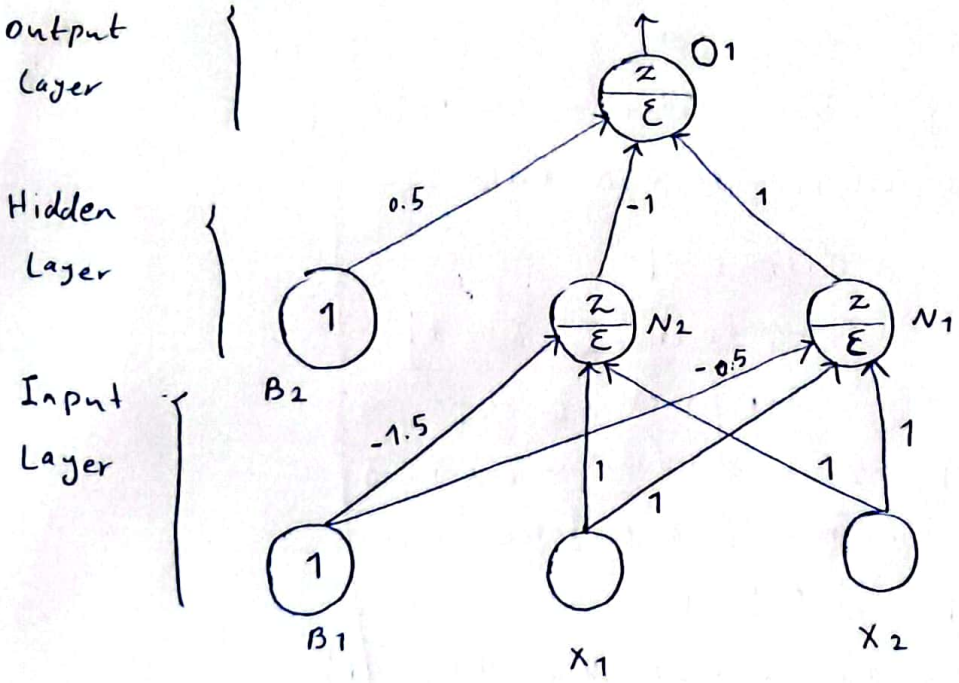
XOR-Gate with Multilayer perceptron

XOR-Gate table :

x_1	x_2	output
0	0	0
1	0	1
0	1	1
1	1	0

we want to get outputs like this table so we should make a multilayer perceptron architecture.

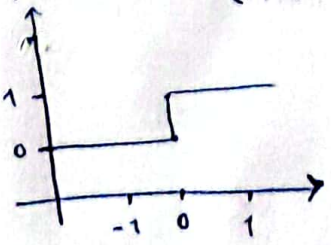


The step function (z), triggers only if the weighted sum is 1 or greater than 1. That is to say,

if $X_1 w_1 + X_2 w_2 \geq 1$, then, step-function $(X_1 w_1 + X_2 w_2) = 1$

if $X_1 w_1 + X_2 w_2 < 1$, then, step-function $(X_1 w_1 + X_2 w_2) = 0$

Step function in graph :



So if we get a value of 1 or greater than 1 for the weighted sum, we will get a value of 1 as an output, otherwise we will get a value of 0 in the output.

Row 1 : $X_1 = 0$ & $X_2 = 0 \longrightarrow$ output = 0

$N_1 = X_2 \times 1 + X_1 \times 1 + B_1 \times (-0.5) = -0.5 \longrightarrow$ step_function $(-0.5) = 0$

$N_2 = 0 \times 1 + 0 \times 1 + 1 \times (-1.5) = -1.5 \longrightarrow$ step function $(-1.5) = 0$

$O_1 = N_2 \times (-1) + N_1 \times (1) + B_2 \times (-0.5) = -0.5 \longrightarrow$ step function $(-0.5) = 0$

⇝ So it is matched with the first row of XOR table.

Row 2 : $X_1 = 1$ & $X_2 = 0 \longrightarrow$ output = 1

$N_1 = 0 \times 1 + 1 \times 1 + 1 \times (-0.5) = 0.5 \longrightarrow$ step function $(0.5) = 1$

$N_2 = 0 \times 1 + 1 \times 1 - 1 \times (-1.5) = -0.5 \longrightarrow$ step function $(-0.5) = 0$

$O_1 = 0 \times (-1) + 1 \times 1 + 1 \times (0.5) = 0.5 \longrightarrow$ step function $(0.5) = 1$

⇝ matched with the second row of XOR table

Row 3 : $X_1 = 0$ & $X_2 = 1 \longrightarrow$ output = 1

$N_1 = 1 \times 1 + 0 \times 1 - 0.5 = 0.5 \longrightarrow$ step function $(0.5) = 1$

$N_2 = 1 \times 1 + 0 \times 1 - 1.5 = -0.5 \longrightarrow$ step function $(-0.5) = 0$

$O_1 = 0 \times (-1) + 1 \times 1 + 1 \times (-0.5) = 0.5 \longrightarrow$ step function $(0.5) = 1$

⇝ matched with the third row of XOR table

Row 4 : $X_1 = 1$ & $X_2 = 1 \longrightarrow$ output = 0

$N_1 = 1 \times 1 + 1 \times 1 + 1 \times (-0.5) = 1.5 \longrightarrow$ step function $(1.5) = 1$

$N_2 = 1 \times 1 + 1 \times 1 + 1 \times (-1.5) = 0.5 \longrightarrow$ step function $(0.5) = 1$

$O_1 = 1 \times (-1) + 1 \times 1 + 1 \times (-0.5) = -0.5 \longrightarrow$ step function $(-0.5) = 0$

⇝ matched with the fourth row of XOR table.

So for each row of the XOR gate table we found our multi layer perceptron structure has given the correct output.