

after first iteration we have,

$$\Delta w^1 = \eta (t^1 - \hat{y}^1) x^1 \quad \text{which } \hat{y}^i = \text{sign}(w \cdot x^i) \text{ in each iteration}$$

then in next iteration we have:

$$\Delta w^2 = \eta (t^2 - \hat{y}^2) x^2$$

$$= \eta (t^2 - \text{sg}(w^1 \cdot x^2)) x^2$$

$$= \eta (t^2 - \text{sg}(w^0 + \Delta w^1) \cdot x^2) x^2$$

$$= \eta (t^2 - \text{sg}(w^0 + \eta (t^1 - \hat{y}^1) x^1) \cdot x^2) x^2$$

$$= \eta (t^2 - \text{sg}(w^0 + \eta (t^1 - \text{sg}(w^0 \cdot x^1)) x^1) \cdot x^2) x^2 \quad (*)$$

if we consider  $w^0 = 0$  the  $*$  will simplify to:

$$\Delta w^2 = \eta (t^2 - \underbrace{\text{sg}(\eta (t^1 - 1) x^1 \cdot x^2)}_{**}) x^2$$

~~$\eta$  is not~~  $\eta$  does not change the function  $**$  since  $\eta > 0$

So we have:

$$\Delta w^2 = \eta (t^2 - \text{sg}((t^1 - 1) x^1 \cdot x^2)) x^2$$

So the scalar  $\eta$  only affect the scale of  $\Delta w^2$  and not the

direction the same can be prove for  $\Delta w^i$  by induction  $\square$