

Show that  $\sigma'(y) = \sigma(y)(1 - \sigma(y))$  :

Answer :

$$\sigma(y) = \frac{1}{1+e^{-y}} \longrightarrow \frac{d}{dy} \sigma(y) = \frac{d}{dy} \left[ \frac{1}{1+e^{-y}} \right]$$

$$= \frac{d}{dy} (1+e^{-y})^{-1} = -(1+e^{-y})^{-2} (-e^{-y})$$

$$= \frac{e^{-y}}{(1+e^{-y})^2} = \frac{1}{(1+e^{-y})} \cdot \frac{e^{-y}}{(1+e^{-y})}$$

$$= \frac{1}{(1+e^{-y})} \cdot \frac{(1+e^{-y}) - 1}{(1+e^{-y})} = \frac{1}{1+e^{-y}} \cdot \left( \frac{1+e^{-y}}{1+e^{-y}} - \frac{1}{1+e^{-y}} \right)$$

$$= \frac{1}{1+e^{-y}} \cdot \left( 1 - \frac{1}{1+e^{-y}} \right) = \boxed{\sigma(y) \cdot (1 - \sigma(y))}$$