

# Jets

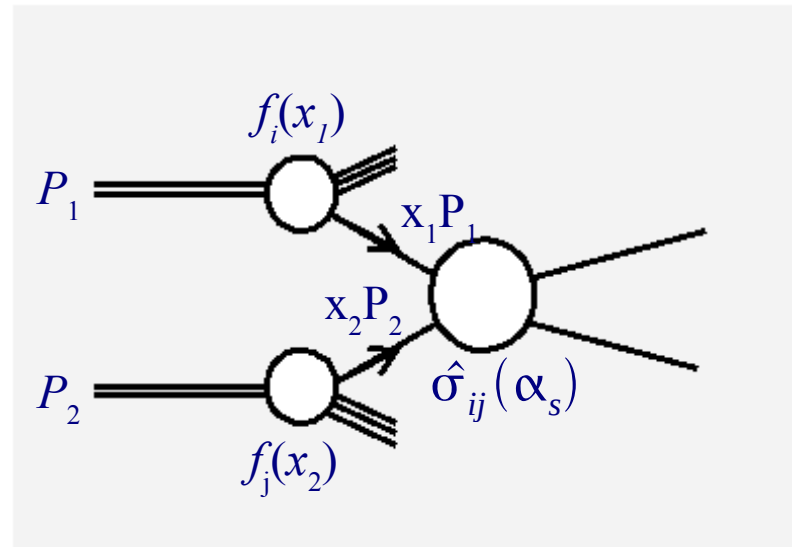
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## Fundamentals about jets

- Hadroproduction of jets
- Kinematics and jets definition
- Two-jet cross section
- Comparison with experiment

# Hadroproduction of jets

- A hard scattering process between two hadrons is the result of an interaction between the quarks and gluons which are the constituents of the incoming hadrons.
- The incoming hadrons provide “broad band” beams of partons which possess varying fractions of the momenta of their parent hadrons.
- Partonic description of a hard-scattering process:



$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, \mu^2) f_j(x_2, \mu^2) \hat{\sigma}_{i,j}(p_1, p_2, \alpha_s(\mu^2), Q^2/\mu^2)$$

# Hadroproduction of jets

- $$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, \mu^2) f_j(x_2, \mu^2) \hat{\sigma}_{i,j}(p_1, p_2, \alpha_s(\mu^2), Q^2/\mu^2)$$

$p_1 = x_1 P_1, p_2 = x_2 P_2$  : partons momenta

$Q$  = energy scale of the hard scattering: i.e.  $M_{\text{IVB}}$  or  $M_Q, p_T^{\text{jet}}$ , etc. ...

$f_i(x, \mu^2)$  = usual  $q$  or  $g$  pdf defined at a factorization scale  $\mu$

$\hat{\sigma}_{i,j}$  = short-distance cross section for the scattering of partons of types  $i$  and  $j$

$\alpha_s$  is small at high energy  $\rightarrow \hat{\sigma}_{i,j}$  can be calculated as a perturbation series in  $\alpha_s$

- The  $(n+k)$ th order approximation is:

$$\hat{\sigma} = \alpha_s^k \sum_{m=0}^n c^{(m)} \alpha_s^m \longrightarrow \text{function of the kinematic variables and factorization scale } \mu$$

- Different processes will contribute with different leading power  $k$ :  
 $W$  production:  $k = 0$ ; jet production at large  $p_T$ :  $k = 2$

# Hadroproduction of jets

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- In **leading approximation** ( $n = 0$ ),  $\hat{\sigma} \equiv$  x-section of the normal parton scattering (calculated in the same way as the QED processes).
- In **higher orders** ( $n > 0$ ),  $\hat{\sigma}$  is derived from the parton scattering cross-section by removing long-distance pieces and factorizing them into the *pdf*.  
So  $\hat{\sigma}$  contains only high transfer momenta (short times and distances), it is insensitive to the physics of low momentum scales.
- The  $\hat{\sigma}$  does not depend on the details of the hadron wave function or the type of the incoming hadron. It is a purely short-distance construct and is calculable in perturbation theory because of asymptotic freedom.
- This factorization property of the x-section can be proved to all orders in perturbation theory.
- **Factorization is a fundamental property of the theory which turns QCD into a reliable calculational tool with controllable approximations.**

# Hadroproduction of jets

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- The **factorization scale**  $\mu$  is an arbitrary parameter.
- It can be thought of as the scale which separates the long- and short-distance physics.
- A parton emitted with  $p_T < \mu$  is considered part of the hadron structure and is absorbed into the *pdf*.
- A parton emitted with  $p_T > \mu$  is part of the  $\hat{\sigma}$
- $\mu$  should be chosen to be  $O(Q^2)$
- The more terms included in the perturbative expansion, weaker the dependence on  $\mu$  will be

# Hadroproduction of jets

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- In  $\sigma(P_1, P_2)$  it was placed  $\mu = \mu_F = \mu_R$ . More generally:

$$\sigma = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, \mu_F^2) f_j(x_2, \mu_F^2) \sum_{m=0}^n (\alpha(\mu_R^2))^{(m+k)} \hat{\sigma}_{ij}^{(m)} \left( p_1, p_2, \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2} \right)$$

- When the scales are changed, the perturbative high order coefficients change in such way that  $\sigma$  at all orders is independent on the energy scale:

$$\frac{\partial \sigma}{\partial \mu_F} = \frac{\partial \sigma}{\partial \mu_R} = 0$$

Usually:  $\mu = \mu_F = \mu_R$  and  $\mu = Q$

- The  $\sigma$  written in the preceding manner is **not** the description of the majority of the events produced in a hadronic collider.  
The major part of the collisions produce particles of small  $p_T$  in the final stage.  
The pQCD describes the most interesting part of the processes involved in a hard interaction.

# Kinematics of jets

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- The scattering of two hadrons provides two broad-band beams of incoming partons. The spectrum of longitudinal momenta of the partons is determined by the *pdf*.
- The centre of mass (CM) of the *parton-parton* scattering is normally boosted with respect to that of the incoming hadrons. It is therefore useful to classify the final state in terms of variables which transform simply under longitudinal boosts

- $y \equiv$  rapidity
- $p_T \equiv$  transverse momentum
- $\phi \equiv$  azimuthal angle

- Therefore the 4-momentum of a particle will be:

$$\begin{aligned} p^\mu &= (E, p_x, p_y, p_z) \\ &= (m_T \cosh y, p_T \sin \phi, p_T \cos \phi, m_T \sinh y) \end{aligned}$$

where:

$$m_T = \sqrt{p_T^2 + m^2}$$

$$y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right)$$

# Kinematics of jets

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- $y$  is additive under the restricted class of Lorentz transformations corresponding to a boost along the  $z$  direction.

$$y' = y + \frac{1}{2} \ln \frac{(1-\beta)}{(1+\beta)}$$

- $\Delta y$  are boost invariant
- $\eta = -\ln \tan \theta/2$  pseudorapidity:  $\eta \rightarrow y$  if  $m \rightarrow 0$
- Pseudorapidity is a more convenient variable experimentally, since the angle  $\theta$  from the beam direction is measured directly in the detector.
- It is also standard to use the transverse energy  $E_T = E \sin \theta$  instead of  $p_T$ , because it is the former quantity which is measured in a hadron calorimeter.

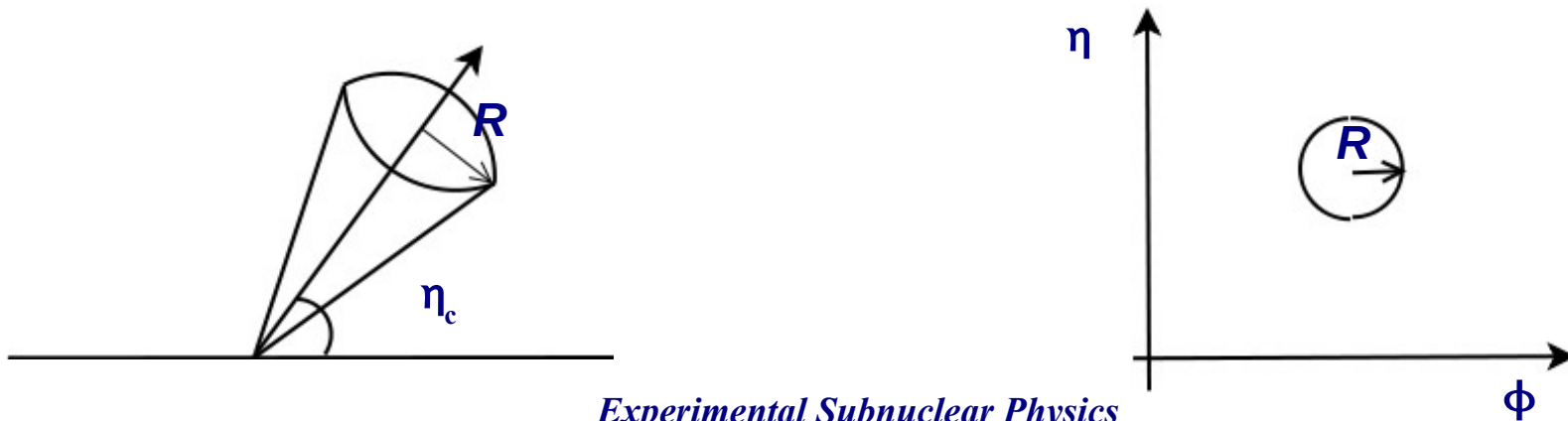


# jets definition

- Many possible definition of hadronic jets.
- There is no “best” definition.
- **To make a proper comparison one must be sure that both theoretical and experimental analyses use the same definition.**
- **Cone algorithm**: energy deposition in an angular region (e.g. the Serman-Weinberg definition)
- **Clustering algorithm**: based on combining particle momenta (e.g. the JADE algorithm)
- For hadron-hadron collisions, the most commonly used definition is of the cone type.
- A jet is a concentration of  $E_T$  in a cone of radius  $R$  where:

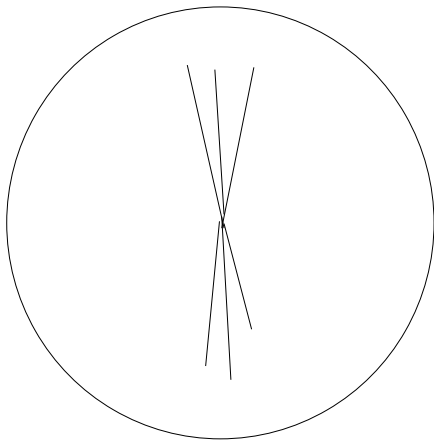
$$R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} = \sqrt{(\eta_i - \eta_c)^2 + (\phi_i - \phi_c)^2}$$

$R$  is invariant under longitudinal boost.



