Goal: To Show the Effect that Overfitting and Unfitting have on Variance and Bias ¶

In [1]:
```
import numpy as np
%matplotlib inline
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.preprocessing import PolynomialFeatures
```

The Sine function was chosen as the model which we would like to predict

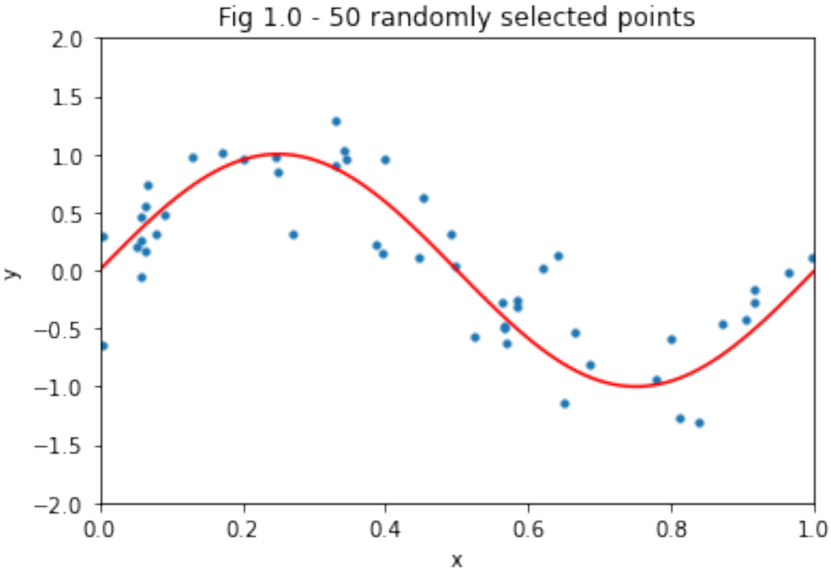
In [2]:
```
def f(x):
    return np.sin(2 * np.pi * x)
```

1 million points were generated around the Sine function. Then a random number generator was used to randomly select 50 points. A visual representation of this can be seen below in Fig 1.0 and Fig 1.1.

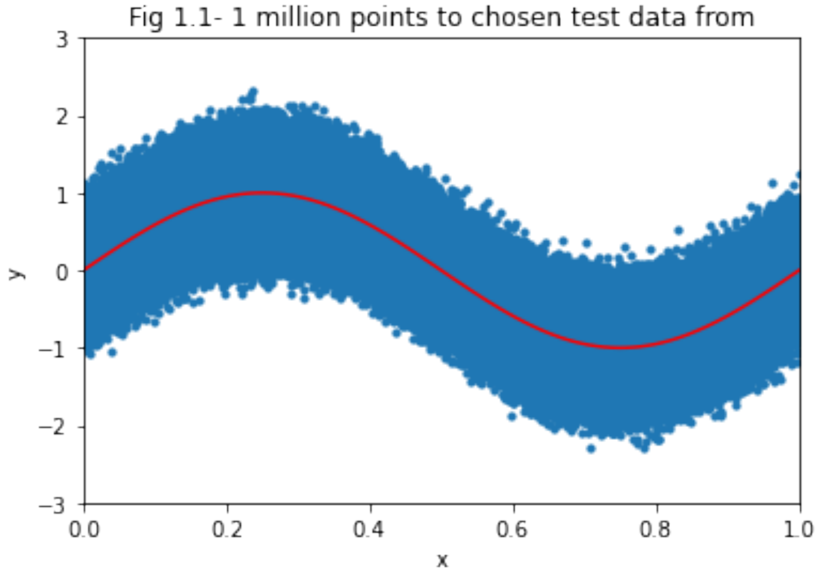
Linear Regression was chosen as the predictor of the Sine function as this will clearly underfit the data. This predictor function f(x) was run for 1000 iterations each with newly selected test data of 50 points. The result of these 1000 liner predictions f(x) where x = {0,1,2....,999} are shown in Fig 1.2.

