COMPUTING ID3 ON LOGIC GATES OF 3 VARIABLES

A	B	C	A∧B∧C	A∨B∨C	A xor B xor C
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	1	1	0

- ENTROPY:

$$E(S) = -\sum_{c=1}^{m} \frac{|S_c|}{|S|} \log\left(\frac{|S_c|}{|S|}\right)$$

INFORMATION GAIN:

$$G(S,a) = E(S) - \sum_{v \in V(a)} \frac{|S_{a=v}|}{|S|} E(S_{a=v})$$

choosing the attribute that has max information gain (optimal)

AND GATE

on level 0 of the tree

$$E(S) = -\left[\frac{7}{8}\log\left(\frac{7}{8}\right) + \frac{1}{8}\log\left(\frac{1}{8}\right)\right] = 0.543$$

- $a = \{A\}$

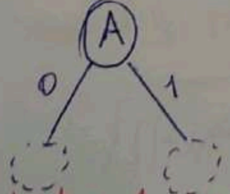
$$G(S,a) = E(S) - \left[\frac{4}{8}\underbrace{1\cdot\log(1)}_{E(S_{a=0})} + \frac{4}{8}\cdot\underbrace{\left[-\left(\frac{3}{4}\log\left(\frac{3}{4}\right) + \frac{1}{4}\log\left(\frac{1}{4}\right)\right)\right]}_{E(S_{a=1})}\right] = 0.135$$

- $a = \{B\}$

$G(S,a) = 0.135 \longrightarrow$ same as above

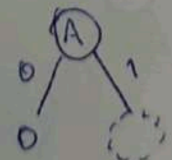
- $a = \{C\}$

$G(S,a) = 0.135 \longrightarrow$ same as above



on level 1 of tree and attribute A has value 0.

$$E(S) = -\left[\frac{4}{4}\log\left(\frac{4}{4}\right)\right] = 0$$



on level 1 of the tree and attribute A = 1

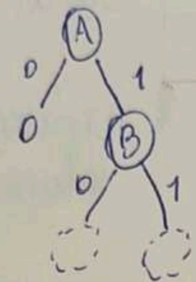
$$E(S) = -\left[\frac{3}{4}\log\left(\frac{3}{4}\right) + \frac{1}{4}\log\left(\frac{1}{4}\right)\right] = 0.811$$

• $a = \{B\}$

$$G(S,a) = E(S) - \Big[\tfrac{1}{2} \underbrace{\left(-\left(\tfrac{1}{2}\cdot 0\right)\right)}_{E(S_{a=0})} + \tfrac{1}{2} \underbrace{\left(-\left(\tfrac{1}{2}\log\left(\tfrac{1}{2}\right) + \tfrac{1}{2}\log\left(\tfrac{1}{2}\right)\right)\right)}_{E(S_{a=1})} \Big] \overset{!}{=} 0.31$$

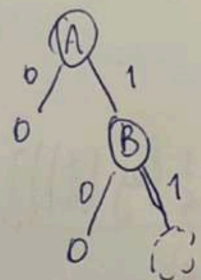
• $a = \{C\}$

$$G(S,a) = E(S) - \Big[\tfrac{1}{2} \underbrace{\left(-\left(\tfrac{1}{2}\log\left(\tfrac{1}{2}\right) + \tfrac{1}{2}\log\left(\tfrac{1}{2}\right)\right)\right)}_{E(S_{a=1})} + \tfrac{1}{2} \underbrace{\left(-\left(1\log(1)\right)\right)}_{E(S_{a=0})} \Big] \overset{!}{=} 0.31$$



on level 2 of the tree and attributes A=1 B=0

$E(S) = 0$



on level 2 of the tree and attributes A=1 B=1

$$E(S) = -\left(\tfrac{1}{2}\log\left(\tfrac{1}{2}\right) + \tfrac{1}{2}\log\left(\tfrac{1}{2}\right)\right) \overset{!}{=} 1$$

• $a = \{C\}$

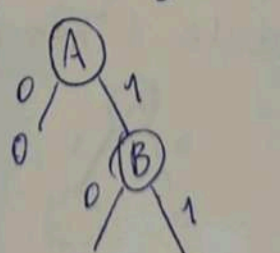
$$G(S,a) = E(S) - \overbrace{\Big[\tfrac{1}{2}\left(-1\log(1)\right) + \tfrac{1}{2}\left(-1\log(1)\right)\Big]}^{\text{obviusly } \emptyset \text{ because is the last node}}$$

on level 3 of the tree and attributes A=1, B=1, C=0

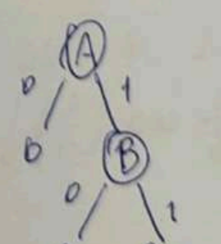
$E(S) = 0$

on level 3 of the tree and attributes A=1, B=1, C=1

$E(S) = 0$



FINAL DT of AND



XOR GATE

on level 0 of the tree

$E(S) = -\left[\frac{2}{8}\log\left(\frac{2}{8}\right) + \frac{6}{8}\log\left(\frac{6}{8}\right)\right] = 0{,}811$