# Two-jet cross sections

- **Two jets events:** when an incoming parton from one hadron scatters off an incoming parton from the other hadron to produce two high-transverse momentum partons which are observed as jets.
- In the CM system: the two final state partons have equal and opposite momenta. If only two partons are produced and if the small transverse momentum of the two incoming partons are not taken into account, then the two jets of the final state are produced in the following manner:



back to back in azimuth and  $p_{T1} = p_{T2}$  in the LAB system

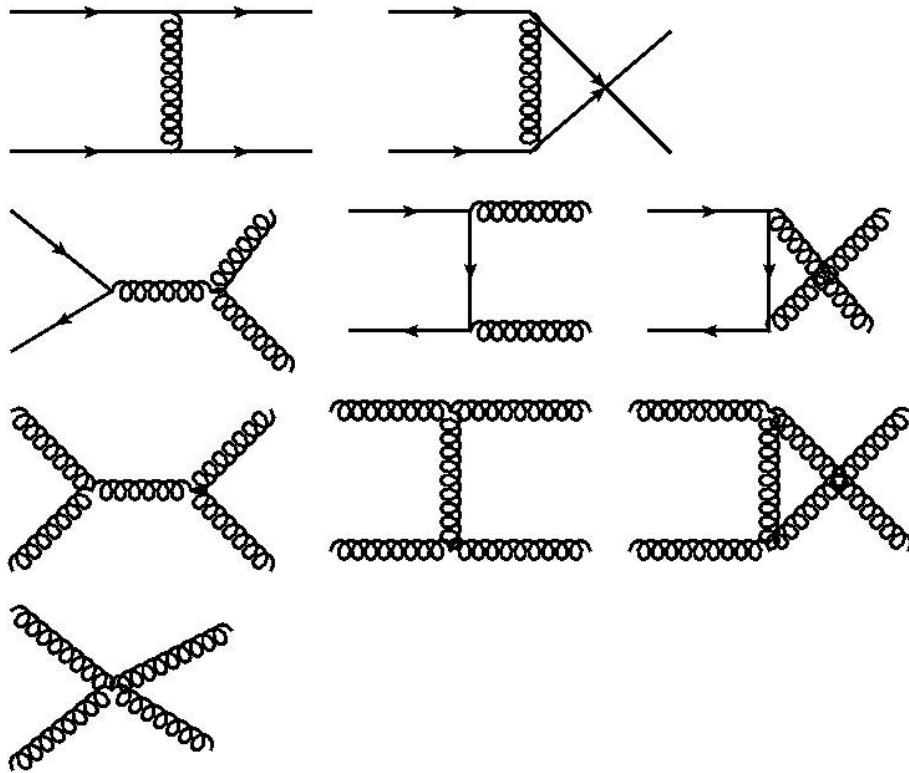
- For a  $2 \rightarrow 2$  parton scattering process:  $p_1 + p_2 \rightarrow p_3 + p_4$

$$\frac{E_3 E_4 d^6 \hat{\sigma}}{d^3 p_3 d^3 p_4} = \frac{1}{2\hat{s}} \frac{1}{16\pi^2} \overline{\sum} |M|^2 \delta^4(p_1 + p_2 - p_3 - p_4)$$

$\overline{\sum} \equiv$  average and sum over the initial- and final-state spins and colours, respectively.

# Two-jet cross sections

- Feynman diagrams for the 2-jet production:



+ crossing diagrams

- Processes with  $g$  in the initial state are more important due to the higher colour charge of the  $g$ .

# Two-jet cross sections

- The preceding expression can be rewritten for initial ( $i,j$ ) and final ( $k,l$ ) expression:

$$\frac{d^3\sigma}{dy_3 dy_4 dp_T^2} = \frac{1}{16\pi s^2} \sum_{i,j,k,l=q,\bar{q},g} \frac{f_i(x_1, \mu^2)}{x_1} \frac{f_j(x_2, \mu^2)}{x_2} \times \sum_{\bar{}} |M(ij \rightarrow kl)|^2 \frac{1}{1+\delta_{kl}}$$

$f_i(x, \mu^2) \equiv$  pdf for the parton  $i$  evaluated at the energy scale  $\mu$   
 $y_3, y_4$  rapidity in the LAB system of the outgoing partons

$$x_1 = \frac{1}{2} x_T (e^{y_3} + e^{y_4}) \quad ; \quad x_2 = \frac{1}{2} x_T (e^{-y_3} + e^{-y_4}) \quad \text{from the momentum conservation}$$

$$x_T = \frac{2p_T}{\sqrt{s}}$$

For massless partons:  $y_i \leftrightarrow \eta_i$  ;  $p_T \leftrightarrow E_T$

$\delta_{kl} \equiv$  statistical factor for partons identical in the final state

- In first approximation: supposing the detector and the algorithm for the identification of the jets 100% efficient:

$$\begin{aligned} \eta(\text{parton}) &\equiv \eta(\text{jets}) \\ E_T(\text{parton}) &\equiv E_T(\text{jets}) \end{aligned}$$

# Comparison with experiment

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- jets well established in the  $p\bar{p}$  colliders:

UA1, UA2 (Spp̄S at CERN:  $\sqrt{s} = 546 \text{ GeV}$  and  $630 \text{ GeV}$ )

CDF, D0 (TEVATRON:  $\sqrt{s} = 1.8 \text{ TeV}$ )

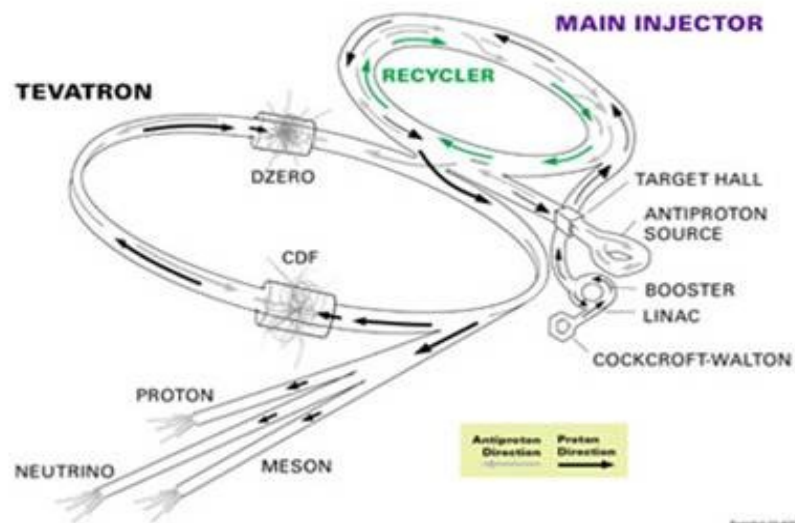
and at LHC ( $pp$  collider)

CMS e ATLAS ( $\sqrt{s} = 14 \text{ TeV}$ )

- only at these high energies it is *easy* to see the jets of the hadrons *underlying* the event.
- problems of bias due to the trigger choice solved using as trigger  $(E_T)_{\text{minimum}}$
- 2 quantities useful to compare theory and experiment:
  - $p_T$  distribution of the jets
  - angular distribution of the jets

# The Tevatron Collider

FERMILAB'S ACCELERATOR CHAIN



The Tevatron was a superconductive ring for proton-antiproton collisions. The interactions were at CM energy of 1.96 TeV with a bunch crossing of 392 ns

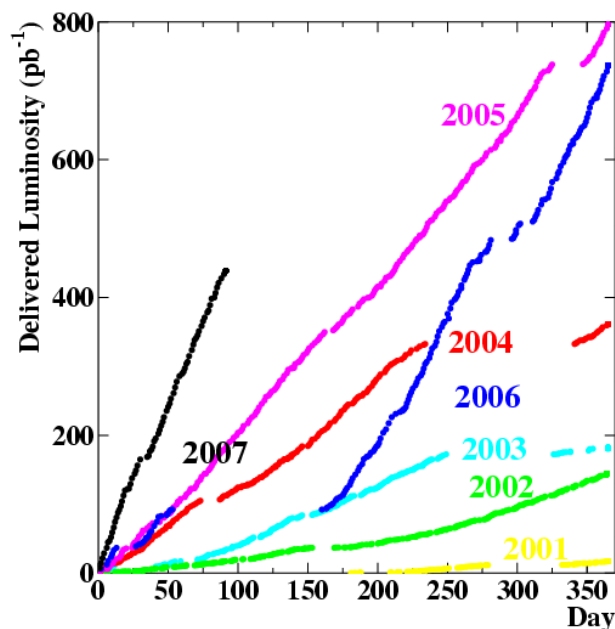
The operation starts with the accumulation of a large number of antiprotons, produced by the collision of protons against a fixed target through the reaction:  
 $pp \rightarrow \bar{p}ppp$  at 120 GeV

In a second step protons and antiprotons are injected into the main ring in small groups (bunches) and then the collisions happen in D0 and CDF detectors,

The luminosity diminishes rapidly at the very beginning, then much more slowly

Between two filling operations 20 hours can pass.

