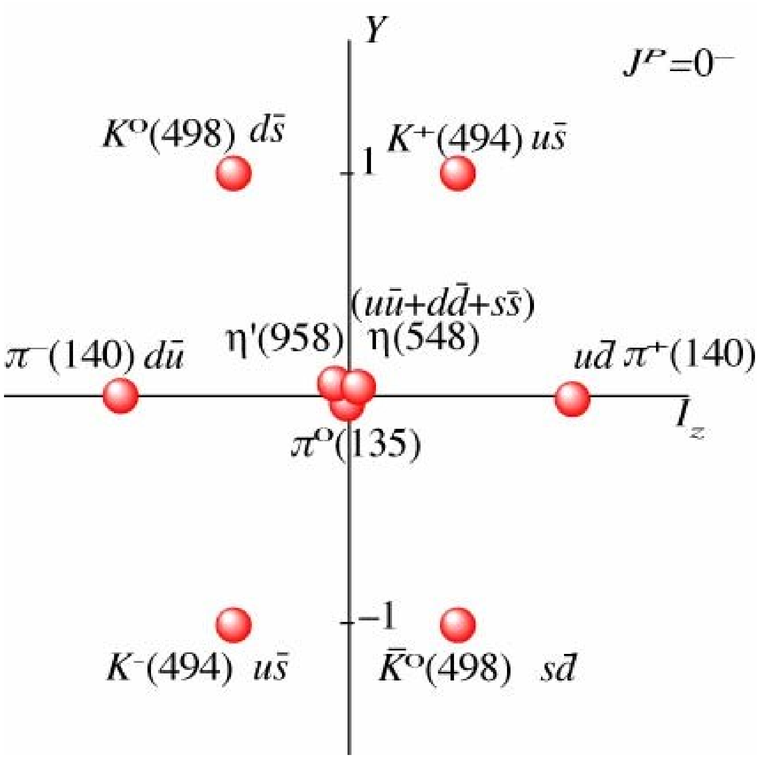
Quantum Chromodynamics (QCD)

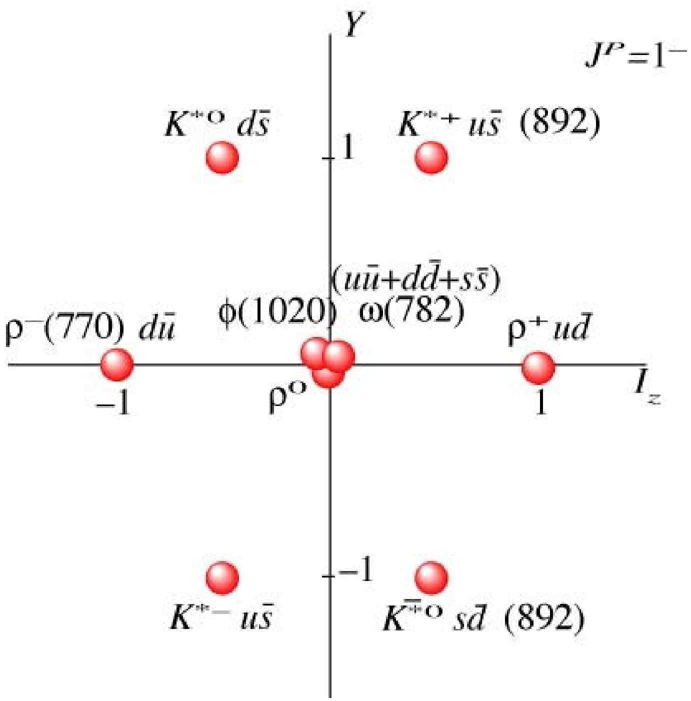
Fundamentals

- Colour SU(3)
- QCD lagrangian
- Local gauge invariance

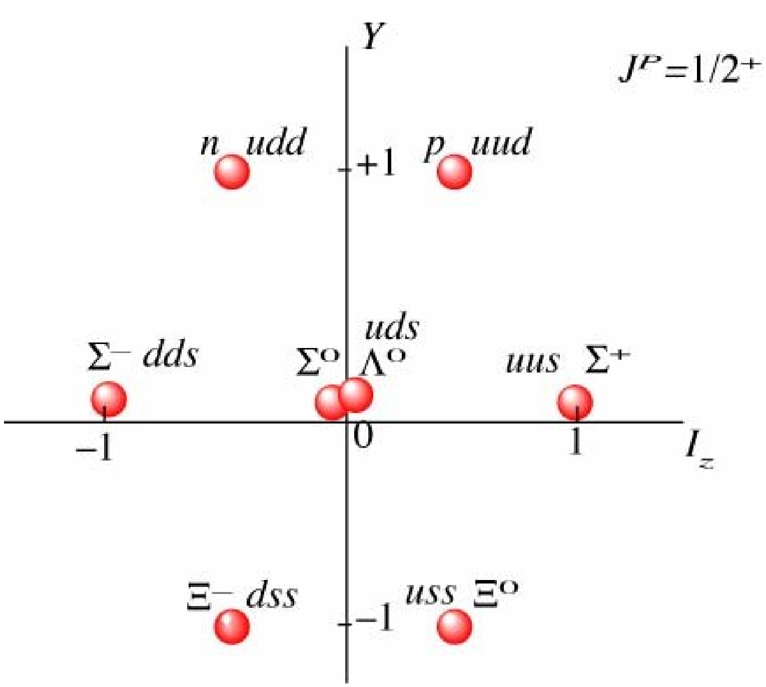
colour SU(3)

