Proof that the derivative of $\sigma(y) := \frac{1}{1+e^{-y}}$ is

$\sigma'(y) = \sigma(y)(1-\sigma(y))$

$$\sigma'(y) = \left(\frac{1}{1+e^{-y}}\right)' = ((1+e^{-y})^{-1})' \overset{\text{Chain rule}}{=} (-1)\cdot(-e^{-y})\cdot(1+e^{-y})^{-2}$$

$$= e^{-y}(1+e^{-y})^{-2} = \frac{e^{-y}}{(1+e^{-y})^2} = \frac{e^{-y}}{1+e^{-y}} \cdot \frac{1}{1+e^{-y}}$$

$$= \frac{e^{-y}}{1+e^{-y}} \cdot \sigma(y)$$

According to equation above $\frac{e^{-y}}{1+e^{-y}} = 1 - \sigma(y)$ must hold

$$\frac{e^{-y}}{1+e^{-y}} = 1 - \sigma(y)$$

$$= 1 - \frac{1}{1+e^{-y}}$$

$$= \frac{1+e^{-y}}{1+e^{-y}} - \frac{1}{1+e^{-y}}$$

$$= \underline{\underline{\frac{e^{-y}}{1+e^{-y}}}}$$

So it holds that $\sigma'(y) = \sigma(y)(1-\sigma(y))$.