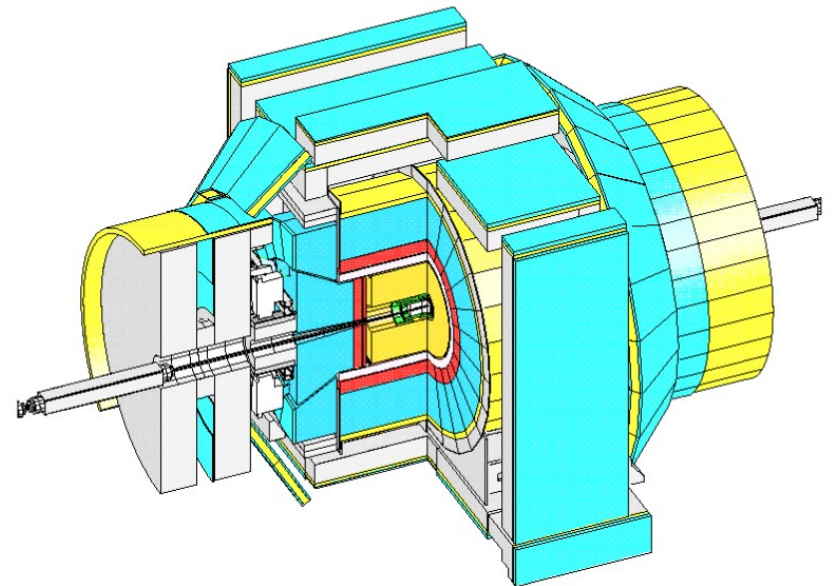
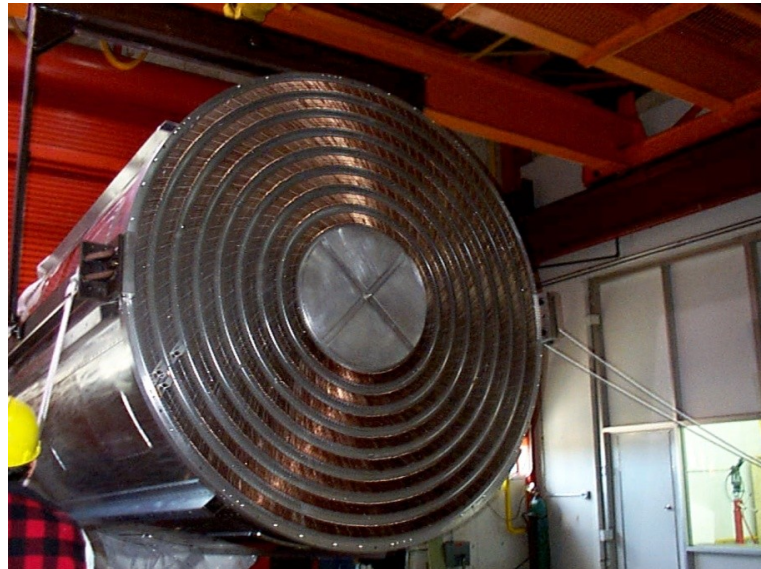
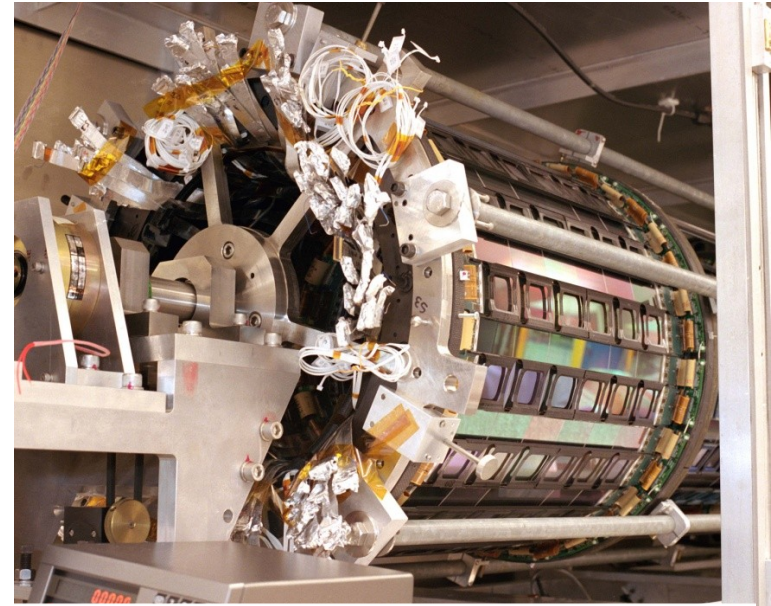
The luminosity record is  $\mathcal{L} = 4 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$



# The CDF Detector

CDF is a magnetic detector ( $B = 1.4\text{T}$ ), built to see everything:

- L00+SVX+ISL: 7 silicon layers
- COT, central tracker to  $|\eta| < 1.1$
- EM calorimeters for electrons ( $|\eta| < 2$ ) and photons; HAD calorimeters
- An extended system of muon chambers covering  $|\eta| < 1.5$



# The CDF Trigger

The interaction rate is of 2.5MHz, but only 100 Hz of events can be recorded

The major part of the interactions are not interesting (soft QCD)

A *perfect* trigger is able to select only interesting events. At 100 Hz the total cross section would be:

$$\sigma = N/\mathcal{L} \rightarrow \text{with } \mathcal{L} = 3 \times 10^{32} \text{cm}^{-2}\text{s}^{-1}, N=100/\text{s} \rightarrow \sigma = \mathbf{1 \text{ mb.}}$$

Comparing with the “interesting” processes:

W production: 20 nb;

Z production: 6 nb

Top pair production: 7 pb

Jets,  $E_T > 100$  GeV: 1-10 nb

J/ψ, B meson production: 10-100 nb

## The trigger is organized in 3 levels

### L1: hardware, synchronous

parallel processing;

Pipeline 42 clock cycles deep

decision in 5 ms;

Max Accepted rate 35 kHz

### L2: hardware and software, asynchronous

decision in ~30 ms;

Max Accept rate 600 Hz

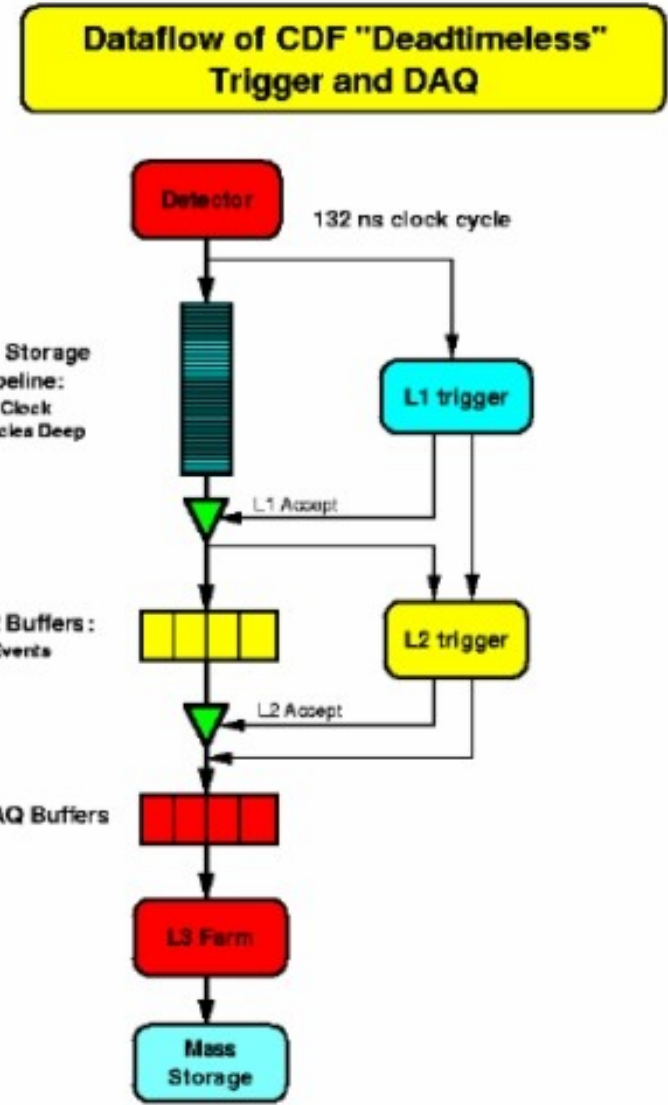
### L3: software

PC Farm;