- $a = \{A\}$

$G(S,a) = E(S) - \left[\frac{4}{8}\left(-\left(\frac{1}{4}\log\left(\frac{1}{4}\right) + \frac{3}{4}\log\left(\frac{3}{4}\right)\right)\right) + \frac{4}{8}\left(-\left(\frac{1}{4}\log\left(\frac{1}{4}\right) + \frac{3}{4}\log\left(\frac{3}{4}\right)\right)\right)\right]$

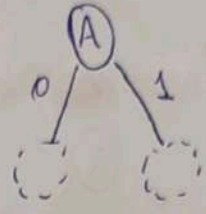
$= 0$

- $a = \{B\}$

$G(S,a) = 0$ ← same as above

- $a = \{C\}$

$G(S,a) = 0$ ← same as above



on level 1 of the tree and attribute $A=0$

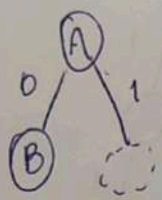
$E(S) = 0{,}811$

- $a = \{B\}$

$G(S,a) = 0{,}311$

- $a = \{C\}$

$G(S,a) = 0{,}311$



on level 1 of the tree and attribute $A=1$

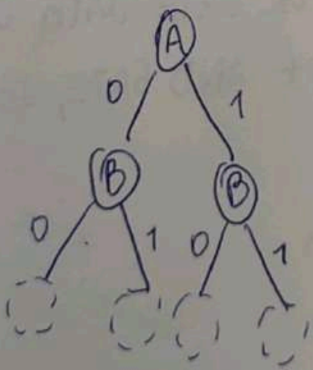
$E(S) = 0{,}811$

- $a = \{B\}$

$G(S,a) = 0{,}311$

- $a = \{C\}$

$G(S,a) = 0{,}311$

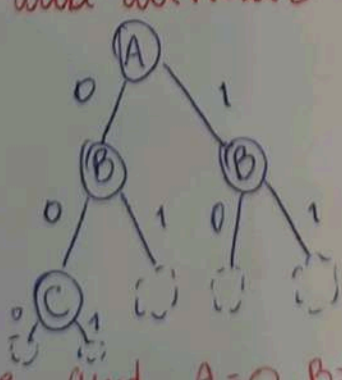


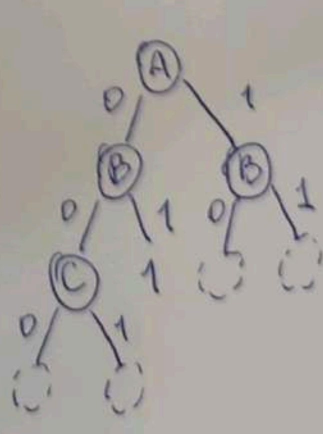
on level 2 of the tree and attribute $A=0$, $B=0$

$E(S) = 1$

- $a = \{C\}$

$G(S,a) = 1$

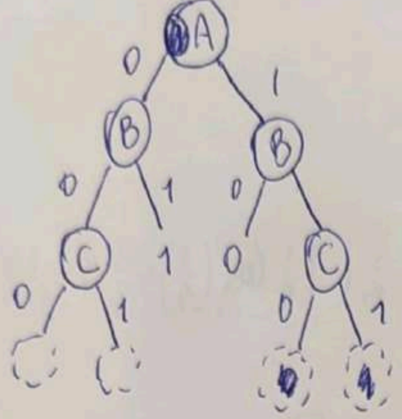




on level 2 of the tree and $A=0$, $B=1$

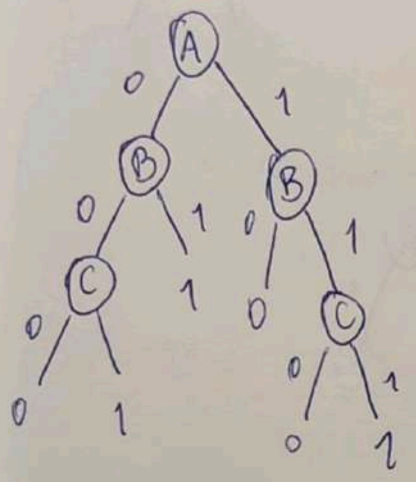
$E(S) = 0$

then the same for the right side of the tree on level 2



then on level 3 there are the leaf nodes, so

FINAL DT of XOR



OR GATE

computing the decision tree of OR GATE with the same methods as above, we find that's the dual dt of AND GATE, as expected.

FINAL DT of OR