- **Hadronic matter is made of quarks**
  The idea of quarks comes from the symmetry present in the spectrum of the lowest-mass mesons and baryons: flavour SU(3) ($SU(3)_f$)

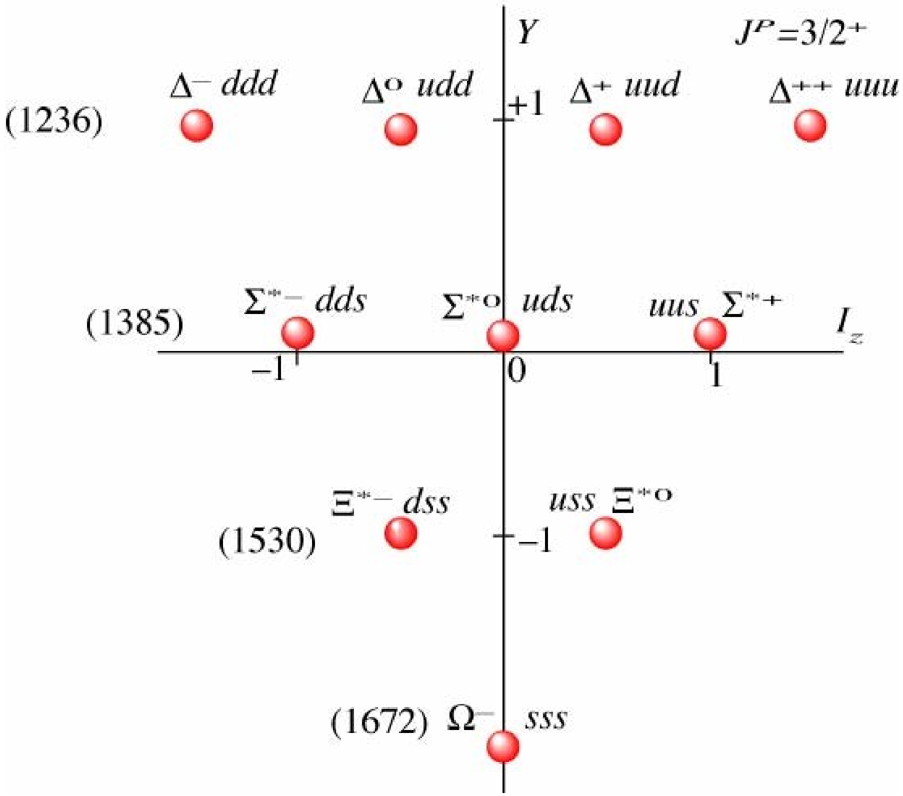




colour SU(3)



Y
$J^P=1/2^+$
n udd
p uud
+1
uds
Σ⁻ dds
Σ° Λ°
uus Σ⁺
−1
0
1
I_z
Ξ⁻ dss
uss Ξ°
−1



Y
$J^P=3/2^+$
(1236)
Δ⁻ ddd
Δ° udd
+1
Δ⁺ uud
Δ⁺⁺ uuu
(1385)
Σ*⁻ dds
Σ*° uds
uus Σ*⁺
I_z
−1
0
1
(1530)
Ξ*⁻ dss
uss Ξ*°
−1
(1672) Ω⁻ sss

colour SU(3)

- Quarks properties:

	$\mathcal{B}$	Q	I	I_z	S	C	B	T	m
d	1/3	-1/3	1/2	-1/2	0	0	0	0	4 - 8 MeV
u	1/3	2/3	1/2	1/2	0	0	0	0	1.5 - 4 MeV
s	1/3	-1/3	0	0	–1	0	0	0	80 - 130 MeV
c	1/3	2/3	0	0	0	+1	0	0	1.15- 1.35 GeV
b	1/3	-1/3	0	0	0	0	–1	0	4.1 - 4.4 GeV
t	1/3	2/3	0	0	0	0	0	+1	173 ± 3 GeV

- For example: baryons are interpreted as: *qqq*
  The 3*q* in the spin-3/2 baryons are in symmetrical state of space, spin and $SU(3)_f$ degrees of freedom
  Problem with the **Fermi-Dirac Statistics** ➔ the wave function must be antisymmetric ➔ **colour** degree of freedom ➔ each *q* has colour with 3 possible values:
  red, green, blue: a = 1, 2, 3