Offline optimized Algorithms

Max Accepted rate 100 Hz

*Experimental Subnuclear Physics*





# Jets

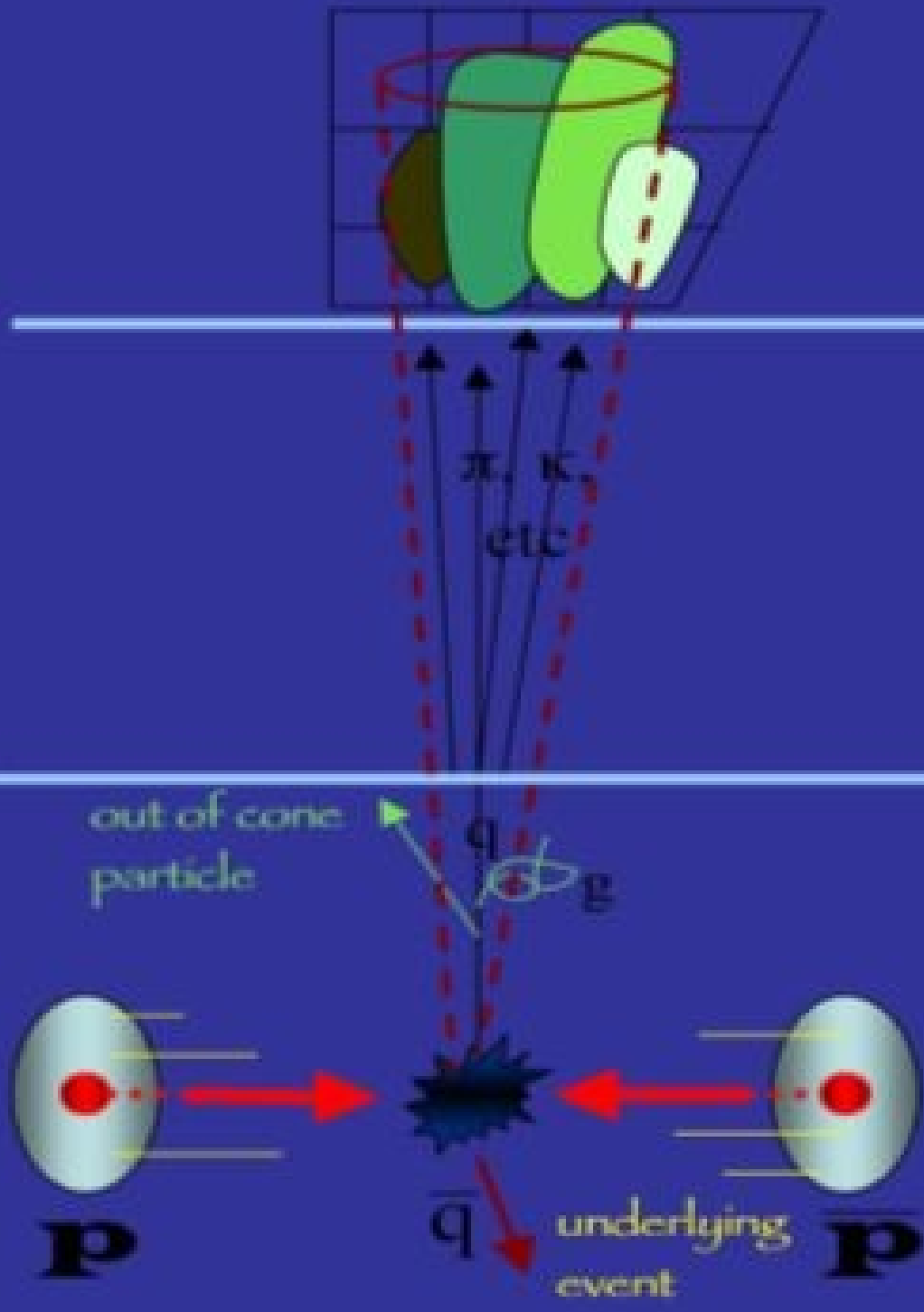
Energy deposits in the calorimeters

Physical particles

Hadronization of the quarks and gluons into real particles

evolution of the quarks and gluons: fragmentation

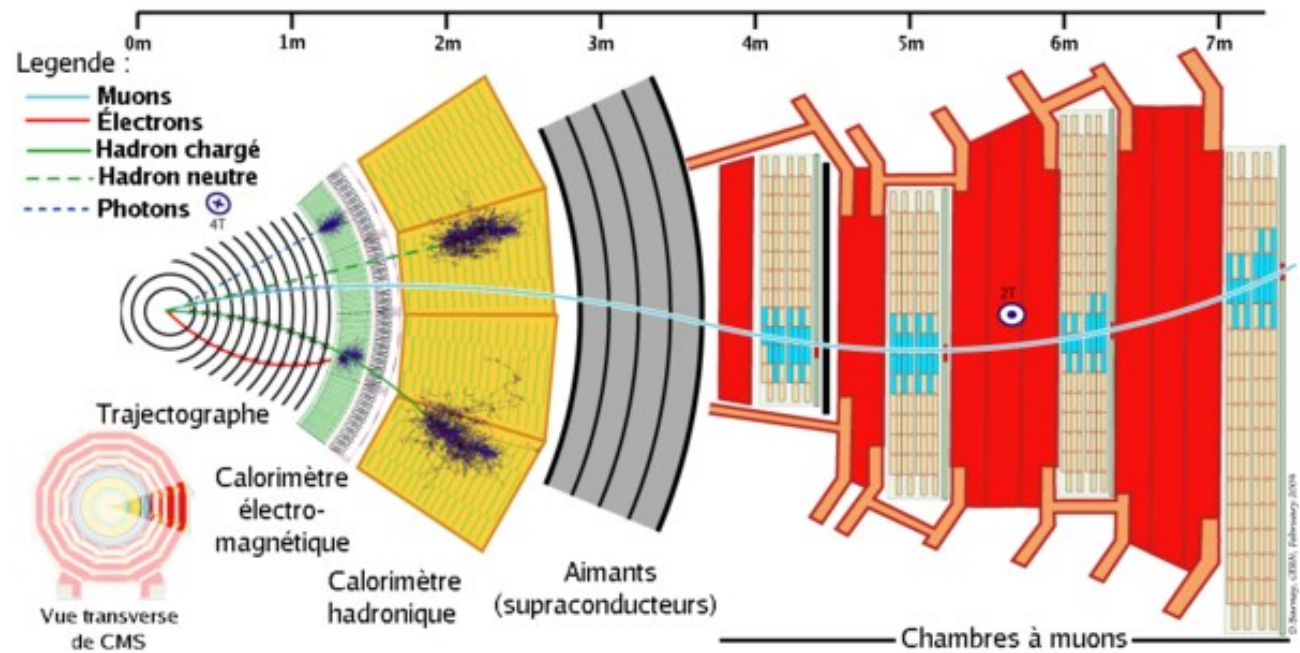
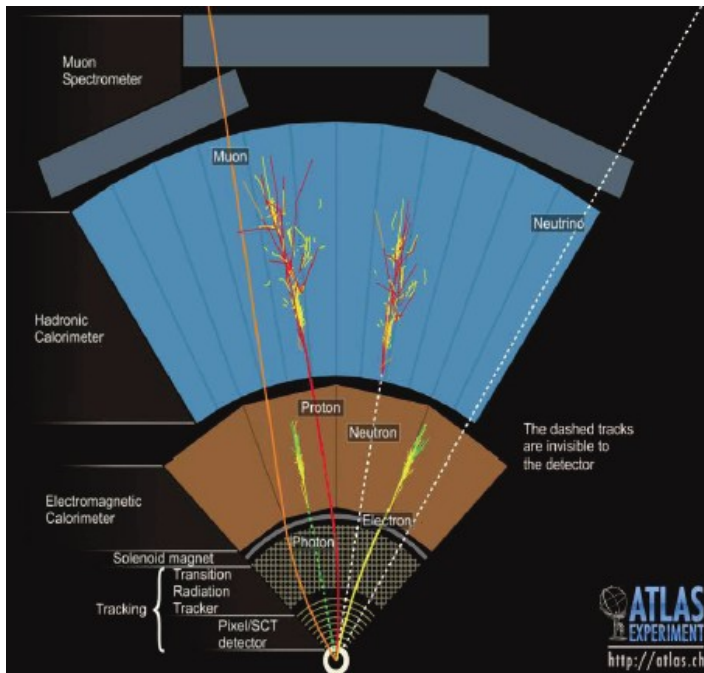
hard interaction between quarks and gluons



# Jets

## How to measure the jets?

- The **calorimeters are sensibles to charged and neutral particles**
- In the e.m. calorimeters the total number of secondary particles inside an e.m. cascade are measured  $\rightarrow E$  is proportional to  $N$
- In the hadronic calorimeters the processes are more complex but the concept is similar
- The correct measurement of the jet energy permits the reconstruction of the massive particle
- The energy measurement is also fundamental to reconstruct the **missing energy**



