Exercise Lecture 7 = prove that if the perceptron algorithm is initialized with the null vector, the the coefficient η does not affect learning.

We know that $\delta(net) = O = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} wi \cdot xi \geq 0 \ (net \geq 0) \\ 0 & \text{if } \sum_{i=1}^{n} wi \cdot xi < 0 \ (net < 0) \end{cases}$

updated w

$w \leftarrow w + \eta(t - O)x$

1) $w^0 x^1 = O^1 \Rightarrow O^1 = \delta(net) = \delta(w^0 x^1)$ $\quad w^0$

2) $w^1 x^2 = O^2 \Rightarrow O^2 = \delta(net) = \delta(w^1 x^2)$ $\quad w^1 = w^0 + \eta(t^1 - O^1)x^1 \Rightarrow \Delta w^1 = w^1 - w^0 = \eta(t^1 - O^1)x^1$

3) $w^2 x^3 = O^3 \Rightarrow O^3 = \delta(net) = \delta(w^2 x^3)$ $\quad w^2 = w^1 + \eta(t^2 - O^2)x^2 \Rightarrow \Delta w^2 = w^2 - w^1 \Rightarrow$

$$\Delta w^2 = w^2 - w^1 = \cancel{w^1} + \eta(t^2 - O^2)x^2 - \cancel{w^1} = \eta(t^2 - \delta(w^1 x^2))x^2 = \eta(t^2 - \delta([w^0 + \eta(t^1 - O^1)x^1][x^2]))x^2$$

$$= \eta(t^2 - \delta([\cancel{w^0} + \eta(t^1 - \underbrace{\delta(w^0 x^1)}_{\delta(0)=1})x^1][x^2])x^2) = \eta(t^2 - \delta(\underbrace{[\eta(t^1 - 1)x^1][x^2]}_{net})x^2) =$$

we know that $\eta > 0 \Rightarrow$ It can't change the sign of the net, so if we remove η, we have =

$$\eta(t^2 - \delta([(t^1 - 1)x^1][x^2])x^2) = \Delta w^2$$