Standard Model

- The Glashow-Weinberg- Salam model
- The Higgs mechanism
- The Glashow-Weinberg- Salam model and the Higgs mechanism
- The fermion masses

Standard Model

- The gauge invariant lagrangian density is:

$$
\begin{aligned}
\mathscr{L} \;=\;& \bar{\Psi}_L \gamma^\mu \left[i\partial_\mu - g\,\vec{I}\cdot\vec{W}_\mu - \frac{g'}{2} Y B_\mu \right] \psi_L \\
+\;& \bar{\Psi}_R \gamma^\mu \left[i\partial_\mu - \frac{g'}{2} Y B_\mu \right] \psi_R \\
& -\frac{1}{4}\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu} \quad -\frac{1}{4} B_{\mu\nu} B^{\mu\nu}
\end{aligned}
$$

Where all fields (fermions, gauge) have no masses.

where:

$$\vec{W}_{\mu\nu} = \underbrace{\partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu}_{\text{as in QED}} \underbrace{- g\,\vec{W}_\mu \times \vec{W}_\nu}_{\text{self-coupling of 3 and 4 gauge bosons}}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Higgs Mechanism

- We add the following density lagrangian:

$$\mathscr{L} = D_\mu \phi^\dagger D^\mu \phi - \mu^2 \phi \phi^\dagger - \lambda (\phi \phi^\dagger)^2$$

where $\mu^2 < 0$; $\lambda > 0$ and

$$D_\mu = \left[i \partial_\mu - g \vec{I} \cdot \vec{W}_\mu - \frac{g'}{2} Y B_\mu \right]$$

Higgs Mechanism

- The 4 scalar fields must be multiplets of SU(2)⊗U(1):

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

$I = ½$; $Y = +1$

$Q_{\phi+} = +1$; $Q_{\phi 0} = 0$

- Then it is chosen a specific direction of SU(2) for the vacuum:

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

and for:

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

Higgs Mechanism

- Subsituting this $\phi(x)$ into the previous $\mathscr{L}$ one obtains (writing up to II order terms):

$$\mathscr{L} = \left[\frac{1}{2}(\partial_\mu H)^2 - \lambda v^2 H^2\right] + \frac{g^2 v^2}{8}\left(W^1_\mu W^{1\mu} + W^2_\mu W^{2\mu}\right)$$
$$+\frac{v^2}{8}\left(gW^3_\mu - g' B_\mu\right)\left(gW^{3\mu} - g' B^\mu\right) + \quad \textit{higher order terms}$$
$$+ \textit{ kinetic energy for } W \textit{ and } B$$

W^1_μ, W^2_μ have conventional mass terms

W^3_μ, B_μ are mixed → but we know that they can be rewritten through: A_μ , Z_μ and $\mathrm{tg}\theta_W = g'/g$

- Therefore now one has (ignoring the interaction terms):

$$\mathscr{L} = \frac{1}{2}\left(\partial_\mu H\right)^2 - \lambda v^2 H^2 \quad ①$$
$$-\frac{1}{4}\left(\partial_\mu W^1_\nu - \partial_\nu W^1_\mu\right)\left(\partial^\mu W^{1\nu} - \partial^\nu W^{1\mu}\right) + \frac{g^2 v^2}{8}\left(W^1_\mu W^{1\mu}\right) \quad ②$$
$$-\frac{1}{4}\left(\partial_\mu W^2_\nu - \partial_\nu W^2_\mu\right)\left(\partial^\mu W^{2\nu} - \partial^\nu W^{2\mu}\right) + \frac{g^2 v^2}{8}\left(W^2_\mu W^{2\mu}\right) \quad ②$$
$$-\frac{1}{4}\left(\partial_\mu Z_\nu - \partial_\nu Z_\mu\right)\left(\partial^\mu Z^\nu - \partial^\nu Z^\mu\right) + \frac{v^2}{8}(g^2 + g'^2)\left(Z_\mu Z^\mu\right) \quad ③$$
$$-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) \quad ④$$

Higgs Mechanism

- One has the desidered mass spectrum:

① Higgs bosons: $m_H = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$

② 2 W bosons: $m_W = \frac{g\,v}{2}$

③ Z boson: $m_Z = \frac{v}{2}\sqrt{g^2+g'^2} = \frac{m_W}{\cos\theta_W}$

④ γ photon: $m_\gamma = 0$

- One starts from a $\mathscr{L}$ with 3 massless W + massless B + 4 scalar fields:
  6 + 2 + 4 = 12 degrees of freedom