In [3]:
```
np.random.seed(56)
n_samples = 1000000
x_plot = np.linspace(0, 1, 100)
X = np.random.uniform(0, 1, size=n_samples)[:, np.newaxis]
y = f(X) + np.random.normal(scale=0.3, size=n_samples)[:, np.newaxi
y_intercept_list = []
slope_list = []
g_x_list = []
y_pred_list = []
y_train_list = []

for i in range(1000):
    X_train, X_test, y_train, y_test = train_test_split(X, y, test_
    model = LinearRegression().fit(X_train, y_train)
    y_intercept = model.intercept_[0]
    slope = model.coef_[0]
    y_intercept_list.append(y_intercept)
    slope_list.append(slope)
    g_x = slope * x_plot + y_intercept
    g_x_list.append(g_x)
    y_pred = model.predict(X_train)
    y_pred_list.append(y_pred)
    y_train_list.append(y_train)
```

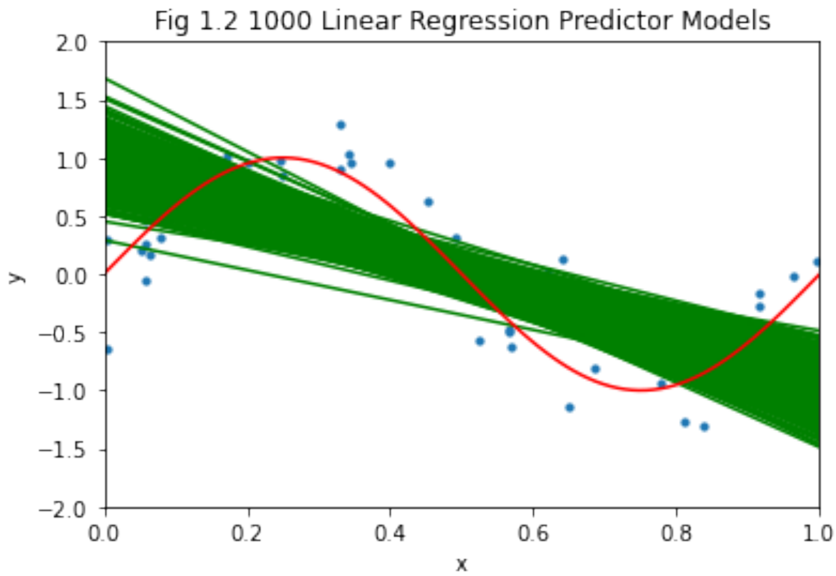
In [4]:
```
x_plot = np.linspace(0, 1, 100)
ax = plt.gca()
ax.plot(x_plot, f(x_plot), color='red')
ax.scatter(X_train, y_train, s=10)
ax.set_ylim((-2, 2))
ax.set_xlim((0, 1))
ax.set_ylabel('y')
ax.set_xlabel('x')
ax.set_title('Fig 1.0 - 50 randomly selected points')
plt.show()
```



In [5]:
```
x_plot = np.linspace(0, 1, 100)
ax = plt.gca()
ax.plot(x_plot, f(x_plot), color='red')
ax.scatter(X_test, y_test, s=10)
ax.set_ylim((-3, 3))
ax.set_xlim((0, 1))
ax.set_ylabel('y')
ax.set_xlabel('x')
ax.set_title('Fig 1.1- 1 million points to chosen test data from')
plt.show()
```



In [6]:
```
ax = plt.gca()
for i in g_x_list:
    ax.plot(x_plot,i, color='green')

ax.scatter(X_train, y_train, s=10)
ax.plot(x_plot, f(x_plot), color='red')


ax.set_ylim((-2, 2))
ax.set_xlim((0, 1))
ax.set_ylabel('y')
ax.set_xlabel('x')
ax.set_title('Fig 1.2 1000 Linear Regression Predictor Models')
plt.show()
```



