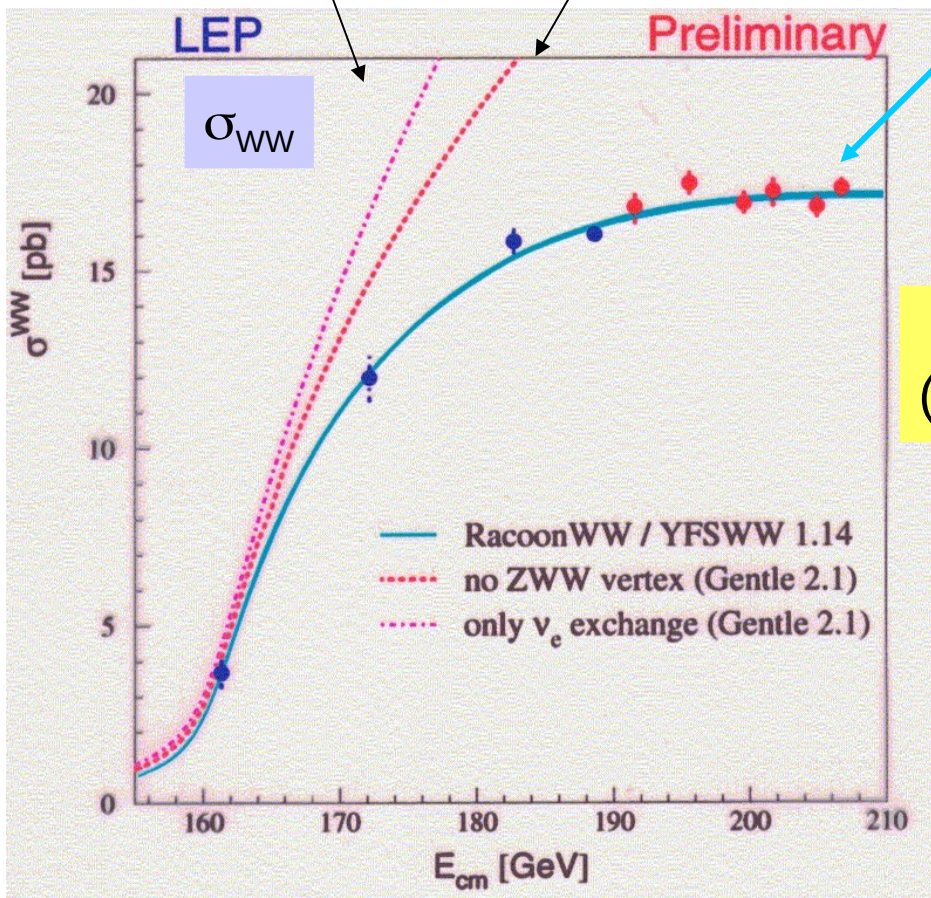
- Also well above the peak of the Z , the S.M. works very well:



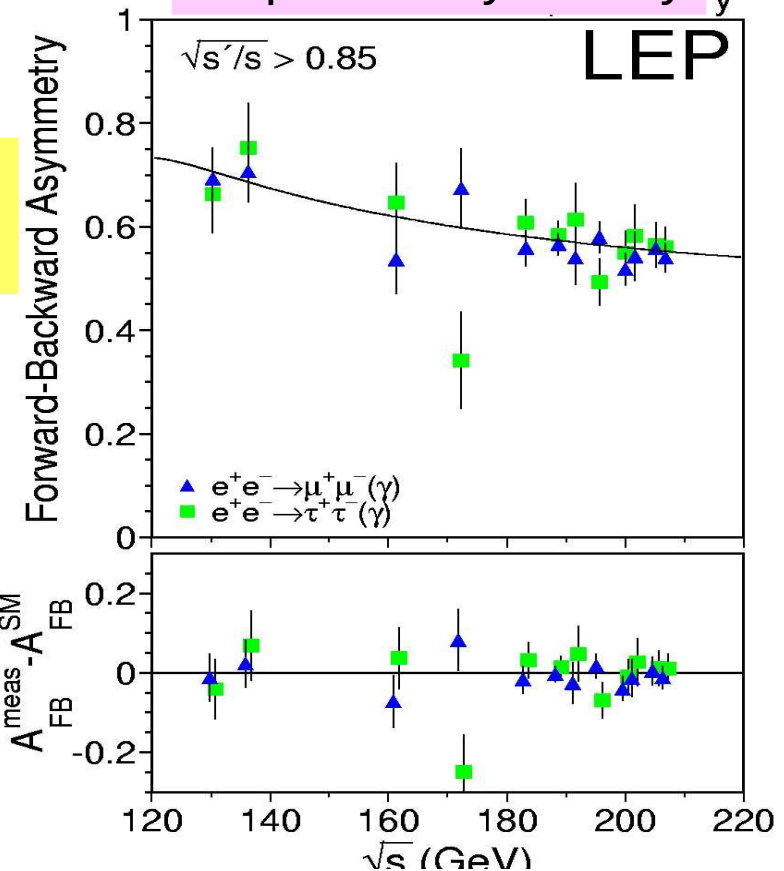
The **cancellations** foreseen by the gauge theory verified at the 1% level

Ad also:

no Z'
($m_{Z'} > 0.8 \text{ TeV}$)



