Lecture 4:

VC-Dimension of other hypothesis spaces, e.g. intervals in R : h(x) = +1 if a ≤ x ≤ b, h(x) = −1 otherwise.

VC dimension of Intervals in $\mathbb{R}$:

Any member of this class is represented by two parameters: $[a,b] \in \mathbb{R}$, which represent an interval between a and b.

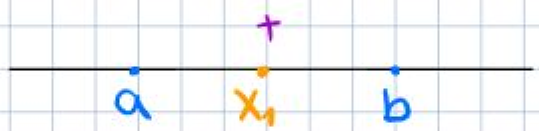
To determine the VC dimension we need to find the smallest set of size n that can not be shattered by $\mathcal{H}$.

for a set of size n, its members are n dots on the real line.

We will show we can implement all possible dichotimes of $n \leq 2$, by giving values to bounds - a, b, but not for $n = 3$.

n = 1:

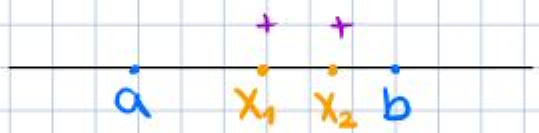
- $h(x_1) = +1$:



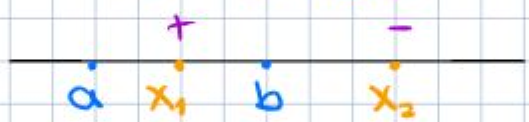
- $h(x_1) = -1$:

n = 2:

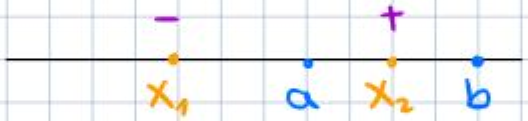
- $h(x_1) = h(x_2) = +1$:



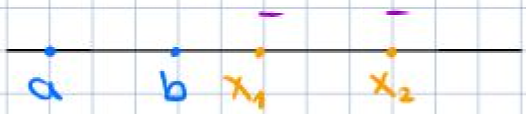
- $h(x_1) = +1,\ h(x_2) = -1$:



- $h(x_1) = -1,\ h(x_2) = +1$:



- $h(x_1) = h(x_2) = -1$:



$n = 3$:

- $h(x_1) = +1,\ h(x_2) = -1,\ h(x_3) = +1,\ x_1 < x_2 < x_3$:

There is no Interval, who can classify this case correctly: $h(x_1) = +1$ and $h(x_2) = -1 \Rightarrow x_1 \leq b < x_2 < x_3 \Rightarrow h(x_3) = -1$, contradiction!

Thus, we conclude $vc(H) = 2$.