

* Exercise Lecture 18 = show that $\delta'(y) = \delta(y)(1 - \delta(y))$

Based on sigmoid function we know that $\delta(y) = \frac{1}{1 + e^{-y}} = \sigma$

$[(e^{mx})' = m e^{mx}]$

$$\delta'(y) = \frac{d}{dy} \left(\frac{1}{1 + e^{-y}} \right) = \frac{d}{dy} (1 + e^{-y})^{-1} = -1(1 + e^{-y})^{-2} \times -1 \times e^{-y} = e^{-y} (1 + e^{-y})^{-2} = \frac{e^{-y}}{(1 + e^{-y})^2}$$

$$= \frac{e^{-y}}{(1 + e^{-y})} \times \frac{1}{(1 + e^{-y})} = \frac{1 + e^{-y} - 1}{1 + e^{-y}} \times \frac{1}{1 + e^{-y}} = \left[\frac{1 + e^{-y}}{1 + e^{-y}} - \frac{1}{1 + e^{-y}} \right] \times \frac{1}{1 + e^{-y}} =$$

$$\left(1 - \frac{1}{1 + e^{-y}} \right) \left(\frac{1}{1 + e^{-y}} \right) = \delta(y)(1 - \delta(y)) = \delta'(y) = \delta(y)(1 - \delta(y))$$