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First, we are going to prove that the logical function XOR can not be implemented using a single Perceptron.

The XOR function is defined as follows:

$$
\begin{aligned}
XOR: \{0,1\}^2 &\longrightarrow \{0,1\} \\
(1,1) &\longmapsto 0 \\
(1,0) &\longmapsto 1 \\
(0,1) &\longmapsto 1 \\
(0,0) &\longmapsto 0
\end{aligned}
$$

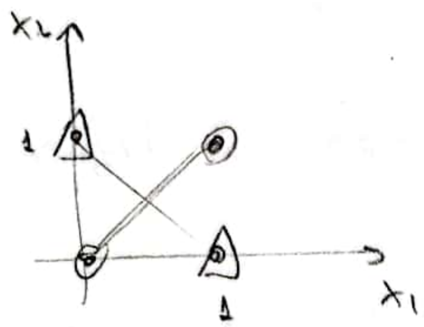
If it were possible to implement it using a single Perceptron, then we could assume that:

$$\exists\, w_1, w_2, b \in \mathbb{R} :$$

$$
\begin{cases}
w_1 + w_2 + b = 0 \\
w_1 + b = 1 \\
w_2 + b = 1 \\
b = 0
\end{cases}
$$

But this system does not have any solution, so we have arrived to a contradiction.

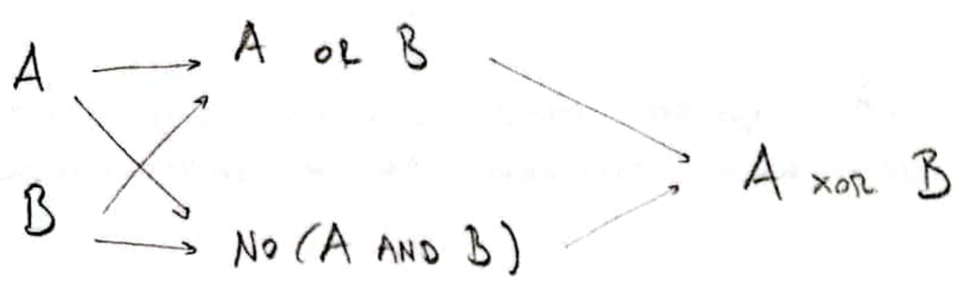
This make sense because it does not exist any hyperplane in $\mathbb{R}^2$ such that it separates/conect the points as we want



We knew it because the VC-dimension of $\mathbb{R}^2$ is 3 (not 4, as we can see here).

Next, we will a possible implementation of the XOR function using multi-layer network with hard thresholds. For this, I found useful to express the XOR function as follows:

$$A \text{ XOR } B = (A \text{ OR } B) \text{ AND } \text{NO}(A \text{ AND } B)$$



Using this reasoning, my implementation of the XOR function is the next: $x_1, x_2 \in \{0, 1\}$