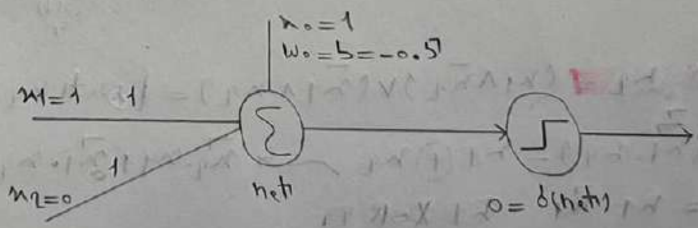
Exercise lecture 7 = give perceptron-based multi-layer networks with hard thresholds and relative weights (without using learning) that implement simple boolean functions such as = A and (not B), A XOR B, ...

1) about "OR" we know that =

$x_1 \vee x_2 = x_1 + x_2 = x_1$ or x_2

truth table =

x_1	x_2	$x_1 \vee x_2$
1	1	1
0	0	0
1	0	1
0	1	1



$net = \sum_{i=0}^{2} w_i x_i = w_0 x_0 + w_1 x_1 + w_2 x_2$, $(w_1 = w_2 = 1, w_0 = b = -0.5, x_0 = 1)$

$net = w_0 x_0 + w_1 x_1 + w_2 x_2 = x_1 + x_2 - 0.5 \Rightarrow x_1 = 1$ and $x_2 = 0 \Rightarrow$

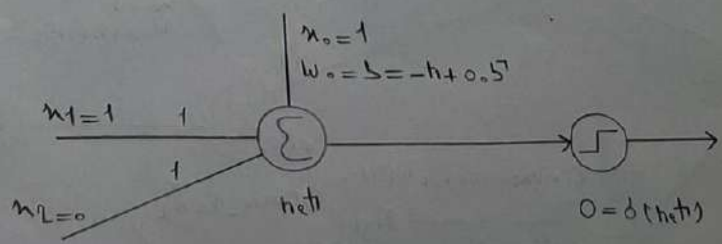
$1 + 0 - 0.5 = 0.5 > 0 \Rightarrow output = 1 \; (x_1 \vee x_2 = 1 \vee 0 = 1)$

2) about "AND" we know that =

$x_1 \wedge x_2 = x_1 . x_2 = x_1$ and x_2

truth table =

x_1	x_2	$x_1 \wedge x_2$
1	1	1
0	0	0
1	0	0
0	1	0



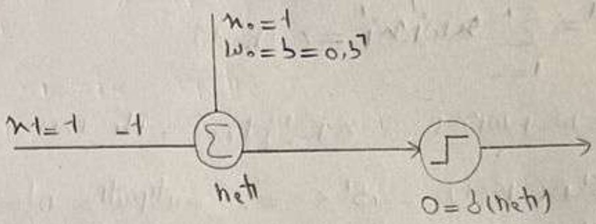
$net = \sum_{i=0}^{2} w_i x_i = w_0 x_0 + w_1 x_1 + w_2 x_2$, $(w_1 = w_2 = 1, w_0 = b = -n + 0.5 = -2 + 0.5 = -1.5, x_0 = 1)$

$net = w_0x_0 + w_1x_1 + w_2x_2 = x_1 + x_2 - 1.5 \Rightarrow x_1 = 1$ and $x_2 = 0 \Rightarrow$
$1 + 0 - 1.5 = -0.5 < 0 \Rightarrow output = 0$ $(x_1 \wedge x_2 = 1 \wedge 0 = 0)$

3) about "NOT" we know that =
$\overline{x_1}$

truth table =

x_1	Not x_1
1	0
0	1



$$net = \sum_{i=0}^{1} w_ix_i = w_0x_0 + w_1x_1 \quad , (w_0 = b = 0.5, \; x_0 = 1, \; w_1 = -1)$$

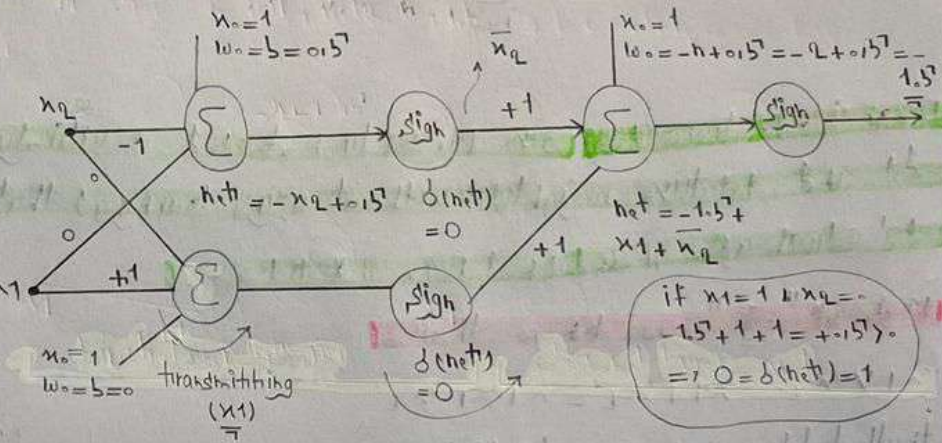
$net = -x_1 + 0.5 \Rightarrow x_1 = 1 \Rightarrow net = -1 + 0.5 = -0.5 < 0 \Rightarrow output = 0$ $(\overline{x_1}$ (if $x_1 = 1) = 0)$

