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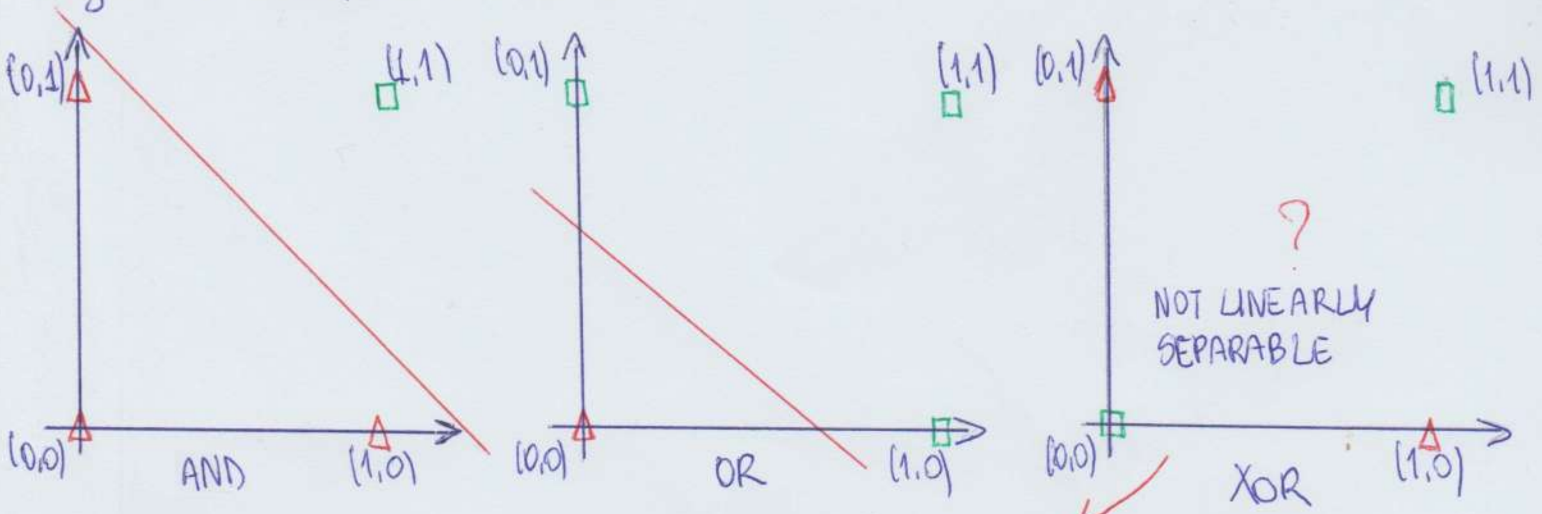
WHY XOR GATE IS NOT COMPUTABLE BY A SINGLE PERCEPTRON

The combination of parameters that can compute the XOR gate using a perceptron doesn't exist because of the non-linear separability of the data.

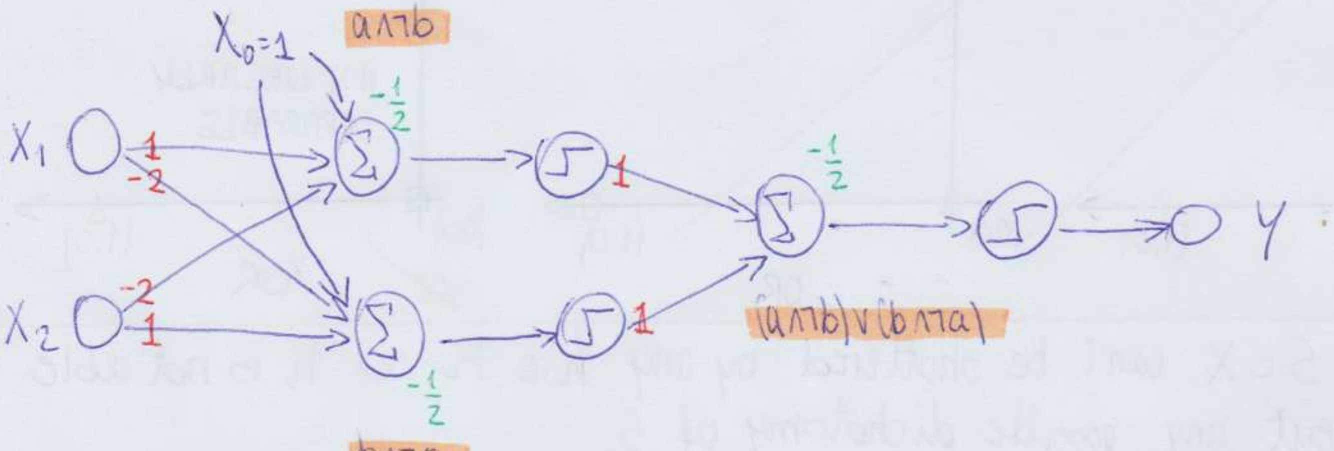
A set of points in $\mathbb{R}^n$ is linearly separable if there's at least a hyperplane in $\mathbb{R}^n$ such that can separate them.

Giving some examples to clarify:



$S \subset X$ can't be shattered by any $h \in \mathcal{H}$, so $\mathcal{H}$ is not able to implement any possible dichotomy of S.

However XOR gate can be implemented as a ~~multi-layered~~ multi-layered network based of perceptrons with hard ~~th~~ tresholds
An example of (a XOR b) with fixed weights can be:



$X_0 = 1$

$(X_1, X_2) \in \{0,1\}$

WEIGHTS

$W_{11}^{1}=1$ $W_{11}^{2}=1$

$W_{12}^{1}=-2$ $W_{21}^{2}=1$

$W_{21}^{1}=-2$

$W_{22}^{1}=1$

BIASES

$b_1^1=-\frac{1}{2}$ $b_1^2=-\frac{1}{2}$

$b_2^1=-\frac{1}{2}$

a XOR b can be written as $(a \wedge \neg b) \vee (b \wedge \neg a)$, here below the truth table

a	b	$\neg b$	$\neg a$	$a \wedge \neg b$	$b \wedge \neg a$	$(a \wedge \neg b) \vee (b \wedge \neg a)$
0	0	1	1	0	0	0
0	1	0	1	0	1	1
1	0	1	0	1	0	1
1	1	0	0	0	0	0

→ that is XOR