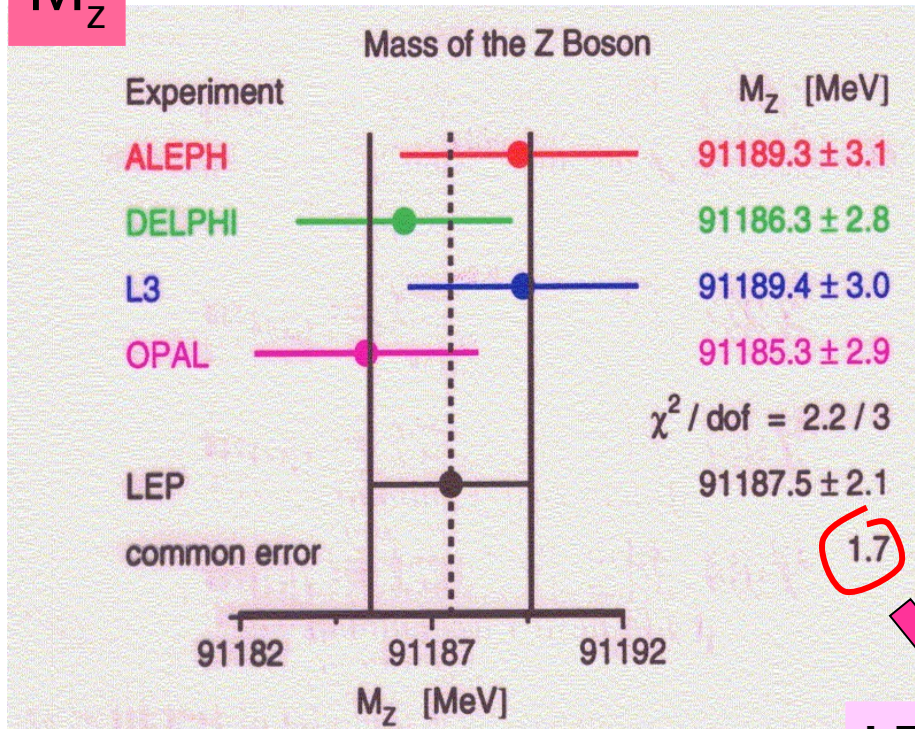
Leptonic Asymmetry A_{FB}^{γ}



Measurement of the intermediate boson masses at LEP

LEP I

M_Z



LEP collider !

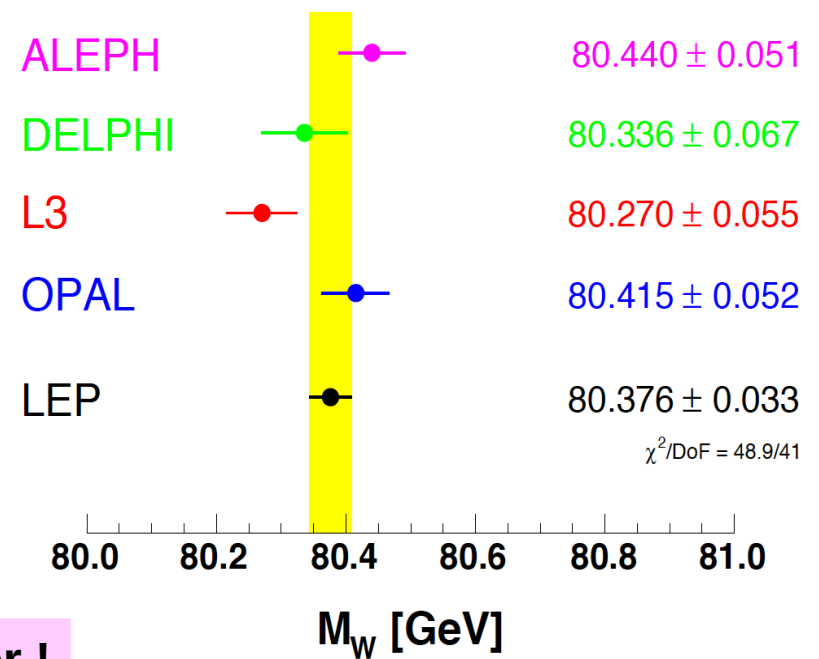
$$\Delta M_Z / M_Z \approx 2.3 \cdot 10^{-5}$$

(cfr. $\delta G_F / G_F \approx 9 \cdot 10^{-6}$, $\delta \alpha_{\text{QED}}(M_Z) / \alpha_{\text{QED}} \approx 2 \cdot 10^{-4}$)

LEP II

M_W

LEP W-Boson Mass



$$\Delta M_W / M_W \approx 4.1 \cdot 10^{-4}$$

given $M_Z, G_F, \alpha_{\text{QED}}(M_Z)$,
depends on $M_{\text{top}}, M_{\text{Higgs}}$

Radiative corrections to M_W

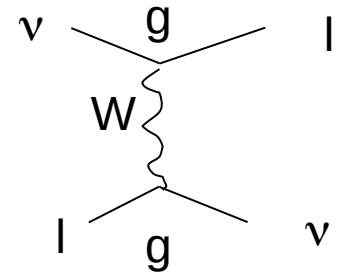
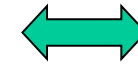
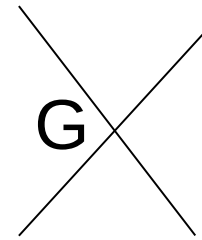
● The tree-level prediction:

Fermi constant (from the muon decay)

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{e^2}{8M_W^2 \sin^2 \theta_W}$$

which gives $(\alpha = \frac{e^2}{4\pi})$:

$$M_W = \left[\frac{\pi\alpha}{\sqrt{2}G \sin^2 \theta_W} \right]^{1/2} \cong \frac{37.2802 \text{ GeV}}{(0.23)^{1/2}} = 77.8 \text{ GeV}$$



$$\left[\left[\frac{\pi\alpha}{\sqrt{2}G} \right]^{1/2} \right] \cong 37.3 \text{ GeV}$$