# Jets

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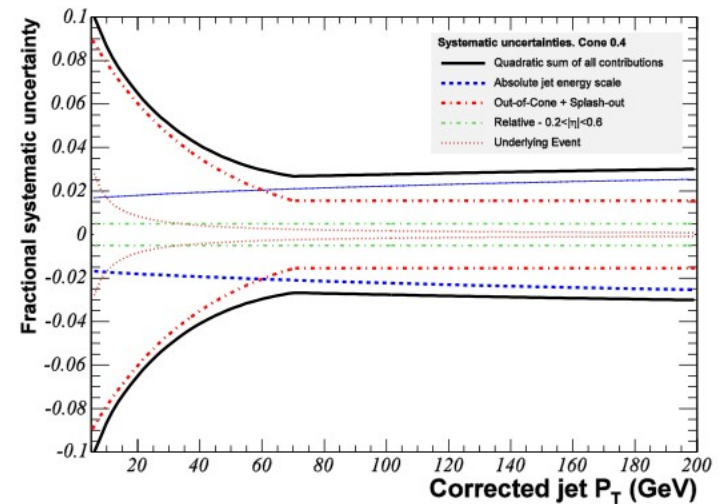
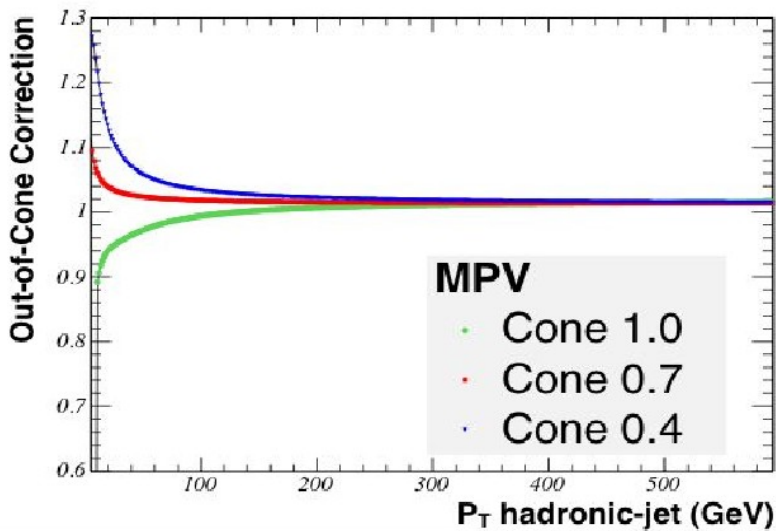
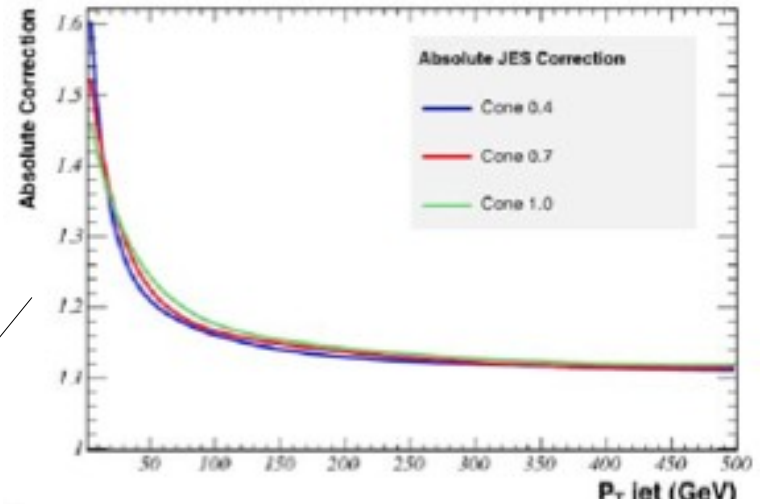
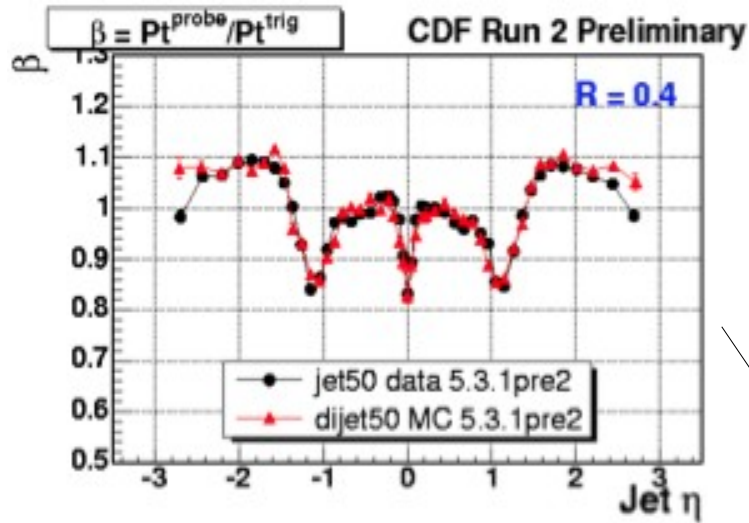
## Jet Energy Calibration

- To calibrate the measurement of the energy in CDF it is used detector-dependent correction, an absolute correction of the energy scale, and then additional corrections for minor effects:
  - ◆ **eta-dependent** correction → dijet balancing
  - ◆ **multiple interaction** correction →  $f(N_{\text{vtx}})$
  - ◆ **absolute scale** correction:  $E/p$  of single tracks is used for the MC “tuning”, it is then used to derive corrections to have a calorimetric measurement nearer to the possible original energy of the hadrons
  - ◆ last, **out of cone** and **underlying event** corrections are made
  - ◆ **Systematic errors reduced to 3%** (data/MC comparisons, g-jet balancing)
  - ◆ **Calorimeter stability, MC (fragmentation, simulation of single particle resp.)**
  - ◆ **Understanding of out-of-cone radiation and underlying event**
  - ◆ **Simulation of response function versus jet rapidity**
- D0 has a calorimeter with almost complete compensation ( $e/p < 1.05$ , linear with energy); disomogeneity and gaps between the cryostats have to be corrected
  - ◆ EM parts is calibrated with  $Z \rightarrow ee$  decays
  - ◆ **U noise** measured *in situ*; other offsets are used to compensate the **pile-up** (energy from previous interactions) and the **underlying event**
  - ◆ The response is measured as a function of the rapidity and  $E_T$  in gamma-jet events
  - ◆ **showering correction**:  $E_T$  flux vs DR off jet cones

# Jets

## CDF Jets Energy Corrections

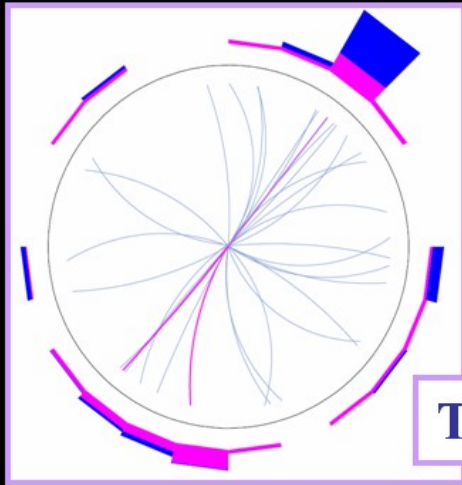
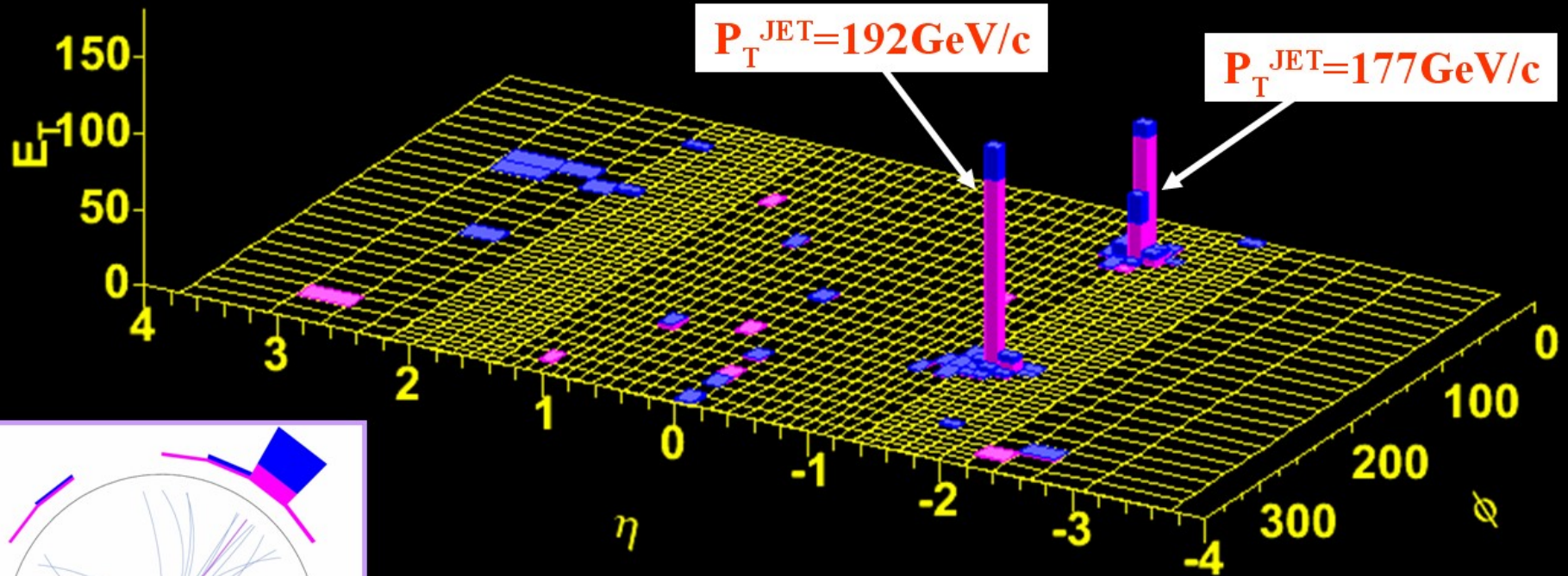
$$P_t^{\text{corr}} = (P_t^{\text{raw}} f_{\text{rel}} - \text{MI}) f_{\text{abs}} - \text{UE} + \text{OOC}$$