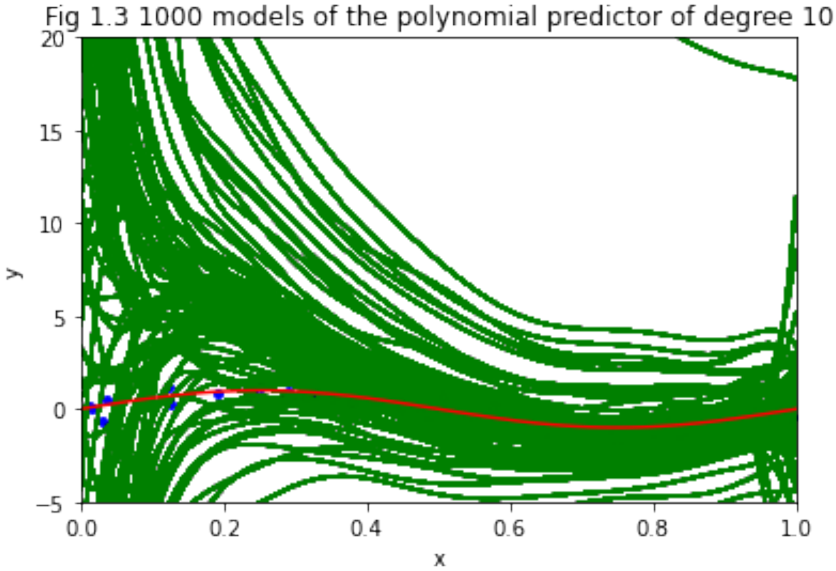
In [7]:
```
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size
```

A polynomial of degree 10 was chosen as the preditor to overfit the data. The graph of these 1000 models are shown in Fig 1.3

In [8]:
```
y_intercept_polynomial_list = []
coefs_polynomial_list = []
g_x_polynomial_list = []
y_pred_polynomial_list = []
y_train_polynomial_list = []
x_train_polynomial_list = []
degree = 10

for j in range(1000):
    X_train, X_test, y_train, y_test = train_test_split(X, y, test_
    x_ = PolynomialFeatures(degree, include_bias=False).fit_transfo
    model = LinearRegression().fit(x_, y_train)
    y_intercept = model.intercept_[0]
    coefs = model.coef_
    y_pred = model.predict(x_)
    y_intercept_polynomial_list.append(y_intercept)
    coefs_polynomial_list.append(coefs)
    y_pred_polynomial_list.append(y_pred)
    y_train_polynomial_list.append(y_train)
    x_train_polynomial_list.append(X_train)
```

In [9]:
```
import numpy as np
from matplotlib import pyplot as plt

def PolyCoefficients(x, coeffs, degree):
    y = 0
    for i in range(degree):
        y += coeffs[i]*x**i
    return y

y_all_list = []
for m in range(100):
    y_list = []
    for l in range(100):
        x_p = x_plot[l]
        y_values = PolyCoefficients(x_p, coefs_polynomial_list[m][0
        y_list.append(y_values)
        y_all_list.append(y_list)
```

In [10]:
```
ax = plt.gca()
for n in y_all_list:
    ax.plot(x_plot, n, color='green')

ax.plot(x_plot, f(x_plot), color='red')
ax.scatter(X_train, y_train, s=15, color='blue')

ax.set_ylim((-5, 20))
ax.set_xlim((0, 1))
ax.set_ylabel('y')
ax.set_xlabel('x')
ax.set_title('Fig 1.3 1000 models of the polynomial predictor of de
plt.show()
```



The sun squared error, the variance and the bias are computed below for the linear regression models and the polynomial of order 10 models. The results of which can be seen below.

In [11]:
```
SSE_list = []
Variance_list = []
Bias_list = []
for i in range(1000):
    SSE = np.mean((np.mean(y_pred_list[i]) - y_train_list[i])** 2)
    Variance = np.var(y_pred_list[i])
    Bias = SSE - Variance
    SSE_list.append(SSE)
    Variance_list.append(Variance)
    Bias_list.append(Bias)
```