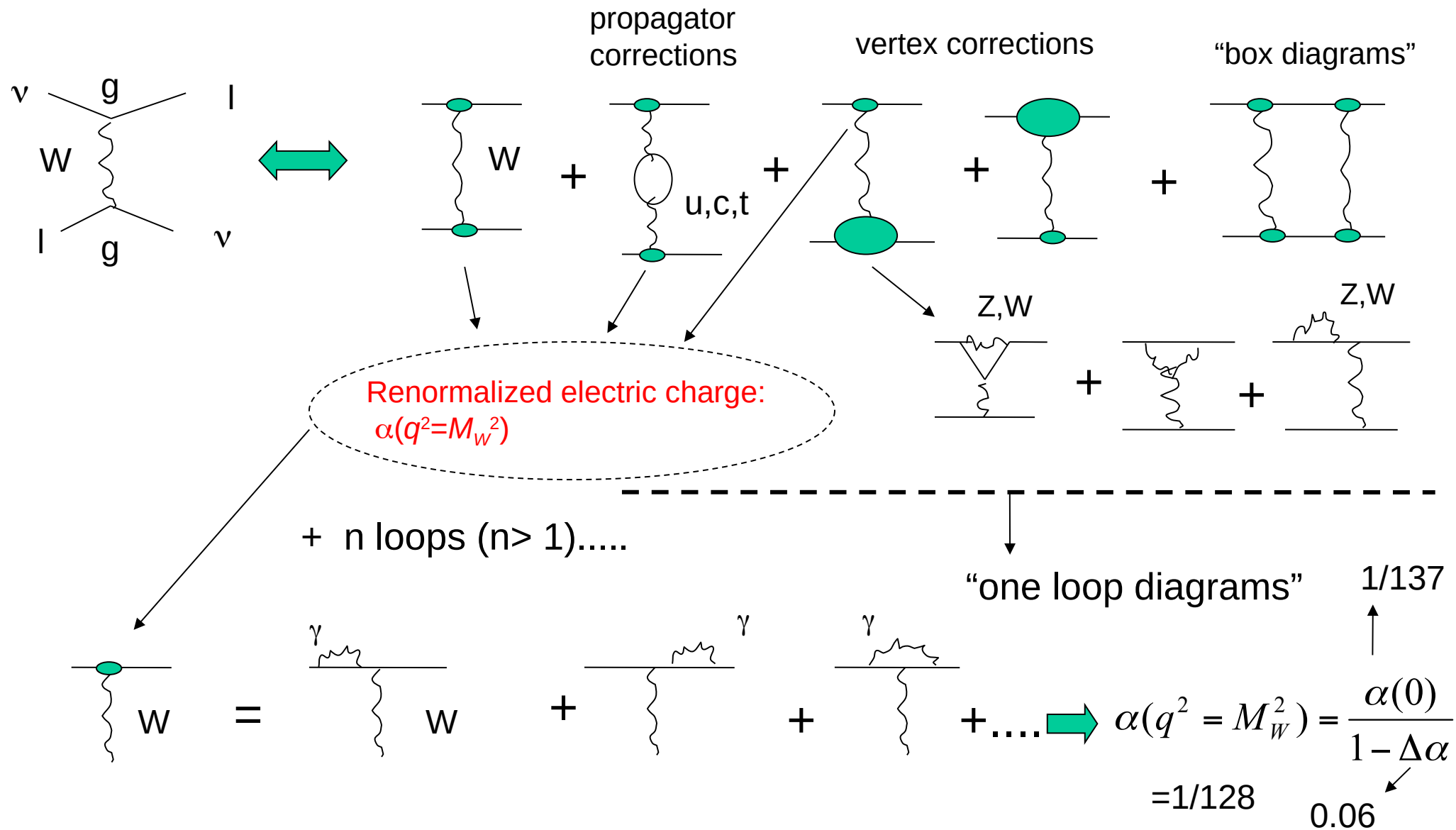
$= 0.228 \pm 0.005$ νN cross section scattering
(CC /NC ratio)

$(1.1666389 \pm .000022) \cdot 10^{-5} \text{ GeV}^{-2}$ from the muon decay:

$$\frac{1}{\tau_\mu} = \frac{G^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right) \left[1 + \alpha(m_\mu^2)(25/4 - \pi^2)/2\pi \right]$$

is modified by the radiative corrections.

Radiative corrections to M_W



Radiative corrections to M_W

➡ The relation:

$$\frac{G}{\sqrt{2}} = \frac{e^2}{8M_W^2 \sin^2 \theta_W} = \frac{\pi\alpha(0)}{2M_W^2 \sin^2 \theta_W}$$

becomes:

$$\frac{G}{\sqrt{2}} = \frac{\pi\alpha(0)}{2M_W^2 \sin^2 \theta_W} \frac{1}{(1 - \Delta\alpha)(1 + \Delta\rho \cot^2 \theta_W + \dots)} \Delta r$$

$\Delta\rho(m_t, m_H)$

[Burgers, Jegerlehner, Phys.LEP vol I, CERN 89-08]

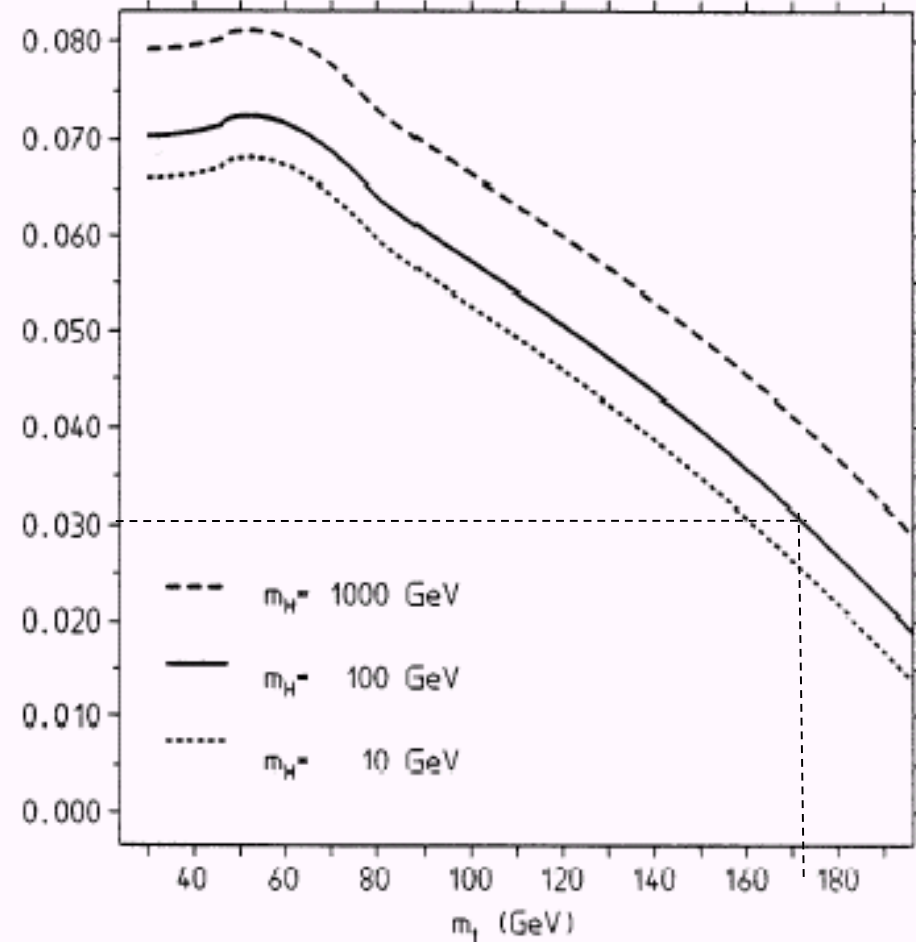
Electroweak correction:

$$\frac{1}{1 - \Delta r} \equiv \frac{1}{(1 - \Delta\alpha)(1 + \Delta\rho \cot^2 \theta_W + \dots)}$$