# Jets

Tower  $E_T > 0.5 \text{ GeV}$

$K_T$   $D=0.7$ : Raw  $P_T^{\text{JET}}$



Track  $p_T > 0.5 \text{ GeV}/c$

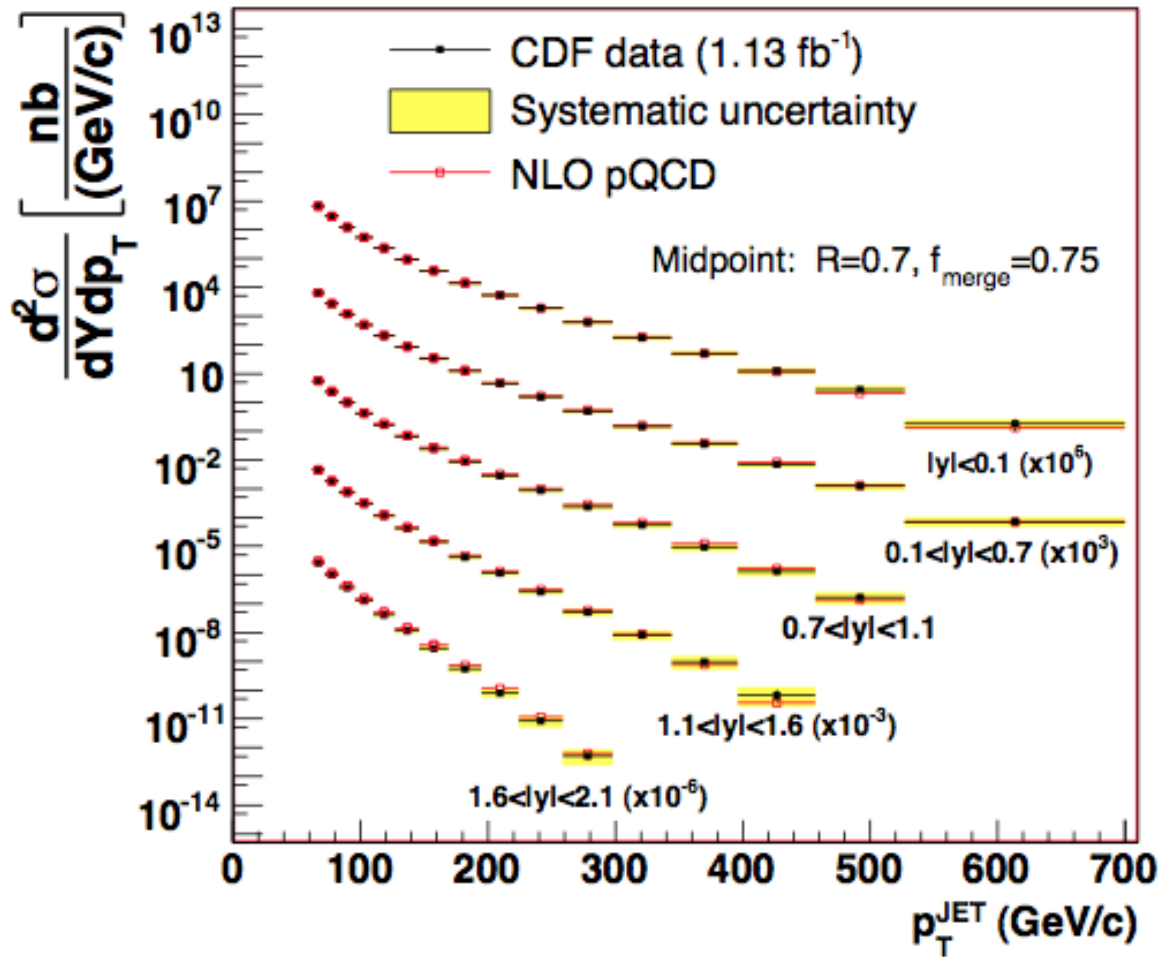
**CDF RUN II**

Run 163064

Event 6753986

# $p_T$ Distribution of the jets

- $p_T$  distribution of the jets at TEVATRON :



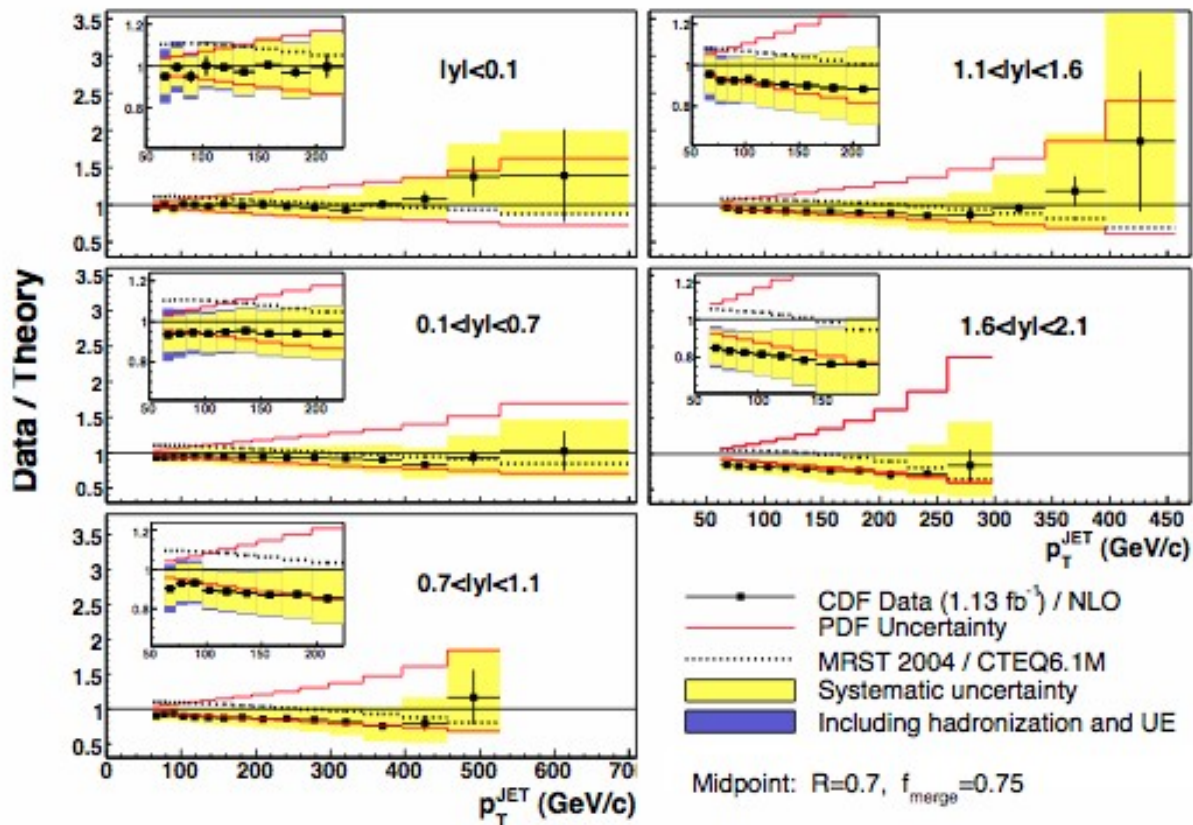
$$\frac{E d^3 \sigma}{d^3 p} \equiv \frac{d^3 \sigma}{d^2 p_T dy} \rightarrow \frac{1}{2\pi E_T} \frac{d^2 \sigma}{d E_T d \eta}$$

( assuming massless jets)

NLO QCD prediction ( $O(\alpha_s^3)$ )

Perfect agreement of the theoretical predictions to the data over 6 orders of magnitude