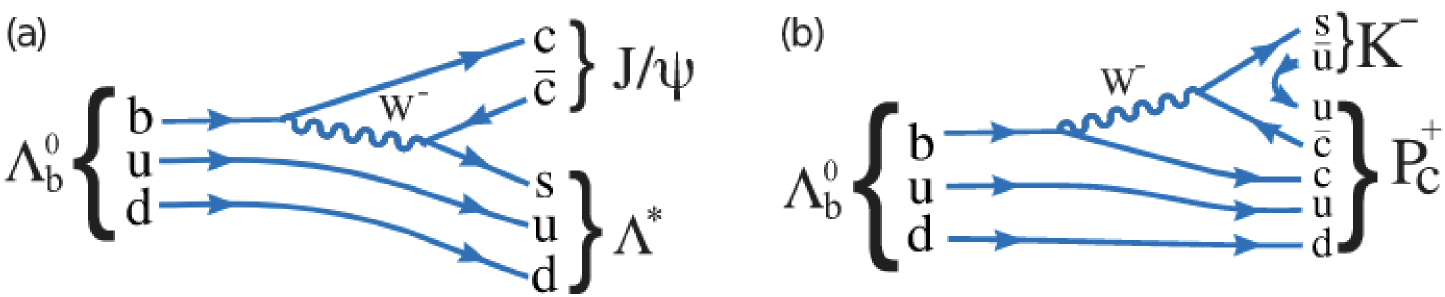
colour SU(3)

- Baryon wave functions are totally antisymmetric in this new index.
- The colour degree of freedom can lead to a proliferation of states → **only colour singlet states can exist in nature**.
- If $SU(3)_c$ is the group of colour transformation, the *q* transform according to the fundamental (3×3 unitary matrix) representation and the *anti-q* according to the complex conjugate representation.
- The basic singlet states are:

$$q_a \bar{q}^a \qquad \textit{for mesons}$$
$$\epsilon^{abc} q_a q_b q_c \qquad \textit{for baryons}$$

colour SU(3)

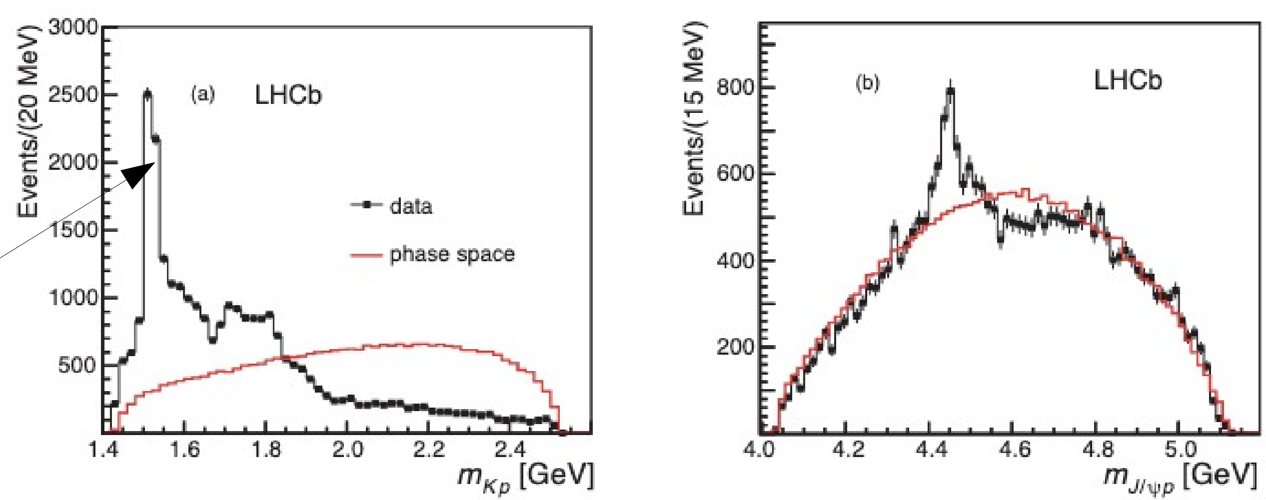
- … but QCD permits also more exotic compositions::



pentaquark candidate

Figure 1: Feynman diagrams for (a) $\Lambda_b^0 \to J/\psi\,\Lambda^*$ and (b) $\Lambda_b^0 \to P_c^+ K^-$ decay.