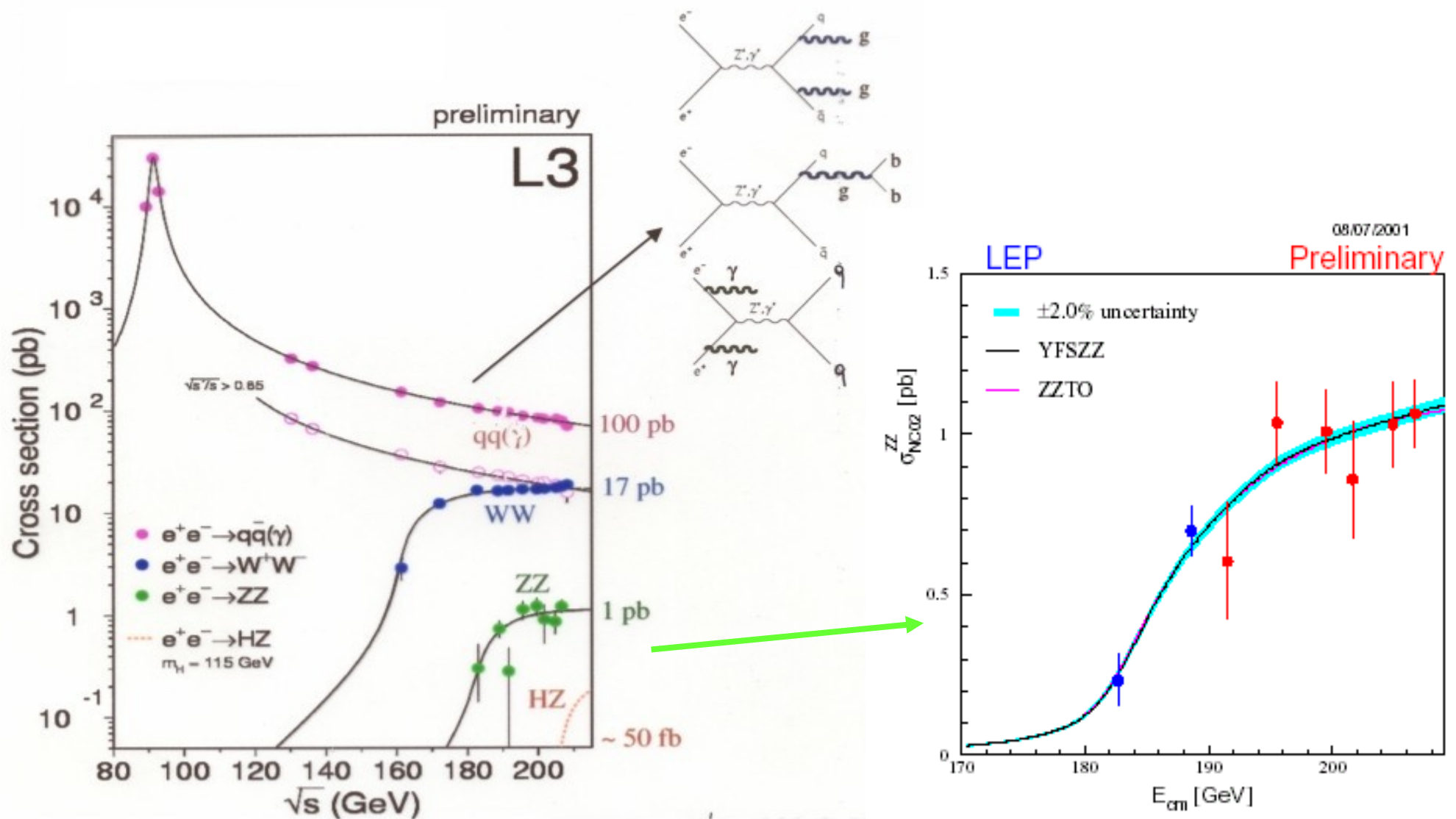
$$M_W = \left[\frac{\pi\alpha}{\sqrt{2}G \sin^2 \theta_W (1 - \Delta r)} \right]^{1/2} = (80.6 \pm 0.8) \text{ GeV} \rightarrow (\text{nel 1983, scoperta del } W \text{ a UA1})$$

$$(80.385 \pm 0.030) \rightarrow (\text{today: } \Delta r = 0.031, m_t = 173 \text{ GeV})$$