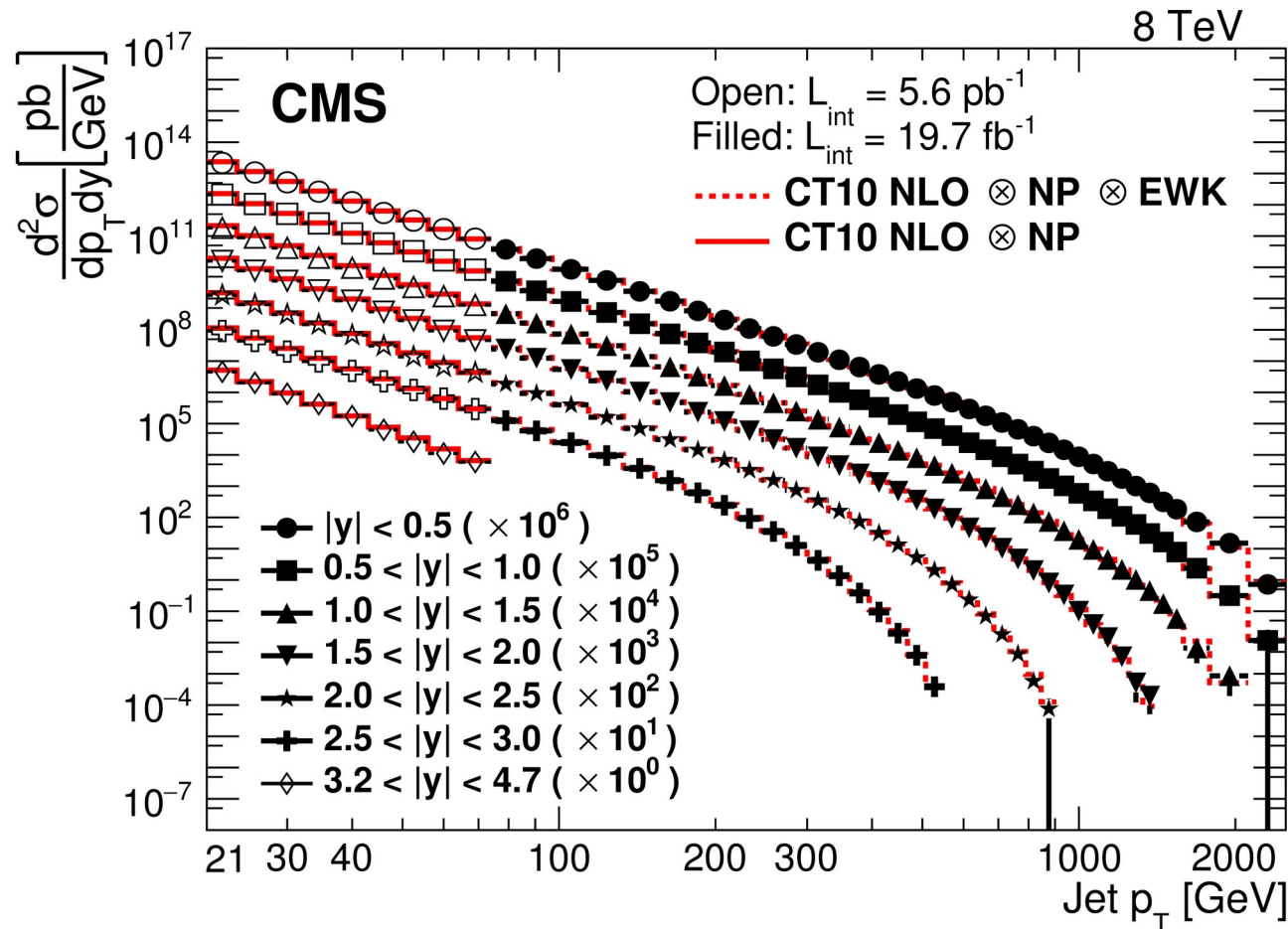
# $p_T$ Distribution of the jets



- Large systematic uncertainties
- A small error in the energetic scale  $\rightarrow$  large variation in  $\sigma$  because the distribution is falling rapidly
- But the systematic uncertainties are correlated, shift all the points up and down
- Search for surplus (respect to the QCD theory) at high  $E_T \rightarrow$  possible indication of quarks substructures or other processes beyond the Standard Model

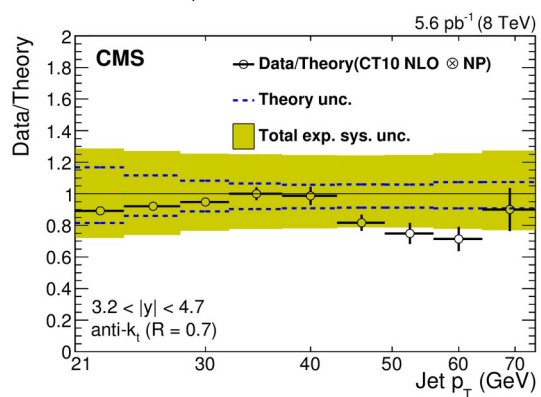
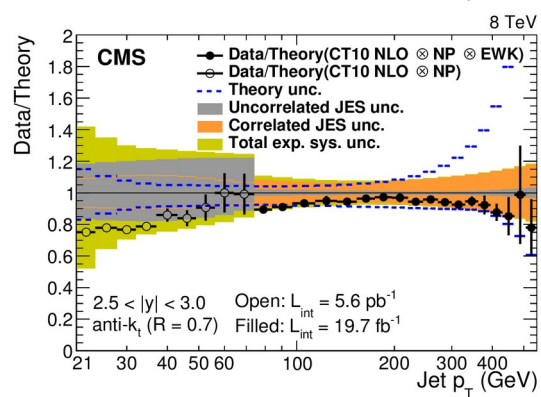
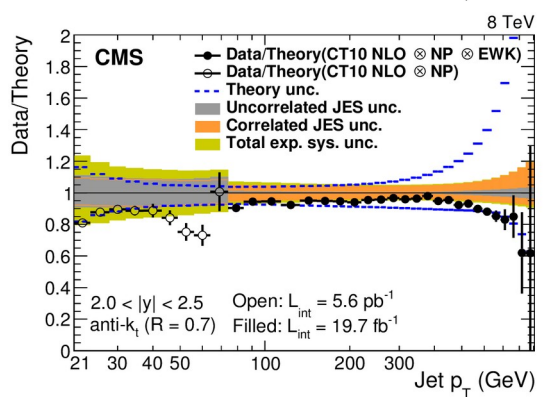
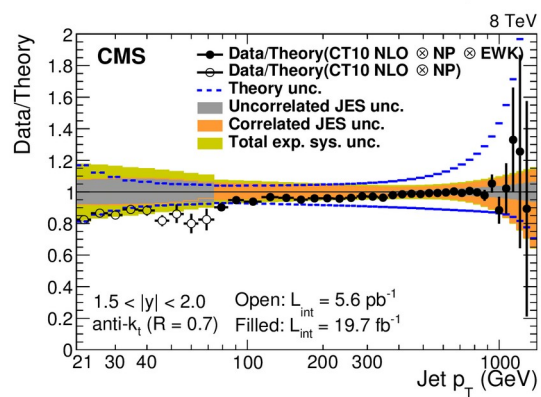
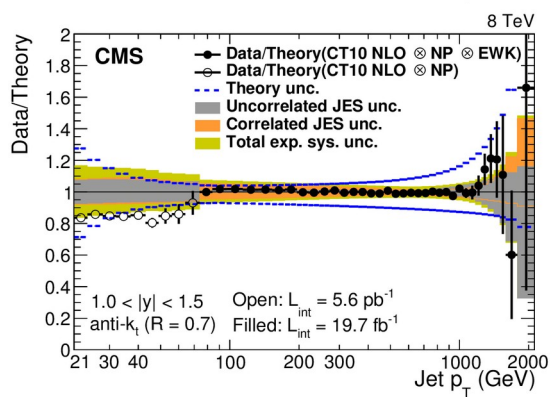
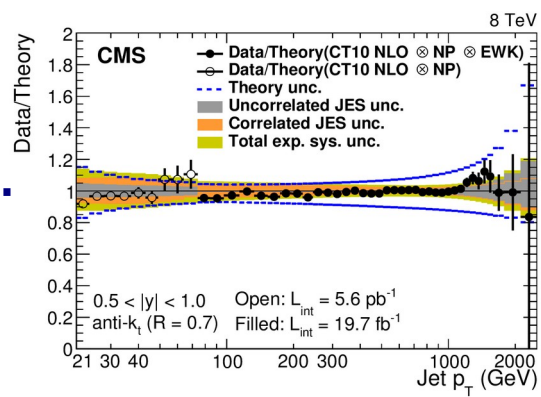
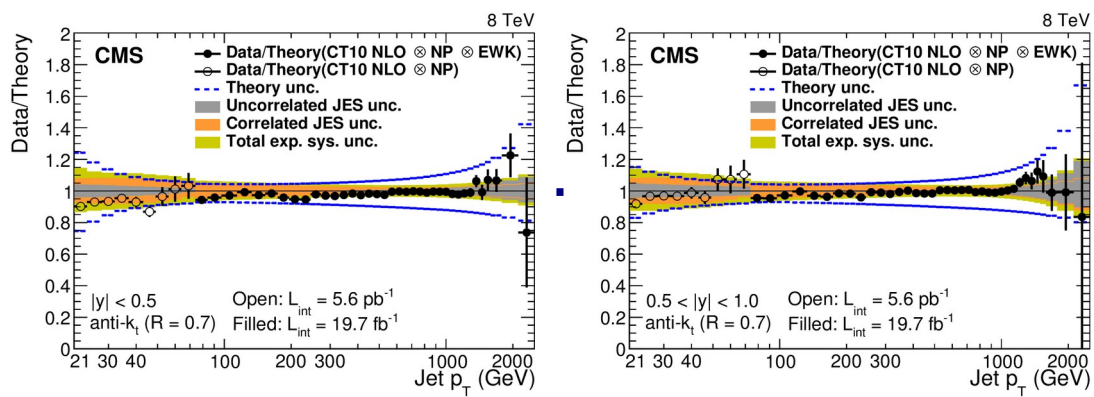
# $p_T$ Distribution of the jets

## ● $p_T$ distribution of the jets at LHC :



Double-differential inclusive jet cross sections as function of jet  $p_T$ . Data (open points for the low- $p_T$  analysis, filled points for the high- $p_T$  one) and NLO predictions based on the CT10 PDF set corrected for the non-perturbative (NP) factor for the low- $p_T$  data (solid line) and the NP and electroweak correction factors for the high- $p_T$  data (dashed line). The comparison is carried out for six different  $|y|$  bins at an interval of  $\Delta|y|=0.5$ .





