

$$b(y) = \frac{1}{1+e^{-y}} \quad (y = w \cdot x)$$

$$\frac{\delta b(y)}{\delta y} = \frac{0 + e^{-y}}{(1+e^{-y})^2} = \frac{e^{-y}}{(1+e^{-y})^2}$$

$$\Rightarrow \frac{y' e^{-y}}{(1+e^{-y})^2} = \frac{1}{1+e^{-y}} \times \frac{e^{-y}}{1+e^{-y}} = \frac{1}{1+e^{-y}} \times \frac{1+e^{-y}-1}{1+e^{-y}}$$

$$= \frac{1}{1+e^{-y}} \times \left(\frac{1+e^{-y}}{1+e^{-y}} - \frac{1}{1+e^{-y}} \right) = b(y)(1-b(y))$$