Experimental Subnuclear Physics $m_H = 125 \text{ GeV}$

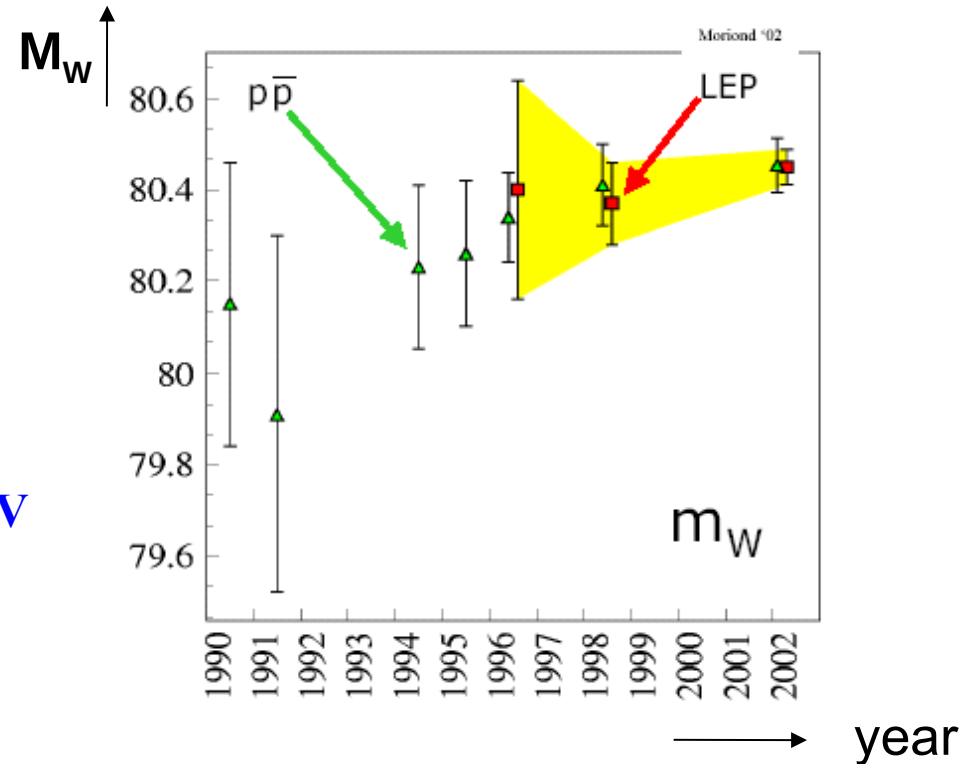
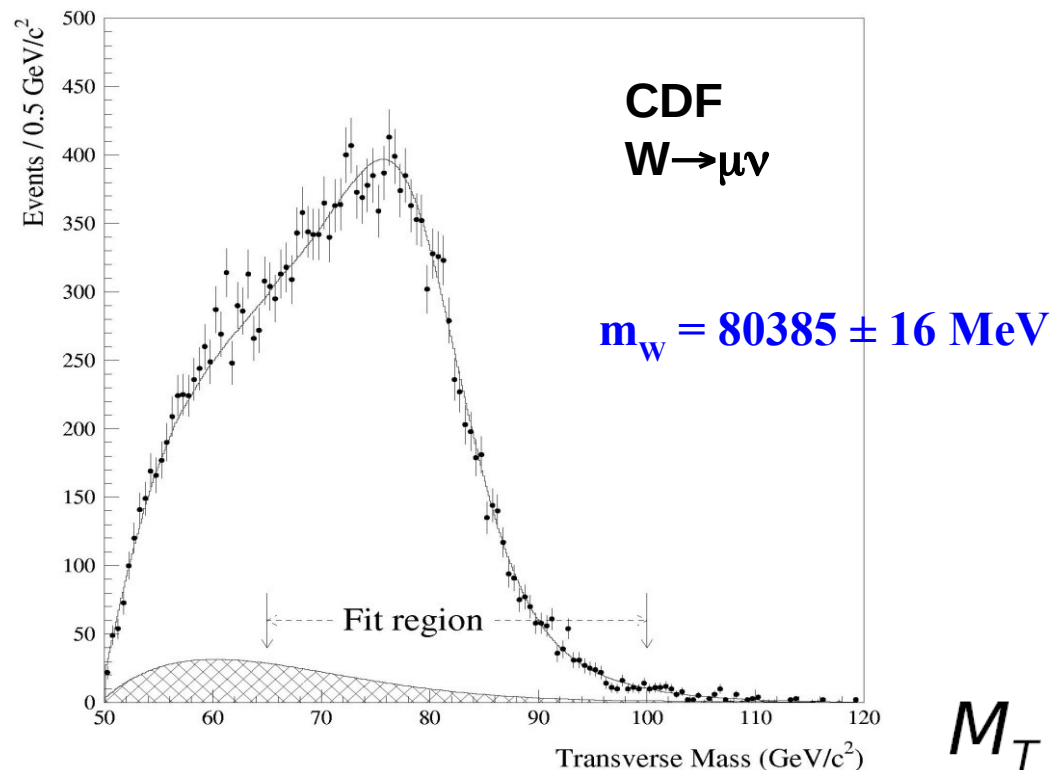


ZZ production at LEP2



M_W measurement at the hadronic colliders

- Independently from LEP, the W mass has been measured with increasing precision at the hadronic colliders (where it was discovered in 1983, at SpS of CERN, by the UA1 experiment):
- Method of measurement based on the reconstruction of the “transverse mass” M_T :

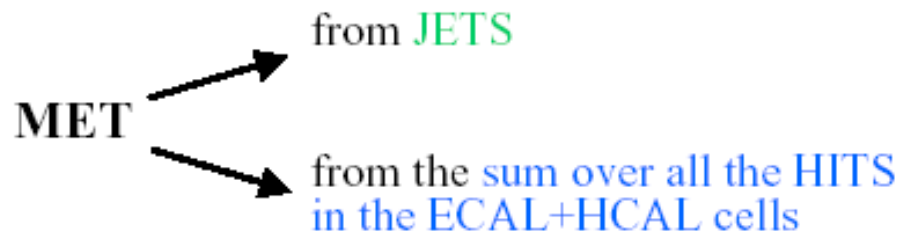


$$M_T = \sqrt{(E_T^\mu + E_T^\nu)^2 - (\vec{P}_T^\mu + \vec{P}_T^\nu)^2} \quad \rightarrow$$

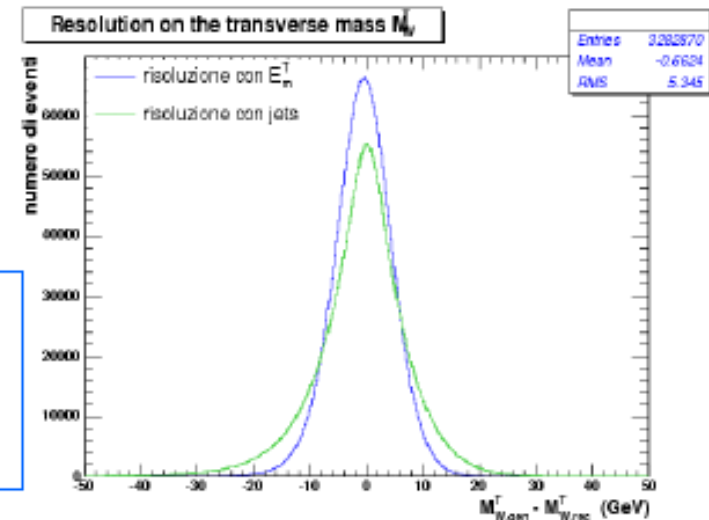
M_W measurement at the hadronic colliders

$$M_W^T = \sqrt{2p_{el}^T p_{\nu}^T (1 - \cos \varphi)}$$

- need to reconstruct the **neutrino**
 - p_{ν}^T from momentum balance in the transverse plane: $\vec{p}_{\nu}^T = -(\vec{p}_{lept}^T + \vec{u})$
where u is the transverse momentum of the hadronic recoil against W
 - **Transverse missing energy (MET)** as an estimate of p_{ν}^T