4) x_1 and not x_2 =
$x_1 \cdot \overline{x_2}$

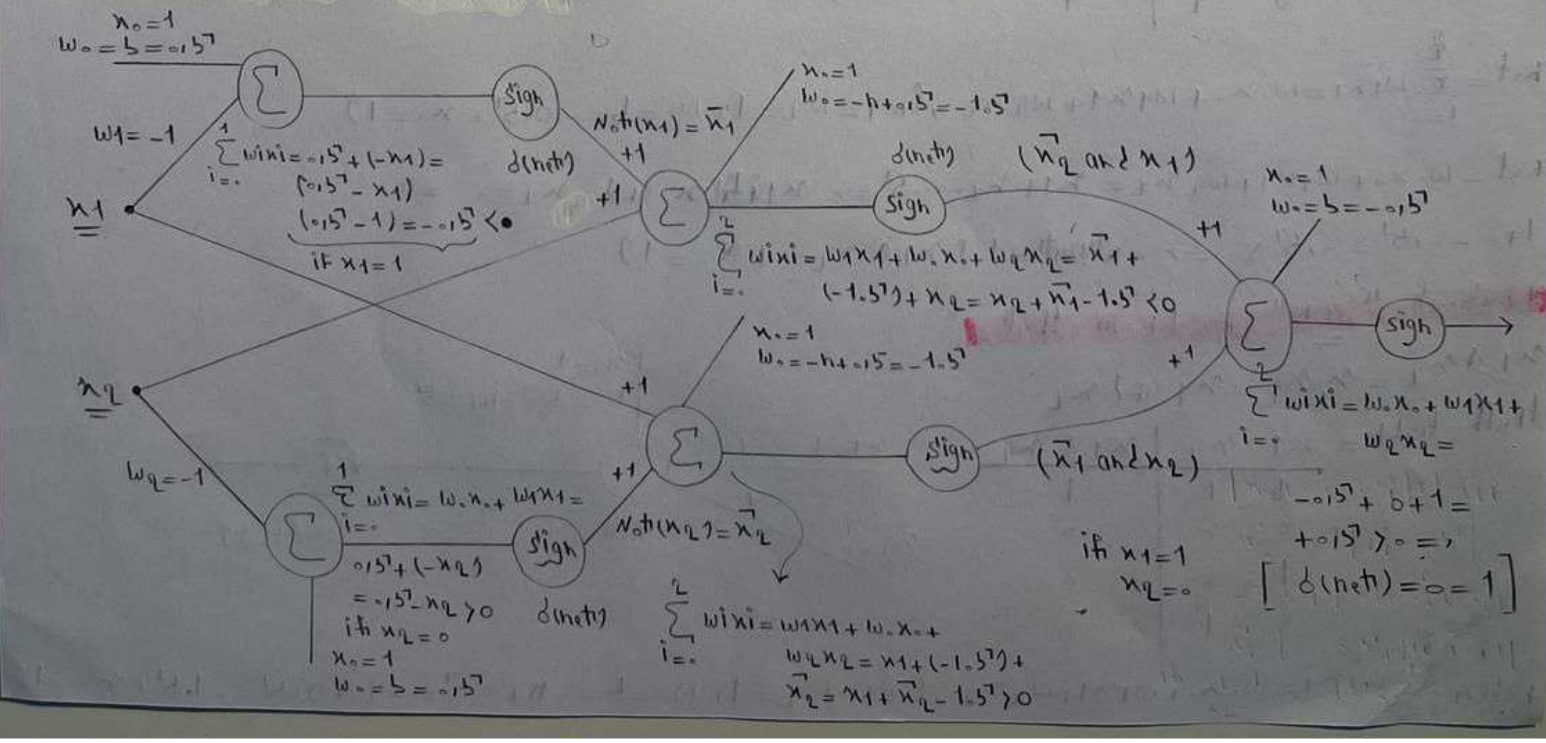
** multilayer = first layer = NOT
second layer = and

truth table =

x_1	x_2	$\overline{x_2}$	$x_1 \wedge \overline{x_2}$
1	1	0	0
0	0	1	0
1	0	1	1
0	1	0	0



5) X_1 XOR X_2 $= (x_1 \wedge \overline{x_2}) \vee (\overline{x_1} \wedge x_2) =$ [Not x_2 and x_1] $\vee$ [Not x_1 and x_2] =
$(x_1 \cdot \overline{x_2}) + (\overline{x_1} \cdot x_2) = x_1 \oplus x_2 \longrightarrow \overline{x_2} \cdot x_1 + \overline{x_1} \cdot x_2 = 1)$ disjunctive normal form