- One ends with a $\mathscr{L}$ with 3 massive bosons ($W^\pm$, Z^0), a massless γ + 1 Higgs scalar field:
  3×3 + 1 ×2 + 1 = 12 degrees of freedom
  3 Goldstone bosons disappeared to give the longitudinal polarization to $W^\pm$, Z^0

Standard Model and Higgs Mechanism

- **Why is γ massless?**
  One has:

$$Q = I_3 + \frac{Y}{2}$$

and then:

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad I = 1/2\,;\; Y = 1$$

- The vacuum fluctuation corresponds to the emission/absorption of the Higgs boson: that is each quantum number of the Higss boson can be created/destroyed
- The charge conservation constrains to choose a vacuum expectation different from zero for ϕ^0 (and not for ϕ^+ which would imply a non conservation of Q)
- ϕ^0 with I_3 = -1/2 and Y = 1 breaks both $SU(2)_L$ and $U(1)_Y$ simultaneously. But if one finds a symmetry of the vacuum which is subgroup of $SU(2)_L \otimes U(1)_Y$ → the gauge boson of such symmetry remains massless.

Standard Model and Higgs Mechanism

- This symmetry exists:

$$Q\phi^0 = \left(I_3 + \frac{Y}{2}\right)\phi^0 = 0 \qquad (*)$$

and:

$$\phi^0 \rightarrow \phi'^0 = \exp\left[i\alpha(x)Q\right]\phi^0 = \phi^0 \qquad \forall\ \alpha(x)$$

- This is a U(1) transformation, its generator Q is a linear combination of the generators of $SU(2)_L \otimes U(1)_Y$
- This is the e.m. transformation $U(1)_{em}$

$$U(1)_{em} \subset SU(2)_L \times U(1)_Y$$

of the 4 generators I and Y only Q satisfies (*) → the γ remains massless

- **The existence of a massless bosons is a necessary consequence of the electric charge conservation which implies a neutral vacuum state.**

Standard Model and Higgs Mechanism

- **Prediction of the masses:**

$$\left.\begin{aligned} m_W &= \frac{1}{2} v\, g \\ \frac{g^2}{8\, m_W^2} &= \frac{G}{\sqrt{2}} \end{aligned}\right\} \quad v = \frac{1}{\sqrt{\sqrt{2}\, G}}$$

The value of the vacuum expectation ϕ^0 depends on the Fermi G constant:

$$v \approx 246 \text{ GeV}$$

- From $g \sin\theta_W = e$ one has:

$$m_W = \left(\frac{\pi\,\alpha}{\sqrt{2}\, G}\right)^{1/2} \frac{1}{\sin\theta_W} \qquad \text{with} \quad \alpha = \frac{e^2}{4\,\pi}$$

one obtains:

$$m_W \approx 80 \text{ GeV}; \qquad m_Z = \frac{m_W}{\cos\theta_W} \approx 90 \text{ GeV}$$

with $\sin^2\theta_W \approx 0.23$

Standard Model and Higgs Mechanism

- The mass of the boson Higgs, $m_H = \sqrt{2\lambda v^2}$, depends on the unknown parameter λ which appears in the potential $V(\phi)$, $\boldsymbol{m}_{\mathbf{H}}$ **cannot be predicted**.

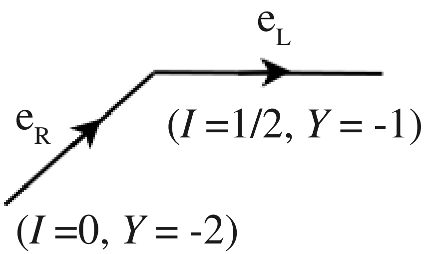
Standard Model and Higgs Mechanism

- **Fermion masses**

Give now mass to the fermions. Give a look to the leptons and quarks of the first generation: (e, ν_e, u, d)

One could write the following mass term for e:

$$\begin{aligned}\mathscr{L}_m &= -m\bar{e}e = -m\bar{e}\left[\frac{1}{2}(1-\gamma_5)+\frac{1}{2}(1+\gamma_5)\right]e \\ &= -m\left(\bar{e}_R e_L+\bar{e}_L e_R\right)\end{aligned}$$

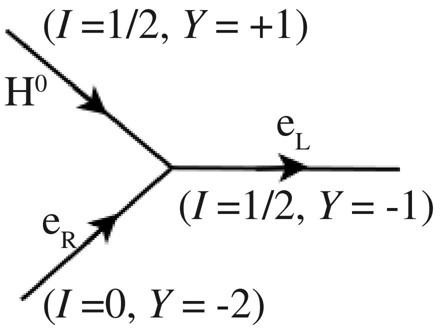


beacuse e_R has $I_W = 0$ and e_L has $I_W = ½$ this mass term breaks the gauge invariance.