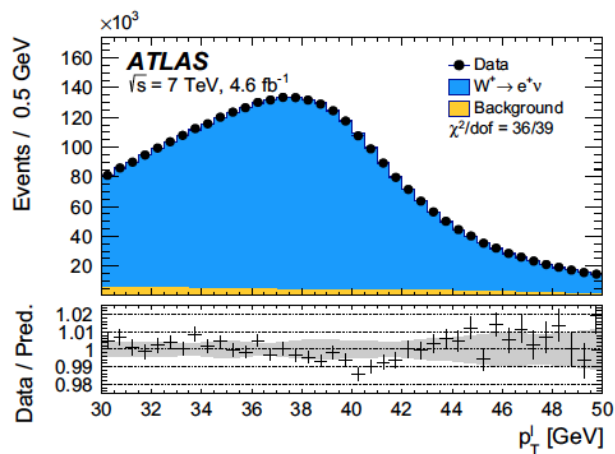


better resolution because accounts also for the hadronic energy contribution not clustered in jets
resulting resolution on M_W^T is $\sigma \sim 5 \text{ MeV}$

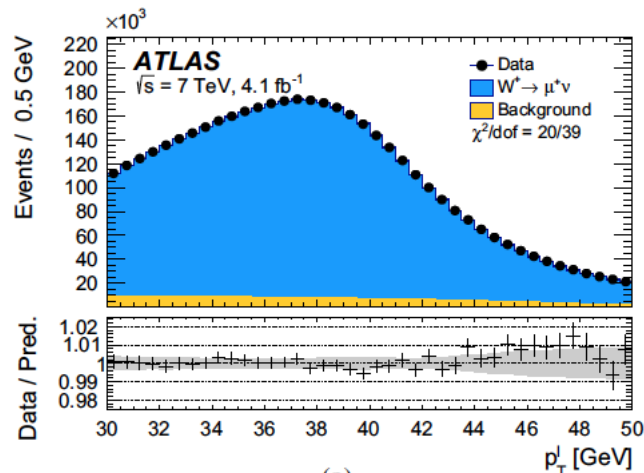


- In general the systematic uncertainty on the energy scale of the calorimeters limits the final precision on M_W . Study of alternative methods for precise measurements at LHC
(goal: $\Delta M_W \sim 10\text{-}15 \text{ MeV}$)

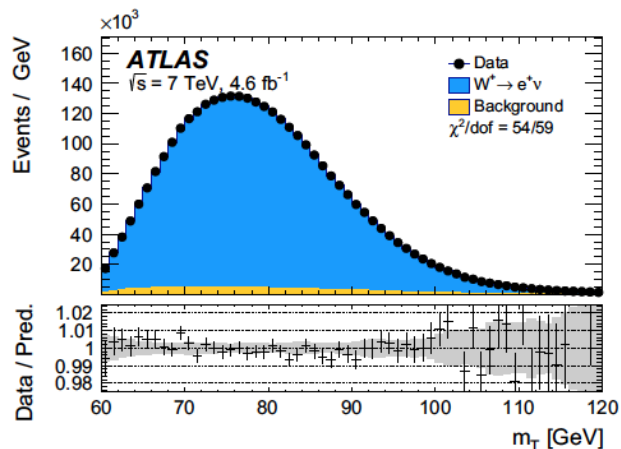




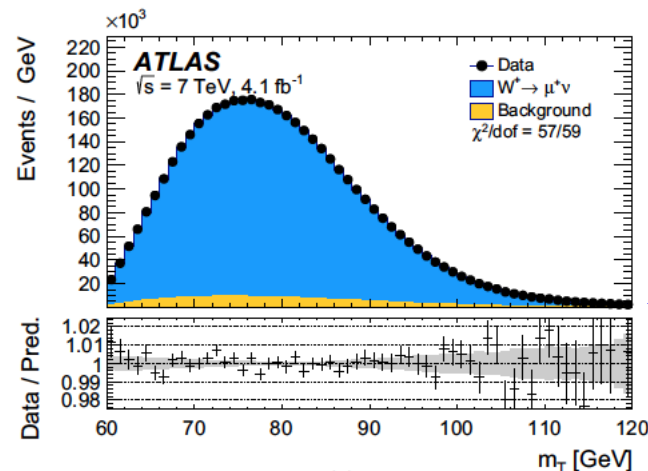
(a)



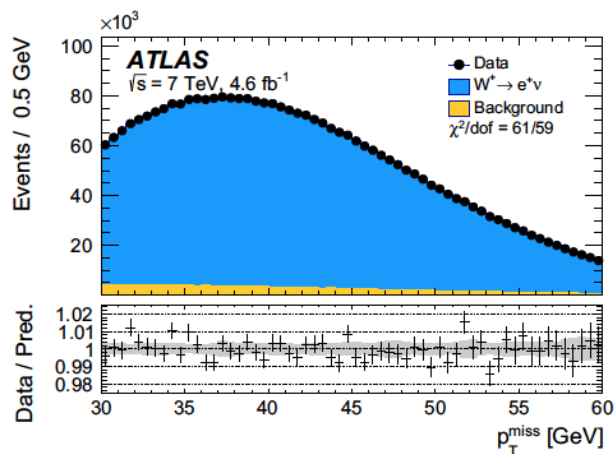
(a)



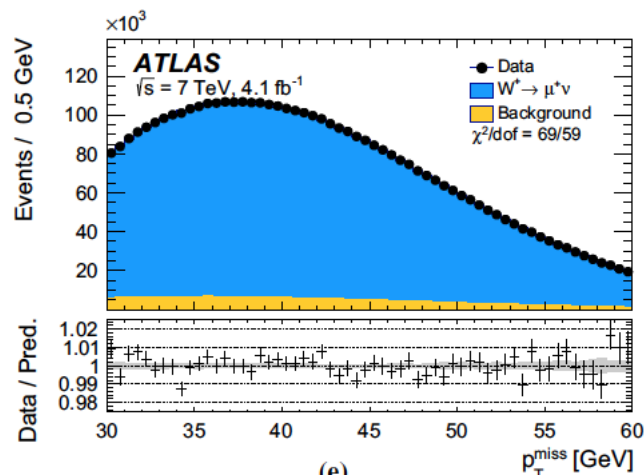
(c)