Ratios of data to the theory prediction using the CT10 PDF set. For comparison, the total theoretical (band enclosed by dashed lines) and the total experimental systematic uncertainties (band enclosed by full lines) are shown as well. The error bars correspond to the statistical uncertainty in the data

# Jets Angular Distribution

- **Jets Angular Distribution (J.A.D.):**

In the parton-parton CM system the **J.A.D.** is sensitive to the form of the  $2 \rightarrow 2$  matrix elements

$$\frac{d^3\sigma}{dy_3 dy_4 dp_T^2} = \frac{1}{16\pi s^2} \sum_{i,j,k,l=q,\bar{q},g} \frac{f_i(x_1, \mu^2)}{x_1} \frac{f_j(x_2, \mu^2)}{x_2} \times \sum |M(ij \rightarrow kl)|^2 \frac{1}{1+\delta_{kl}}$$

(knowing that:  $dp_T^2 dy_3 dy_4 = \frac{1}{2} s dx_1 dx_2 d\cos\theta^*$  )

$$\frac{d^2\sigma}{dM_{jj}^2 d\cos\theta^*} = \sum_{i,j=q,\bar{q},g} \int_0^1 dx_1 dx_2 f_i(x_1, \mu^2) f_j(x_2, \mu^2) \delta(x_1 x_2 s - M_{jj}^2) \frac{d\hat{\sigma}^{ij}}{d\cos\theta^*}$$

with ( $\theta^*$  is the polar angle of one of the jets in dijet CM system):

$$\frac{d\hat{\sigma}^{ij}}{d\cos\theta^*} = \sum_{k,l} \frac{1}{32\pi M_{jj}^2} \sum |M(ij \rightarrow kl)|^2 \frac{1}{1+\delta_{kl}}$$

# Jets Angular Distribution

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- In proton-antiproton collisions the most important subprocesses are:

$$gg \rightarrow gg ; gq \rightarrow gq; q\bar{q} \rightarrow q\bar{q}$$

for each of them there is a  $\theta^*$  distribution similar to the Rutherford scattering at small angle (characteristic of the exchange of a massless vector boson in the  $t$ -channel):

$$\frac{d\hat{\sigma}}{d\cos\theta^*} \sim \frac{1}{\sin^4(\theta^*/2)} = \frac{1}{(1-\cos\theta^*)^2}$$

- To study the deviations from this behaviour one uses the following variable:

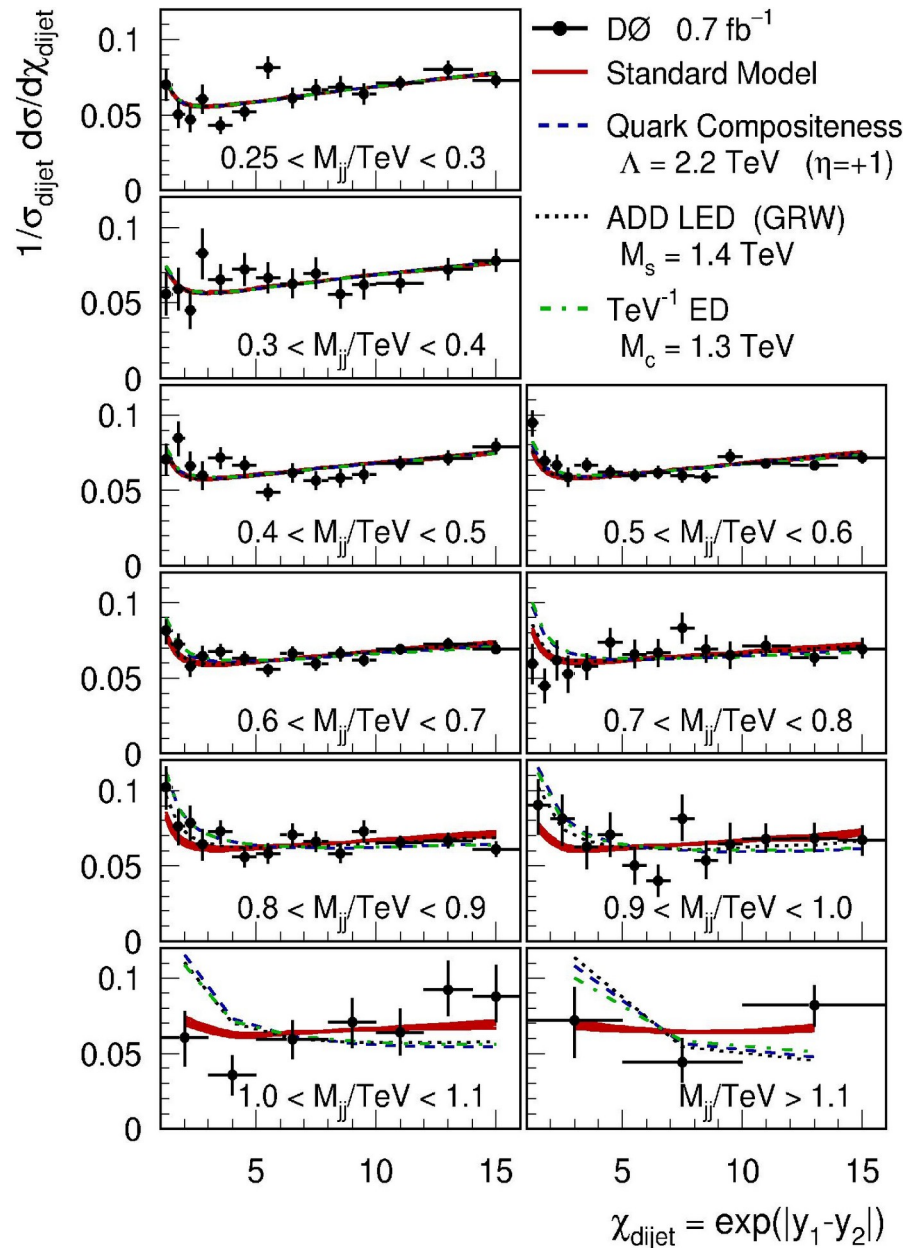
$$\chi = \frac{1+\cos\theta^*}{1-\cos\theta^*} = \exp(|y_1 - y_2|)$$

which removes the Rutherford singularity.

For small angles ( $\chi \rightarrow \infty$ ) one has:

$$\frac{d\hat{\sigma}}{d\chi} \simeq \text{costante} \quad \text{ignoring the dependence on the coupling constant and the parton distributions}$$

# Jets Angular Distribution



The data reject some types of scattering between partons, for example exchange of **scalar** gluons:

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or quark compositeness etc. etc.  
( exotic physics)

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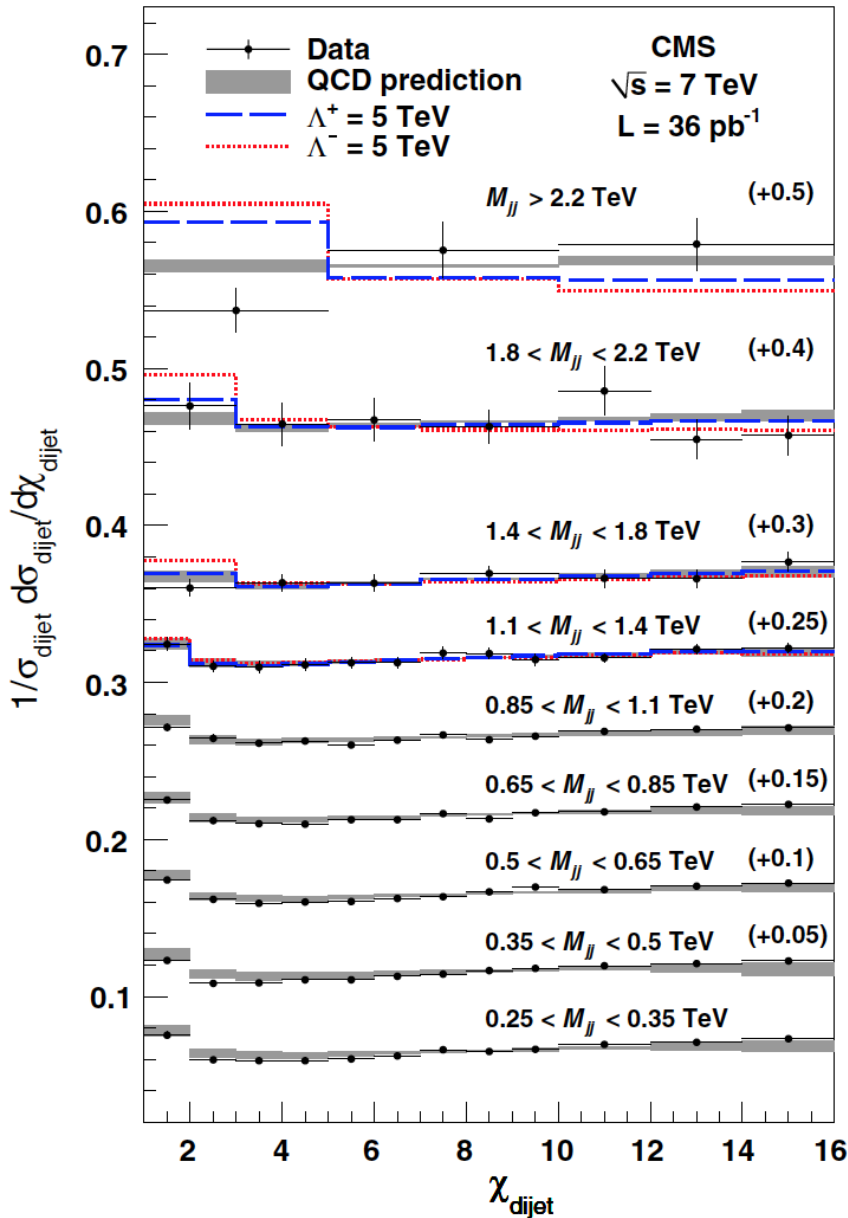


FIG. 1 (color online). Normalized dijet angular distributions in several  $M_{jj}$  ranges, shifted vertically by the additive amounts given in parentheses in the figure for clarity. The data points include statistical and systematic uncertainties. The results are compared with the predictions of pQCD at NLO (shaded band) and with the predictions including a contact interaction term of compositeness scale  $\Lambda^+ = 5$  TeV (dashed histogram) and  $\Lambda^- = 5$  TeV (dotted histogram). The shaded band shows the effect on the NLO pQCD predictions due to  $\mu_r$  and  $\mu_f$  scale variations and PDF uncertainties, as well as the uncertainties from the nonperturbative corrections added in quadrature.

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