Show that $\sigma'(y) = \delta(y)(1-\delta(y))$

we know that $\delta(y) = \dfrac{1}{1+e^{-y}}$, so we just have to compute its derivat

by following the next rools, we can solve the problem:

$(x^a)' = ax^{a-1}$; $a' = 0$; $x' = 1$; $fg = f'g + fg'$

$(e^x)' = e^x$

$f(g(x))' = f'(g(x))g'(x) \quad \Rightarrow$

$$\left(\frac{1}{1+e^{-y}}\right)' = \left((1+e^{-y})^{-1}\right)' = -1(1+e^{-y})^{-2} \cdot (1+e^{-y})' =$$

$$= \frac{-1}{(1+e^{-y})^2} \cdot e^{-y} \cdot \underbrace{(-y)'}_{-1 \cdot y} = \frac{-e^{-y}}{(1+e^{-y})^2} \cdot \overbrace{(0 \cdot y - 1 \cdot 1)}^{-1} = \frac{e^{-y}}{(1+e^{-y})^2} =$$

$$= \underbrace{\frac{1}{1+e^{-y}}}_{\delta(y)} \cdot \frac{e^{-y}}{1+e^{-y}} = \delta(y) \cdot \left(\frac{1+e^{-y}-1}{1+e^{-y}}\right) = \delta(y)\left(\cancel{\frac{1+e^{-y}}{1+e^{-y}}} - \underbrace{\frac{1}{1+e^{-y}}}_{\delta(y)}\right) =$$

$$\boxed{= \delta(y)(1-\delta(y))}$$