(c)



(e)



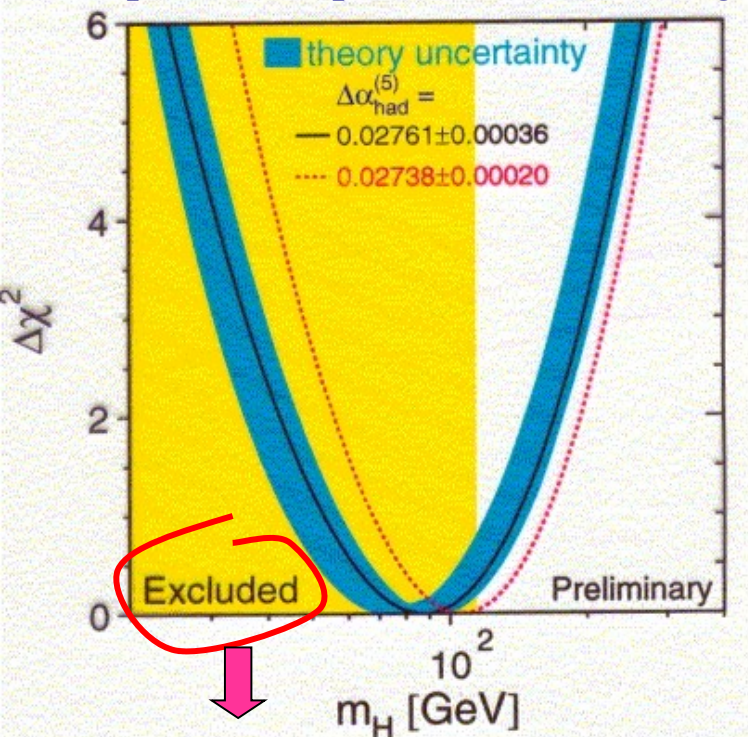
(e)

The mass of the W boson is determined from fits to the transverse momentum of the charged lepton, p_{Tl} , and to the transverse mass of the W boson, m_T and to the transverse momentum of the missing neutrino, p_{Tmiss}

$$M_W = 80369.5 \pm 6.8 \text{ (stat.)} \\ \pm 10.6 \text{ (exp. syst.)} \\ \pm 13.6 \text{ (mod. syst.) MeV} \\ = 80369.5 \pm 18.5 \text{ MeV}$$

M_{Higgs} prediction

- All the precision measurements, through the M_H dependence of the experimental observables, permits to predict the following value:



$$\rightarrow m_H = 91_{-16}^{+18} \text{ GeV}$$

that is: $65 \text{ GeV} < m_H < 122 \text{ GeV} \text{ (90\% CL)}$

[with the **top** it worked ...:

1993: $m_{\text{top}}^{\text{EW}} = 166 \pm 18 \pm 20 \text{ GeV}$ $\nearrow m_H = 60-700$
 LEP, EPS
 Marseille

1994: $m_{\text{top}} = 174 \pm 10^{+13}_{-23} \text{ GeV}$ \nwarrow
 CDF, ICHEP
 Glasgow

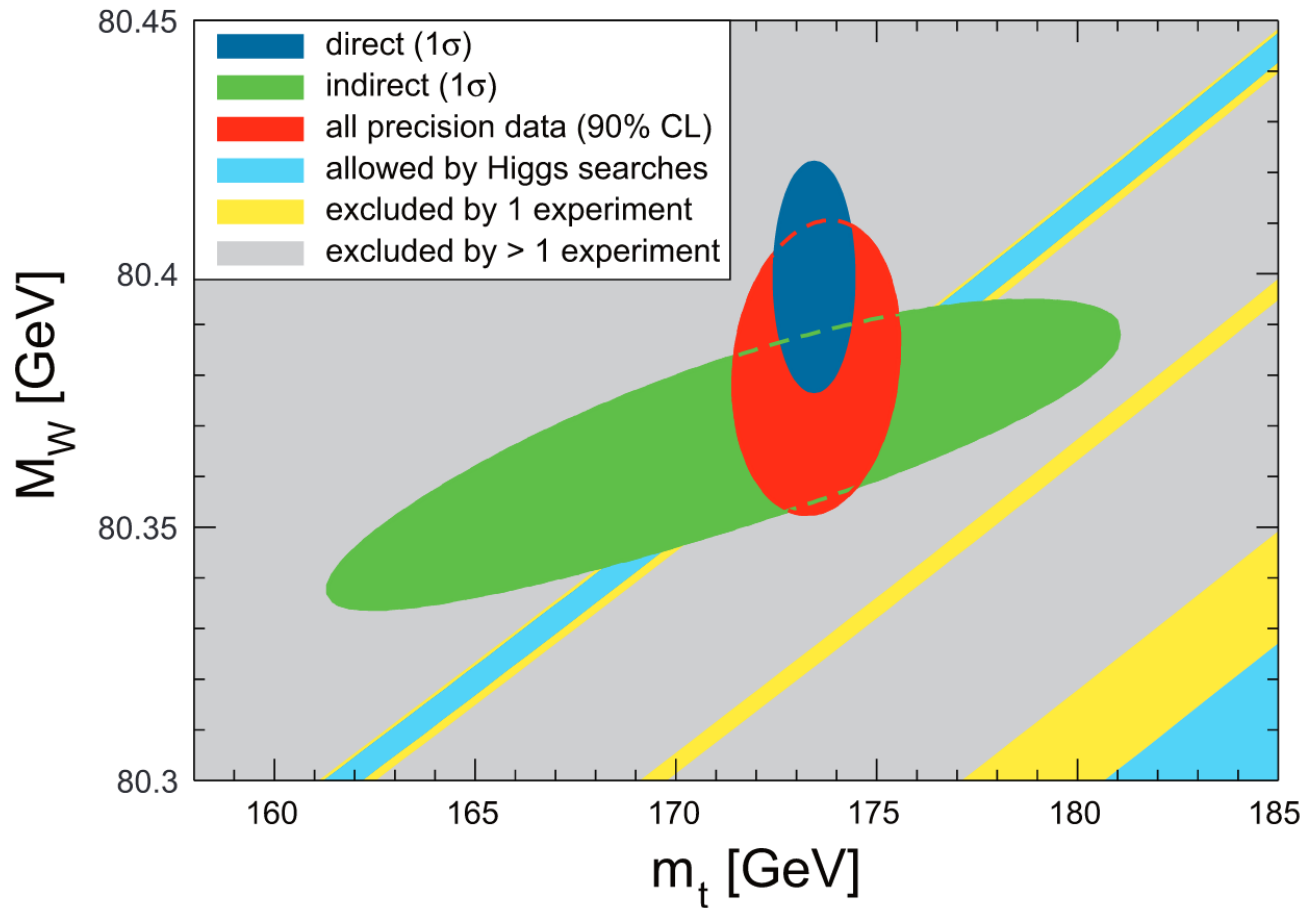
today:

