VC-Dimension of other hypothesis spaces, e.g intervals in $\mathbb{R}$:

$$h(x) = \begin{cases} 1 & \text{if } a \leq x \leq b \\ -1 & \text{otherwise} \end{cases}$$

First we consider 2 points x_1 & x_2 such that $x_1 < x_2$. Then there are $2^2 = 4$ possible labelings

1. $x_1 : 1$, $x_2 : 1$
2. $x_1 : -1$, $x_2 : -1$
3. $x_1 : 1$, $x_2 : -1$
4. $x_1 : -1$, $x_2 : 1$:

All the labelings can be achieved through the classifier h by setting the parameters $a < b \in \mathbb{R}$ such that

1. $a < x_1 < x_2 < b$
2. $x_1 < x_2 < a < b$
3. $a < x_1 < b < x_2$
4. $x_1 < a < x_2 < b$

respectively (Actually $x_1 < x_2$ can be assumed w.l.o.g but it is enough to find one set that can be shattered.)

Now consider 3 arbitrary points x_1, x_2, x_3 & w.l.o.g assume $x_1 < x_2 < x_3$ then you can't achive the labling $(1, -1, 1)$.

As in case 3 above, the lables $x_1 : 1$ & $x_2 : -1$ imply $a < x_1 < b < x_2$ which implies $x_3 > b$ & therefore the lable of x_3 has to be -1.

Thus the classifier cannot shatter any set of three points & therefore the VC-Dimension is $\underline{2}$.

Scanned with CamScanner