

LQC 08 NN II (Derivative of Sigmoid function)

Lets denote $\sigma(x) = \frac{1}{1+e^{-x}}$ \rightarrow the derivative of the sigmoid is $\sigma(x)(1-\sigma(x))$

$$\frac{d}{dx} \left[\frac{1}{1+e^{-x}} \right] = \frac{d}{dx} (1+e^{-x})^{-1} = -(1+e^{-x})^{-2} (-e^{-x}) = \frac{e^{-x}}{(1+e^{-x})^2}$$

$$= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}} = \frac{1}{1+e^{-x}} \cdot \frac{(1+e^{-x})^{-1}}{(1+e^{-x})} = \frac{1}{1+e^{-x}} \cdot \left[\frac{1+e^{-x}}{1+e^{-x}} - \frac{1}{1+e^{-x}} \right]$$

$$= \frac{1}{1+e^{-x}} \left[1 - \frac{1}{1+e^{-x}} \right] = \sigma(x)(1-\sigma(x)) \text{ proved!}$$