$$2022: m_{\text{top}}^{\text{EW}} = 173.13 \pm 0.56 \text{ GeV}$$

$$m_{\text{top}} = 172.69 \pm 0.30 \text{ GeV} \quad]$$

Big success of the SM !

M_{Higgs} prediction and Standard Model consistency



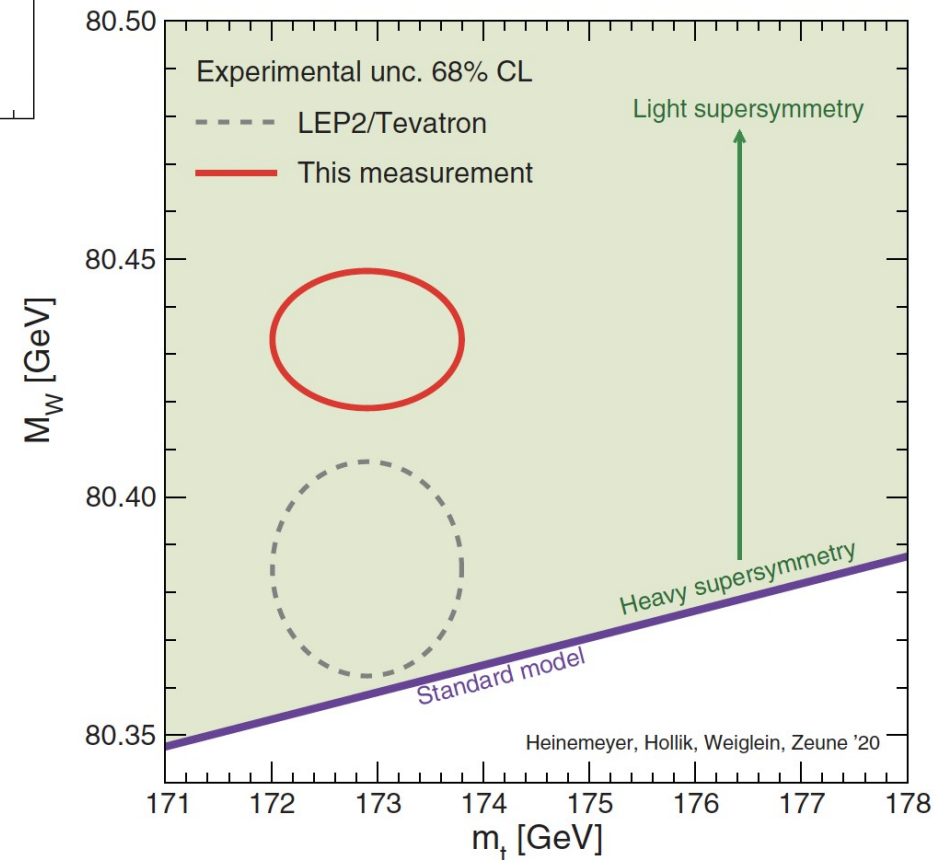
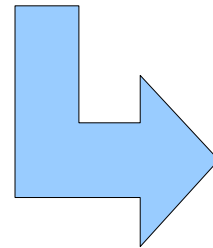
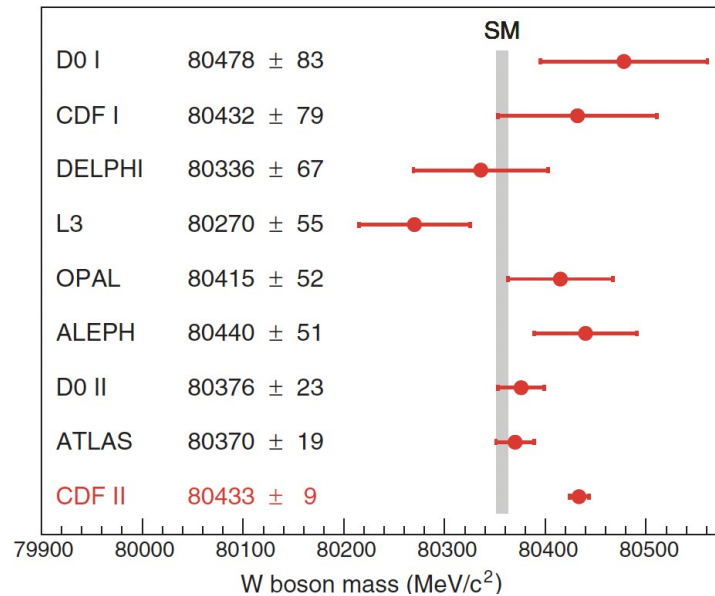
PDG-2012
situation before the
Higgs discovery

Figure 10.6: One-standard-deviation (39.35%) region in M_W as a function of m_t for the direct and indirect precision data, and the 90% CL region ($\Delta\chi^2 = 4.605$) allowed by all precision data. The SM predictions are also indicated, where the blue bands for Higgs masses between 115.5 and 127 GeV and beyond 600 GeV are currently allowed at the 95% CL. The bright (yellow) bands are excluded by one experiment and the remaining (gray) regions are ruled out by more than one experiment (95% CL).

.... but new M_W measurement by CDF

Fig. 5. Comparison of this CDF II measurement and past M_W measurements with the SM expectation. The latter includes

the published estimates of the uncertainty (4 MeV) due to missing higher-order quantum corrections, as well as the uncertainty (4 MeV) from other global measurements used as input to the calculation, such as m_t , c , speed of light in a vacuum.



Experimental Subn