

Theorem: ~~Prove~~ Show that $\frac{d}{dx} \sigma(x) = \sigma(x)(1 - \sigma(x))$

~~Prove~~ which $\sigma(x) = \frac{1}{1 + e^{-x}}$

Proof: base on derivation rules we have:

$$\frac{d}{dx} \sigma(x) = \frac{\frac{d}{dx} (1 + e^{-x})}{(1 + e^{-x})^2} = \frac{-1 - e^{-x}}{(1 + e^{-x})^2} = \frac{e^{-x}}{(1 + e^{-x})^2} = \frac{1}{(1 + e^{-x})^2} - \frac{1}{(1 + e^{-x})}$$

$$+ \frac{e^{-x}}{(1 + e^{-x})^2} = \frac{-1}{(1 + e^{-x})^2} + \frac{1 + e^{-x}}{(1 + e^{-x})^2} = -\frac{1}{(1 + e^{-x})^2} + \frac{1}{1 + e^{-x}} = -\sigma^2(x) + \sigma(x)$$

$$= \sigma(x)(1 - \sigma(x)) \quad \square$$