Here, I will prove the formula:

$$E[(y-\hat{f}(x))^2] = (\text{Bias}[\hat{f}(x)])^2 + \text{var}[\hat{f}(x)] + \sigma^2$$

Where:

$$\text{Bias}[\hat{f}(x)] = E[\hat{f}(x)] - f(x)$$

$$\text{var}[\hat{f}(x)] = E[(E(\hat{f}(x)) - \hat{f}(x))^2]$$

$$y = f(x) + \varepsilon \quad \text{with} \quad E[\varepsilon] = 0$$

$$\text{Var}[\varepsilon] = \sigma^2$$

Prove:

$$E[(y-\hat{f}(x))^2] = E[(f(x)+\varepsilon-\hat{f}(x))^2] =$$

$$= E[(f(x)+\varepsilon-\hat{f}(x)+E(\hat{f}(x))-E(\hat{f}(x)))^2] =$$

$$= E[\varepsilon(f(x)-E[\hat{f}(x)])] + E[\varepsilon(E[\hat{f}(x)]-\hat{f}(x))]$$

$$+ E[(f(x)-E[\hat{f}(x)])^2] + E[(E[\hat{f}(x)]-\hat{f}(x))^2] +$$

$$+ E[(f(x)-E(\hat{f}(x))(E[\hat{f}(x)]-\hat{f}(x))] + E[\varepsilon^2] =$$

$$= 0 + 0 + (\text{Bias}[\hat{f}(x)])^2 + \text{var}[\hat{f}(x)] +$$

$$+ 0 + \sigma^2 =$$

$$= (\text{Bias}[\hat{f}(x)])^2 + \text{var}[\hat{f}(x)] + \sigma^2$$

$\square$