Lecture 08 Cristiano Panighel 2058351

- Show that $\sigma'(y) = \sigma(y)(1-\sigma(y))$

We know that $\sigma(y) = \frac{1}{1+e^{-y}}$

So, I first compute the derivative $\sigma'(y) = \frac{\partial}{\partial y}\left(\frac{1}{1+e^{-y}}\right)$ using the chain rule, $D(g(f(y))) = g'(f(y)) \cdot f'(y)$ with functions $f(y) = 1+e^{-y}$ and $g(y) = \frac{1}{f(y)}$.

For the first factor, I know that $\frac{\partial}{\partial v}\left(\frac{1}{v}\right) = -\frac{1}{v^2}$, so if I consider $1+e^{-y}$ as v, I get $g'(f(y)) = -\frac{1}{(1+e^{-y})^2}$

The second factor, knowing that a derivate of a costant is 0, I obtain $f'(y) = -1 \cdot e^{-y}$

Putting all together, I have

$$g'(f(y)) \cdot f'(y) = -\frac{1}{(1+e^{-y})^2} \cdot -e^{-y} = \frac{e^{-y}}{(1+e^{-y})^2} = \sigma'(y)$$

$$\sigma(y)(1-\sigma(y)) = \frac{1}{(1+e^{-y})} \cdot \left(1 - \frac{1}{(1+e^{-y})}\right) = \frac{1}{(1+e^{-y})} - \frac{1}{(1+e^{-y})^2} = \frac{1+e^{-y}-1}{(1+e^{-y})^2} = \frac{e^{-y}}{(1+e^{-y})^2}$$

From these results, I can say that $\sigma'(y) = \sigma(y)(1-\sigma(y))$