- One can build a gauge invariant $\mathscr{L}$ coupling the leptons with the Higgs bosons:

$$\mathscr{L}_{e\phi} = -g_e\left[\bar{L}\phi e_R+\bar{e}_R\bar{\phi}L\right]$$

I: ½ ½ 0 0 ½ ½ = 1



where:

$$L = \begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L ; \qquad \phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}$$

Standard Model and Higgs Mechanism

- for example:

$$\bar{L}\phi = \left(\bar{\nu}_e \quad e^-_L\right)_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \left(\bar{\nu}_{eL}\phi^+ + e^-_L\phi^0\right)$$

is invariant for SU(2), if one multiplies by e_R, which is a singlet, nothing happens (it remains gauge invariante).
The term $\bar{e}_R\bar{\phi}L$ is hermitian coniugate of the first term (the gauge invariant).
$g_e \equiv$ arbitrary constant

Standard Model and Higgs Mechanism

- If the symmetry is spontaneously broken:

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H(x) \end{pmatrix}$$

one obtains:

$$\begin{aligned}
\mathscr{L}_{e\phi} &= -\frac{g_e}{\sqrt{2}}\left[(\bar{\nu}_e \ \ \bar{e})_L \begin{pmatrix} 0 \\ v+H \end{pmatrix} e_R + \bar{e}_R (0 \quad v+H) \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \right] \\
&= -\frac{g_e}{\sqrt{2}}\left[v\,\bar{e}_L e_R + \bar{e}_L e_R H + v\,\bar{e}_R e_L + \bar{e}_R e_L H\right] \\
&= -\frac{g_e v}{\sqrt{2}}\left[\bar{e}_L e_R + \bar{e}_R e_L\right] - \frac{g_e}{\sqrt{2}}\left[\bar{e}_L e_R + \bar{e}_R e_L\right] H \\
&= \mathscr{L}_m + \mathscr{L}_{int} = -m_e \bar{e} e - \frac{m_e}{v} \bar{e} e H
\end{aligned}$$

$$\boxed{m_e = \frac{g_e v}{\sqrt{2}}} \longrightarrow$$

g_e costante arbitrary constant not predicted by the theory

$$\boxed{\frac{m_e}{v} = \frac{1}{2} g \frac{m_e}{m_W}} \longrightarrow$$

coupling between *e*-Higgs (proportional to m_e)

Standard Model and Higgs Mechanism

- There is no coupling between the Higgs boson and the neutrinos.

  This is due to the fact that it was put $m_\nu = 0$, that is having required only the presence of ν_L, there are no ν_R term and then it is not possible to write terms of the type:

$$-g_\nu\left[\bar{L}\,\phi\,\nu_R + \bar{\nu}_R\,\bar{\phi}\,L\right]$$

- Now we give a look to the up and down quarks:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad ; \quad u_R, d_R$$

new respect to the leptonic case

One builds the conjugate Higgs doublet:

$$\tilde{\phi} = i\tau_2\phi^* = i\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} \phi^{+*} \\ \phi^{0*} \end{pmatrix}\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} \phi^{+*} \\ \phi^{0*} \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} \phi^{0*} \\ -\phi^{+*} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \bar{\phi}^{0} \\ -\phi^{-} \end{pmatrix}$$

Standard Model and Higgs Mechanism

- $\tilde{\phi}$ trasforms in the same manner of ϕ
  If $Y\phi=+\phi$ one has that $Y\tilde{\phi}=-\tilde{\phi}$ so $Q = I_3 + Y/2$ is satisfied.
- After the breaking:

$$\tilde{\phi} = \frac{1}{\sqrt{2}}\begin{pmatrix} v+H \\ 0 \end{pmatrix}$$

$$\mathscr{L}_{q\phi} = -g_d \bar{L}_q \phi d_R - g_u \bar{L}_q \tilde{\phi} u_R + h.c.$$

is gauge invariant, where:

$$L_q = \begin{pmatrix} u \\ d \end{pmatrix}_L$$

g_d, g_u random costant.

- After the breaking la rottura:

$$\mathscr{L}_{q\phi} = -m_d \bar{d} d - m_u \bar{u} u - \frac{m_d}{v} \bar{d} d H - \frac{m_u}{v} \bar{u} u H$$

… in effect one should use the quark mixing matrix…