$\Lambda^* \to K^- p$

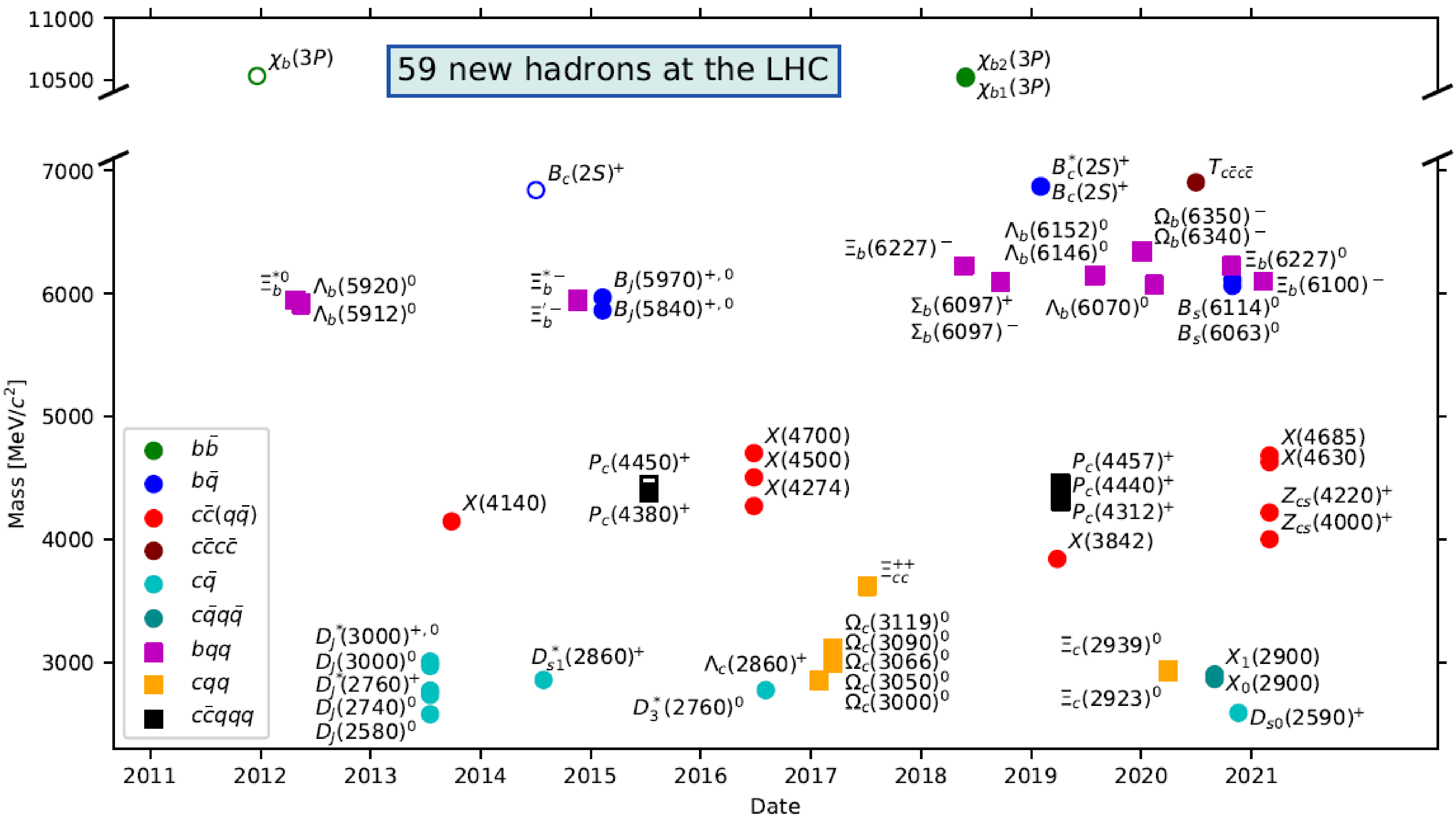


LHCb experiment
arXiv:1507:03414

in reality the situation is more complex with more than one peak.
see:
LHCb experiment
arXiv:1904:03947

Figure 2: Invariant mass of (a) K^-p and (b) $J/\psi p$ combinations from $\Lambda_b^0 \to J/\psi K^- p$ decays. The solid (red) curve is the expectation from phase space. The background has been subtracted.

colour SU(3)

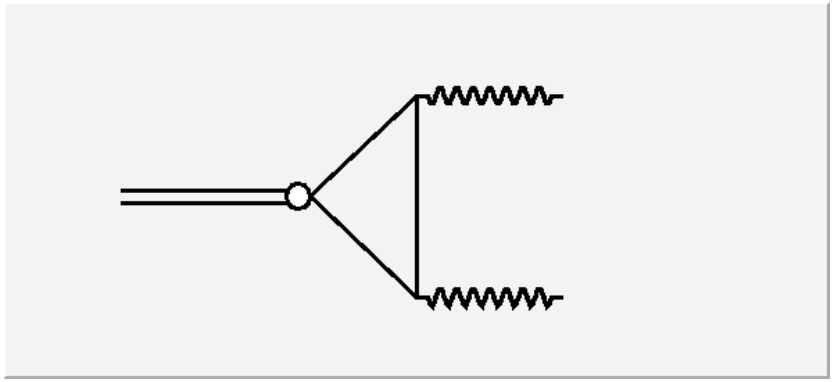

59 new hadrons at the LHC
Mass [MeV/c^2]
Date
$b\bar{b}$
$b\bar{q}$
$c\bar{c}(q\bar{q})$
$c\bar{c}c\bar{c}$
$c\bar{q}$
$c\bar{q}q\bar{q}$
bqq
cqq
$c\bar{c}qqq$
$\chi_b(3P)$
$\chi_{b2}(3P)$
$\chi_{b1}(3P)$
$B_c(2S)^+$
$B_c^*(2S)^+$
$T_{c\bar{c}c\bar{c}}$
$\Omega_b(6350)^-$
$\Omega_b(6340)^-$
$\Lambda_b(6152)^0$
$\Lambda_b(6146)^0$
$\Xi_b(6227)^-$
$\Xi_b(6227)^0$
$\Xi_b(6100)^-$
Ξ_b^{*0}
$\Lambda_b(5920)^0$
$\Lambda_b(5912)^0$
Ξ_b^{*-}
$\Xi_b^{'-}$
$B_J(5970)^{+,0}$
$B_J(5840)^{+,0}$
$\Sigma_b(6097)^+$
$\Sigma_b(6097)^-$
$\Lambda_b(6070)^0$
$B_s(6114)^0$
$B_s(6063)^0$
$X(4700)$
$X(4500)$
$X(4274)$
$P_c(4450)^+$
$P_c(4380)^+$
$X(4140)$
$P_c(4457)^+$
$P_c(4440)^+$
$P_c(4312)^+$
$X(3842)$
$X(4685)$
$X(4630)$
$Z_{cs}(4220)^+$
$Z_{cs}(4000)^+$
Ξ_{cc}^{++}
$\Omega_c(3119)^0$
$\Omega_c(3090)^0$
$\Omega_c(3066)^0$
$\Omega_c(3050)^0$
$\Omega_c(3000)^0$
$\Xi_c(2939)^0$
$\Xi_c(2923)^0$
$X_1(2900)$
$X_0(2900)$
$D_{s0}(2590)^+$
$D_J^*(3000)^{+,0}$
$D_J(3000)^0$
$D_J^*(2760)^+$
$D_J(2740)^0$
$D_J(2580)^0$
$D_{s1}^*(2860)^+$
$\Lambda_c(2860)^+$
$D_3^*(2760)^0$