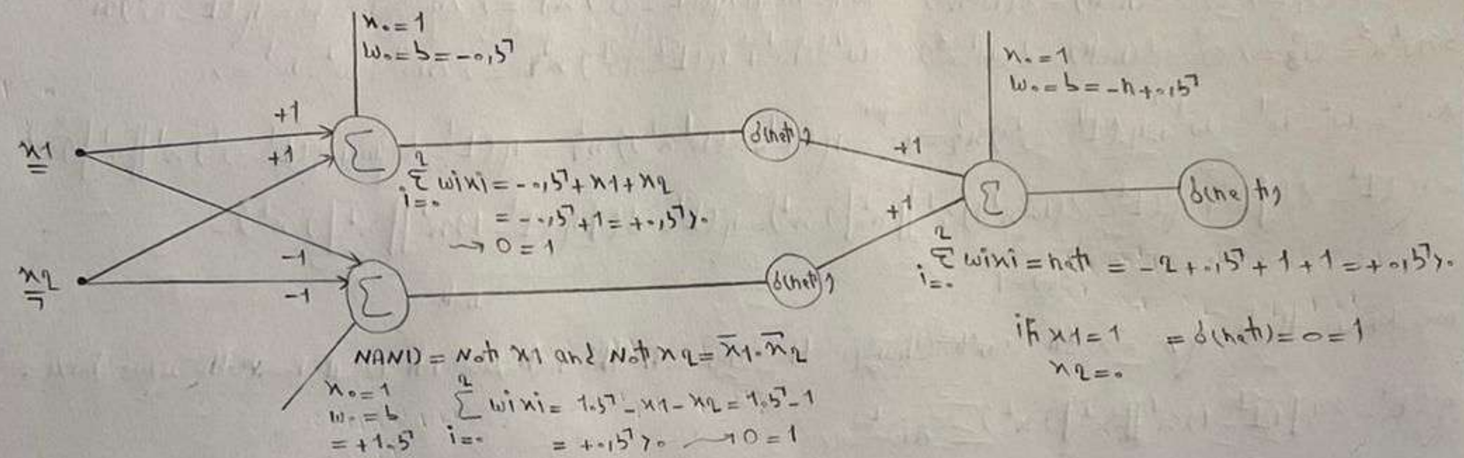


We know that if $x_1=1$ and $x_2=0$ the output of x_1 XOR x_2 is 1 not 0 so the Conjunctive normal form of XOR is $= (\bar{x}+\bar{y}).(x+y)$

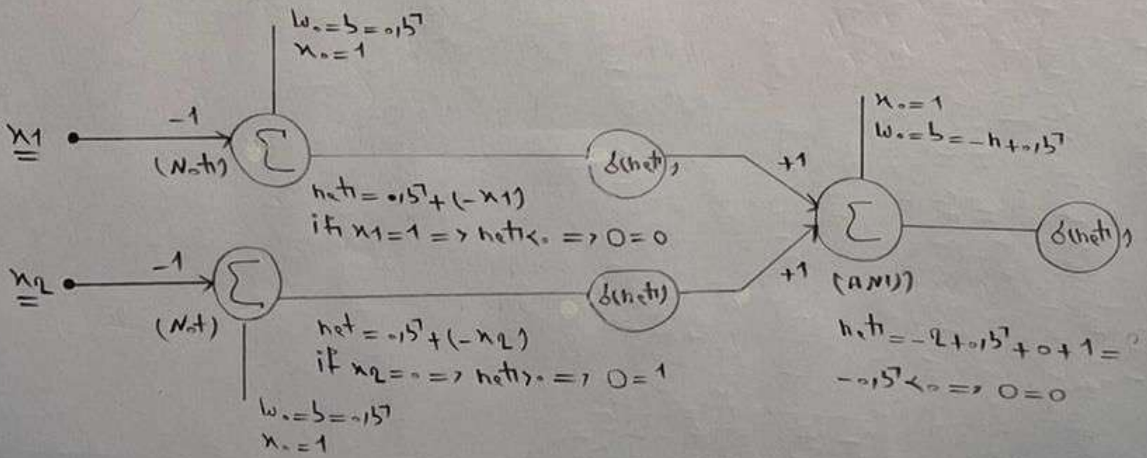
prove = we know that $\bar{x}_1.x_1=0$ and $\bar{x}_2.x_2=0 \Rightarrow$

$$x_1 \text{ XOR } x_2 = \bar{x}_2.x_1+\bar{x}_1.x_2+\bar{x}_1.x_1+\bar{x}_2.x_2 = x_1(\bar{x}_2+\bar{x}_1)+x_2(\bar{x}_1+\bar{x}_2) =$$

$$(\bar{x}_2+\bar{x}_1)(x_1+x_2) = \underbrace{(x_1+x_2)}_{\text{or}}\underset{\text{and}}{.}\underbrace{(\bar{x}_1+\bar{x}_2)}_{\text{nand}}$$



6) x_1 NOR x_2 = $\bar{x}_1$ and $\bar{x}_2$ = (Not x_1) and (Not x_2) = $\bar{x}_1 . \bar{x}_2$



7) x_1 XNOR x_2 = (x_1 and x_2) or (x_1 NOR x_2) = $(x_1.x_2)+(\bar{x}_1.\bar{x}_2)$