

Exercise

Show that $\sigma'(y) = \sigma(y)(1 - \sigma(y))$.

Solution

$$\begin{aligned}\sigma(y) &= (1 + \exp(-y))^{-1} \\ \sigma'(y) &= -(1 + \exp(-y))^{-2} \cdot \frac{d}{dy} (1 + \exp(-y)) \\ &= -(1 + \exp(-y))^{-2} [-\exp(-y)] \\ &= \sigma(y) \left[\frac{\exp(-y)}{1 + \exp(-y)} \right] \\ &= \sigma(y) \left[\frac{1 + \exp(-y) - 1}{1 + \exp(-y)} \right] \\ &= \sigma(y) \left[1 - \frac{1}{1 + \exp(-y)} \right] \\ &= \sigma(y) (1 - \sigma(y))\end{aligned}$$