colour SU(3)

Experimental results (*important from an historical point of view)*) about the colour

- $\pi^0 \to \gamma\gamma$

$$\Gamma(\pi^o \to \gamma\gamma) = \xi^2 \left(\frac{\alpha}{\pi}\right)^2 \frac{1}{64\pi} \frac{m_\pi^2}{f_\pi^2} = 7.6\,\xi^2\, eV$$



where $f_\pi \sim 93$ MeV ≡ pion decay constant
Experimental value Γ = 7.7 ± 0.6 eV
The value of ξ is:

$$\xi = 3\left[\left(\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2\right] = 1$$

- but the decay rate was calculated by Steinberger (1949) well before the discovery of quarks, using *p* e *n* as costituents:

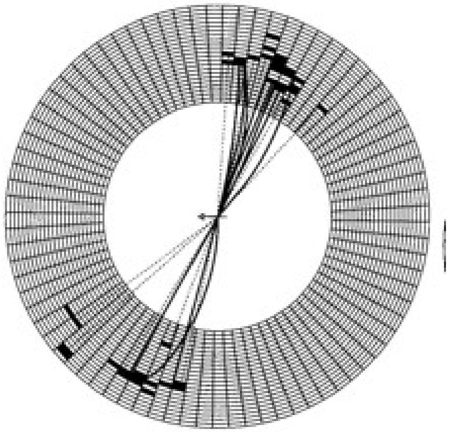
$$\xi = \left[(1)^2 - (0)^2\right] = 1$$

- **the result is not conclusive.**

colour SU(3)

- **R ratio:**

$$R = \frac{\sigma(e^+ e^- \to hadrons)}{\sigma(e^+ e^- \to \mu^+ \mu^-)} = 3 \sum_i q_i^2$$



JADE experiment

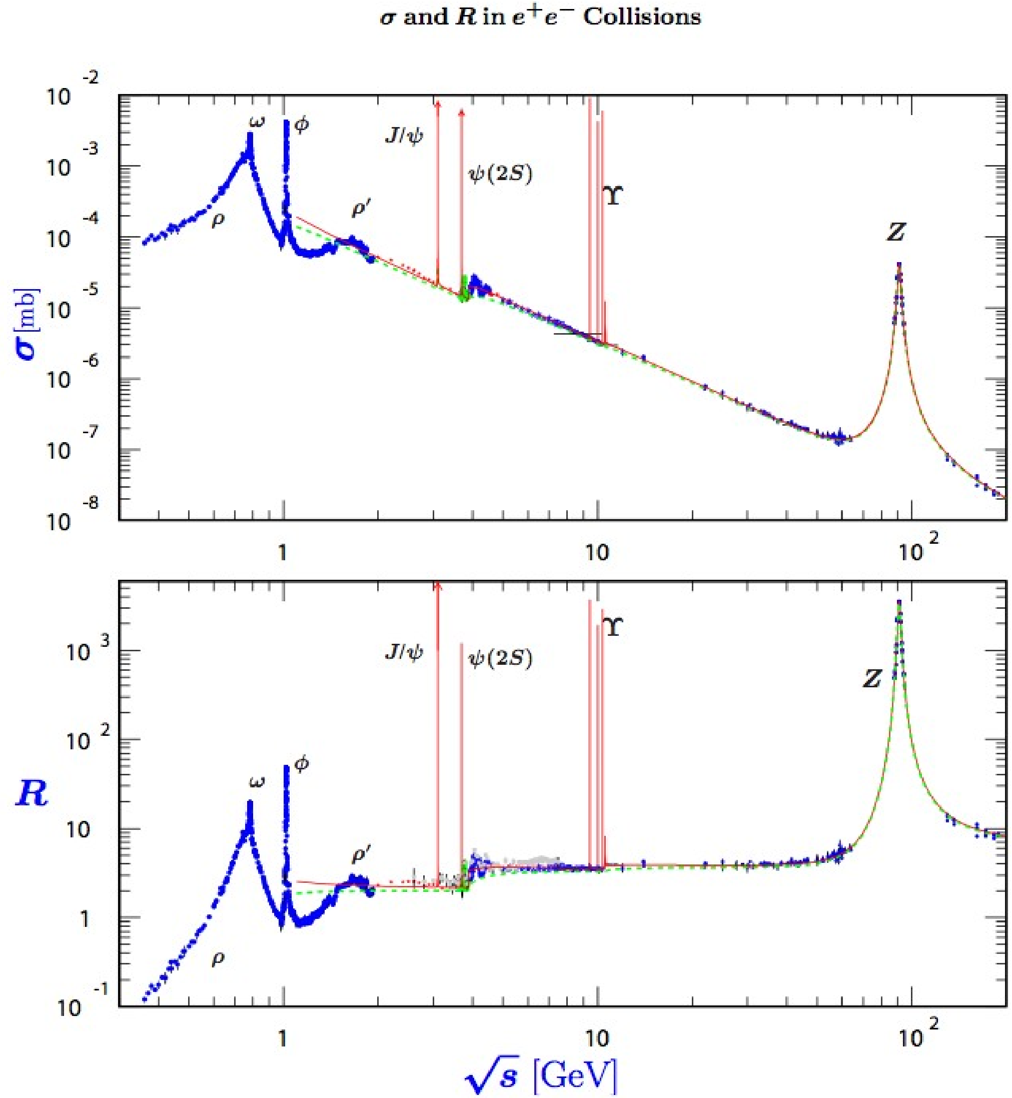


Figure 46.6: World data on the total cross section of $e^+e^- \to hadrons$ and the ratio $R(s) = \sigma(e^+e^- \to hadrons, s)/\sigma(e^+e^- \to \mu^+\mu^-, s)$. $\sigma(e^+e^- \to hadrons, s)$ is the experimental cross section corrected for initial state radiation and electron-positron vertex loops, $\sigma(e^+e^- \to \mu^+\mu^-, s) = 4\pi\alpha^2(s)/3s$. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are an educative guide: the broken one (green) is a naive quark-parton model prediction, and the solid one (red) is 3-loop pQCD prediction (see "Quantum Chromodynamics" section of this *Review*, Eq. (9.7) or, for more details, K. G. Chetyrkin *et al.*, Nucl. Phys. **B586**, 56 (2000) (Erratum *ibid.* **B634**, 413 (2002)). Breit-Wigner parameterizations of J/ψ, $\psi(2S)$, and $\Upsilon(nS), n = 1, 2, 3, 4$ are also shown. The full list of references to the original data and the details of the R ratio extraction from them can be found in [arXiv:hep-ph/0312114]. Corresponding computer-readable data files are available at http://pdg.lbl.gov/current/xsect/. (Courtesy of the COMPAS (Protvino) and HEPDATA (Durham) Groups, May 2010.)

colour SU(3)

- **R ratio:**

$$R = \frac{\sigma(e^+ e^- \rightarrow hadrons)}{\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)} = 3 \sum_i q_i^2$$

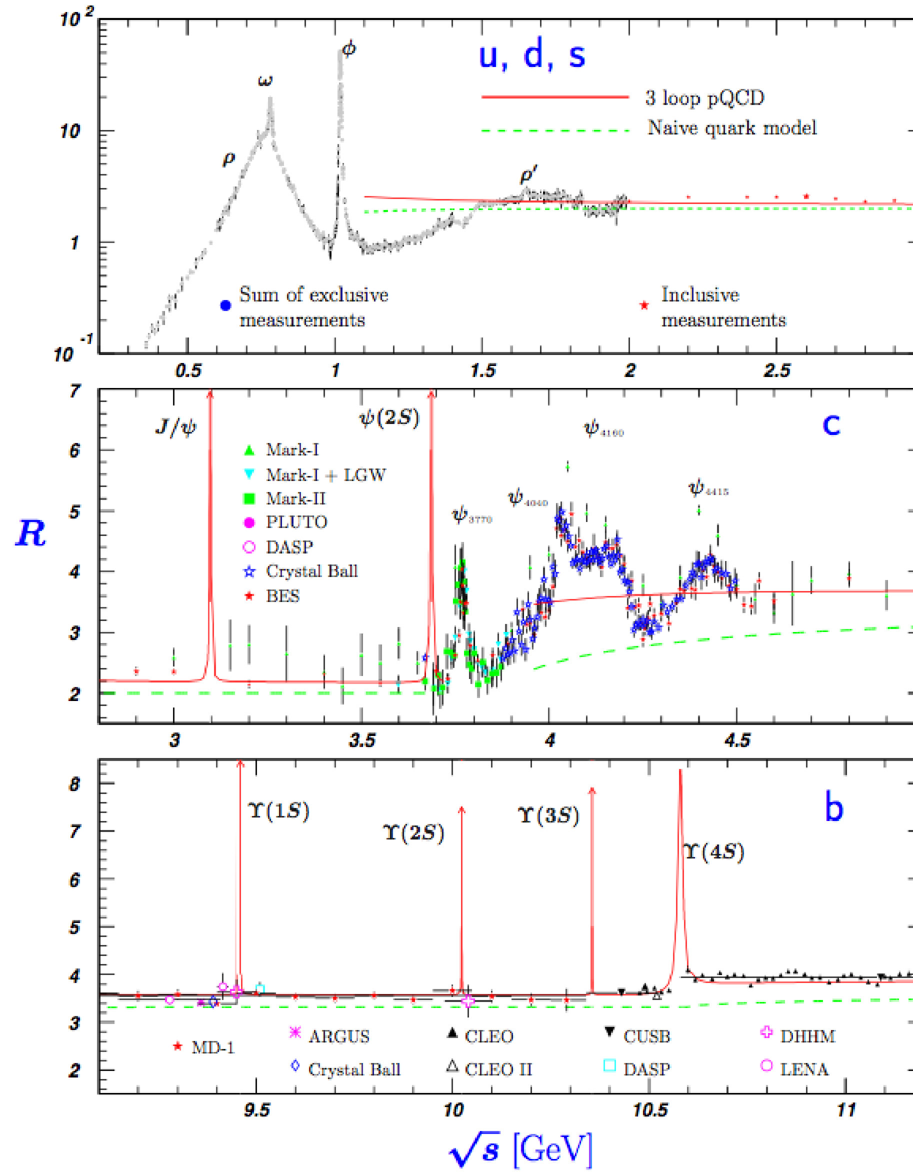
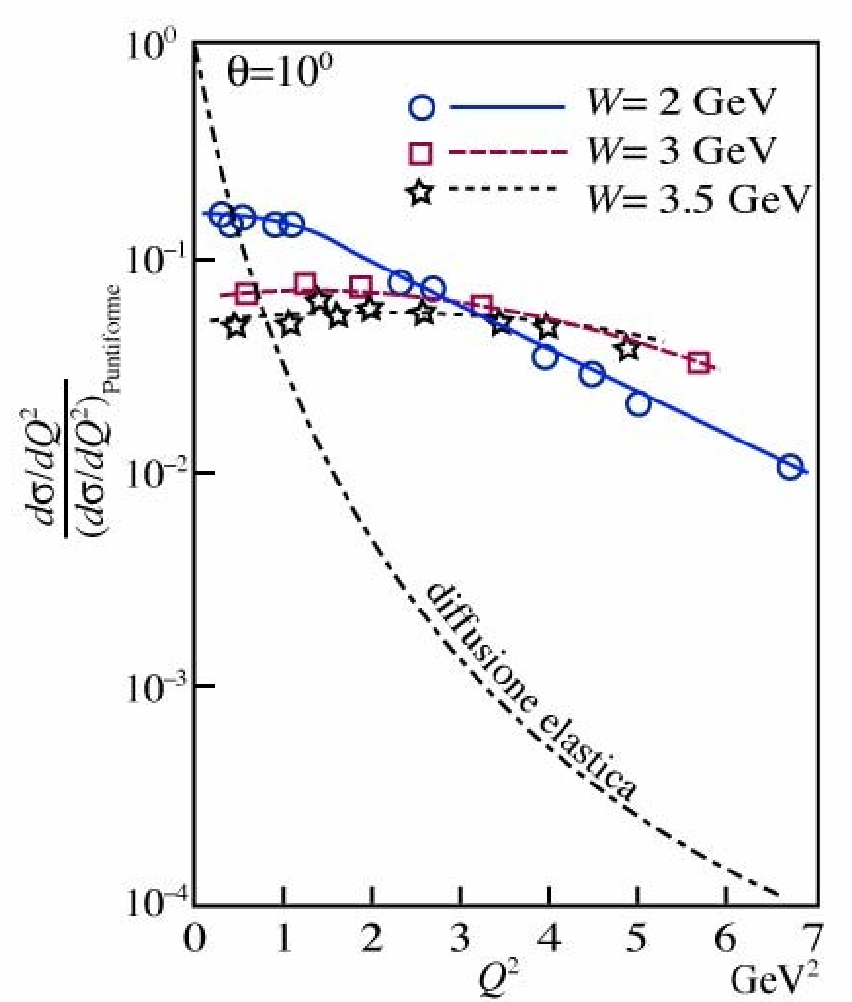
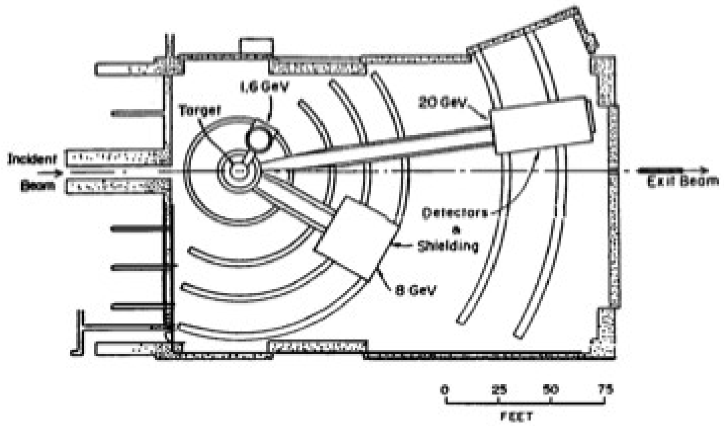


Figure 46.7: R in the light-flavor, charm, and beauty threshold regions. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are the same as in Fig. 46.6. **Note:** CLEO data above $\Upsilon(4S)$ were not fully corrected for radiative effects, and we retain them on the plot only for illustrative purposes with a normalization factor of 0.8. The full list of references to the original data and the details of the R ratio extraction from them can be found in [arXiv:hep-ph/0312114]. The computer-readable data are available at http://pdg.lbl.gov/current/xsect/. (Courtesy of the COMPAS (Protvino) and HEPDATA (Durham) Groups, May 2010.)