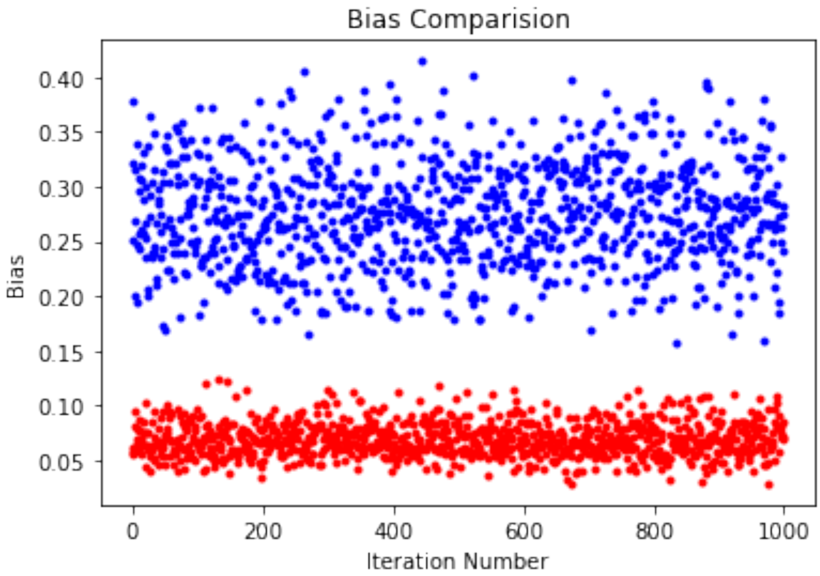
In [12]:
```
SSE_polynomial_list = []
Variance_polynomial_list = []
Bias_polynomial_list = []
for i in range(1000):
    SSE = np.mean((np.mean(y_pred_polynomial_list[i]) - y_train_pol
    Variance = np.var(y_pred_polynomial_list[i])
    Bias = SSE - Variance
    SSE_polynomial_list.append(SSE)
    Variance_polynomial_list.append(Variance)
    Bias_polynomial_list.append(Bias)
```

The bias measures the distortion of an estimate. The linear regression model is represented by the blue dots and the polynomial of order 10 is represented by the red dots.

It is clear that a linear regression cannot capture the complexity of the Sine function and underfits the data. This gives it a high bias in comparision to the polynomial of order 10 which has little bias as it greatly overfits the data.

In [13]:
```
itle('Bias Comparision')
lot(Bias_list,'.', color='blue');
lot(Bias_polynomial_list, '.',color='red', );
label("Iteration Number")
label("Bias")

('The mean bias of the linear regression is ',np.mean(Bias_list), '\
```

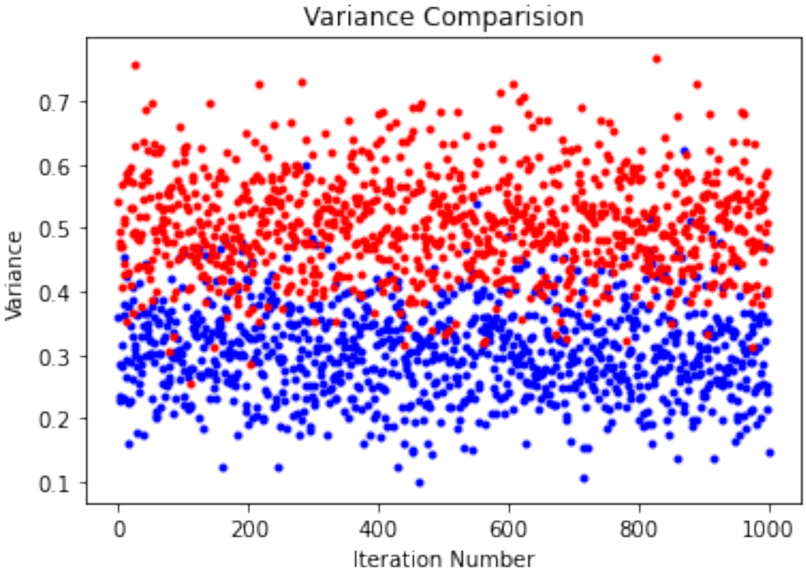
```
The mean bias of the linear regression is  0.27586501344444864
 while the mean bias of the polynomial of order 10 is  0.069673699
31498046
```



It can also be seen that overfitting the data causes a high variance while underfitting the data causes a lower variance as seen below.

In [14]:
```
plt.title('Variance Comparision')

plt.plot(Variance_list, '.',color='blue');
plt.plot(Variance_polynomial_list, '.' ,color='red');
plt.xlabel("Iteration Number")
plt.ylabel("Variance")
print('The mean variance of the linear regression is ',np.mean(Vari
```

```
The mean variance of the linear regression is  0.305992708423779244
 while the mean variance of the polynomial of order 10 is  0.50586
81103237923
```



In [ ]: