

$$\frac{d\sigma(y)}{dy} = ?$$

$$\frac{d}{dy} (f(g(y))) = f'(g(y)) * g'(y)$$

$$\frac{d\sigma(y)}{dy} = \frac{d}{dy} \left(\frac{1}{1+e^{-y}} \right) = \frac{d}{dy} (1+e^{-y})^{-1} = -(1+e^{-y})^{-2} * (-e^{-y}) = \frac{e^{-y}}{(1+e^{-y})^2}$$

$$= \frac{e^{-y}}{(1+e^{-y})(1-e^{-y})} = \frac{1}{1+e^{-y}} * \frac{e^{-y}}{1+e^{-y}} = \sigma(y) * \frac{e^{-y} + (-1+1)}{1+e^{-y}}$$

$$= \sigma(y) * \left(\frac{1+e^{-y}}{1+e^{-y}} - \frac{1}{1+e^{-y}} \right) = \boxed{\sigma(y) * (1 - \sigma(y))}$$