colour SU(3)

- **Evidence of the Quarks**
  Deep inelastic scattering: $e^- N \to e^- X$ at SLAC





colour SU(3)

- **Asymptotic freedom**
  The q are not observed as free particles. They suffer strong interactions which bind them into the hadrons.
  The asymptotic freedom predicts:

  $\alpha_s \gg 1$ for large *distances* ➜ **confinament**

  $\alpha_s < 1$ for small *distances* ➜ **free** ***q*** (at asymptotically large energies)

- The approach to asymptotia is very slow.
- At any finite energy there are calculable corrections to the free quark result which are unambiguous predictions of the theory.

Lagrangian of QCD

- The Feynman rules describing the interactions of quarks and gluons are derived by the **lagrangian density:**

$$\mathscr{L}_{\text{QCD}} = \mathscr{L}_{\text{classical}} + \mathscr{L}_{\text{gauge-fixing}} + \mathscr{L}_{\text{ghost}}$$

- where:

$$\mathscr{L}_{classical} = -\frac{1}{4} F^{A}_{\alpha\beta} F^{\alpha\beta}_{A} + \sum_{flavors} \bar{q}_a \left(i \gamma_\mu D^\mu - m \right)_{ab} q_b$$

$$F^{A}_{\alpha\beta} = \partial_\alpha A^{A}_{\beta} - \partial_\beta A^{A}_{\alpha} - g f^{ABC} A^{B}_{\alpha} A^{C}_{\beta}$$

the first two terms are similar to the QED.
The third term gives rise to the **self-interactions** of the gluons (vertices with 3 or 4 gluons). It gives origin to the property of **asymptotic freedom**.
f^{ABC} (A,B,C = 1, …, 8) are the **structure constants** of the $SU(3)_c$ colour group;
g is the coupling constant which detemines the strength of the interaction between coloured quanta
The quarks q_a (a = 1, 2, 3) are in a triplet representation of the colour group.

Local gauge invariance

- $\mathcal{L}_{\text{classical}} \equiv$ **invariant under local gauge transformations.**
  One can perform a redefinitions of the quark fields independently at every point in space and time, without changing the physical content of the theory.

$$q_a(x) \to q'_a(x) = \exp(it\cdot\theta(x))_{ab}\, q_b(x) \equiv \Omega(x)_{ab}\, q_b(x)$$

$$D_\alpha q(x) \to D'_\alpha q'(x) \equiv \Omega(x) D_\alpha q(x)$$

  The covariant derivative is so called because it transforms under a local gauge transformation in the same way as the quark field itself. (Where t is a matrix in the fundamental representation of $SU(3)_c$)

- from the last equation one can derive the properties of the gauge field A (**gluon**):

$$D'_\alpha q'(x) = (\partial_\alpha + igt\cdot A'_\alpha)\,\Omega(x) q(x) =$$
$$\equiv (\partial_\alpha \Omega(x))\, q(x) + \Omega(x)\,\partial_\alpha q(x) + ig(t\cdot A'_\alpha)\,\Omega(x) q(x)$$
$$t\cdot A_\alpha \equiv \sum_A t^A \cdot A_\alpha^A$$

- with: $$D'_\alpha q' \equiv \Omega\, D_\alpha q$$

- one has: $$t\cdot A'_\alpha = \Omega\,(t\cdot A_\alpha)\,\Omega^{-1} + \frac{i}{g}(\partial_\alpha \Omega)\,\Omega^{-1}$$

Local gauge invariance

- The transformation property of the non-Abelian field strength tensor, $F_{\alpha\beta}$, is:

$$t \cdot F_{\alpha\beta} \rightarrow t \cdot F'_{\alpha\beta} = \Omega(x) t \cdot F_{\alpha\beta} \Omega^{-1}(x)$$

- Gluons have no mass because there is no gauge invariant way of including a mass. A term such as:

$$m^2 A_\alpha A^\alpha$$

is not gauge invariant. This is very similar to the QED requirement of a massless photon.

- On the other hand the mass term for the quarks is gauge-invariant:

$$m \bar{q} q = m \bar{q}' q'$$

Feynman Rules

- **To obtain the Feynman rules of QCD, $\mathscr{L}_{\text{classical}}$ is not enough.**
- The key point is that **it is impossible to define the propagator for the gluon field without making a choice of gauge.**
- The choice:

$$\mathscr{L}_{gauge-fixing} = -\frac{1}{2\lambda}(\partial^{\alpha} A_{\alpha}^{A})^2$$

  fixes the class of **covariant gauges** with gauge parameter λ
- In a non-Abelian theory such as QCD this covariant gauge must be supplemented by a ghost Lagrangian, which is given by:

$$\mathscr{L}_{ghost} = \partial_{\alpha} \eta^{A\dagger} (D_{AB}^{\alpha} \eta^{B})$$

  $\eta^A \equiv$ complex scalar field which obeys Fermi statistics.
  The ghost fields cancel unphysical degrees of freedom which would otherwise propagate in covariant gauges.