Lecture 04

VC - Dimension

VC - Dimension of other hypothesis spaces, e.g. intervals in $\mathbb{R}$:

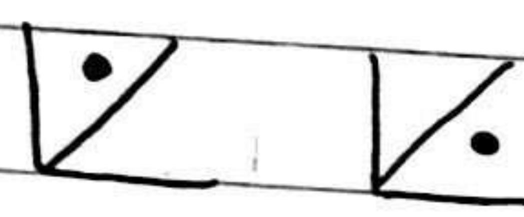
$$h(x) = +1 \text{ if } a \leq x \leq b,\ h(x) = 0 \text{ otherwise.}$$

$$h(x) = \begin{cases} 1 & a \leq x \leq b \\ 0 & \text{o.w} \end{cases}$$

First we Consider One point m: $VC(H) \geq 1$

↳ There will be two Cases : 1. $h(m) = 1 \Rightarrow a \leq m \leq b$

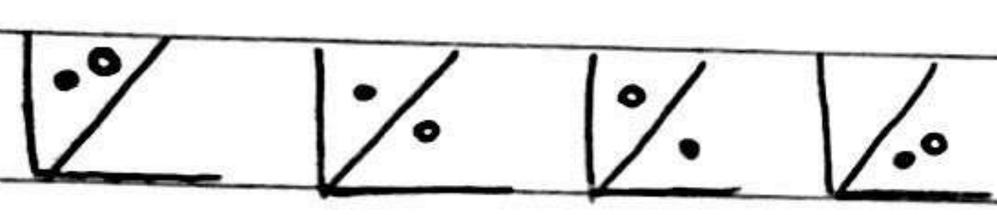
2. $h(m) = 0 \Rightarrow m < a$ or $b < m$



Then we Consider two points m and n (where $m < n$) : $VC(H) \geq 2$

↳ There will be four Cases because $2^2 = 4$:

1. $h(m) = 1$, $h(n) = 1 \Rightarrow a < m < n < b$
2. $h(m) = 0$, $h(n) = 0 \Rightarrow a < b < m < n$ or $m < n < a < b$
3. $h(m) = 0$, $h(n) = 1 \Rightarrow m < a < n < b$
4. $h(m) = 1$, $h(n) = 0 \Rightarrow a < m < b < n$



Then we Consider three points m, n and p (where $m < n < p$) :

↳ There will be eight Cases because $2^3 = 8$:

1. $h(m) = 1$, $h(n) = 1$, $h(p) = 1 \Rightarrow a < m < n < p < b$
2. $h(m) = 0$, $h(n) = 0$, $h(p) = 0 \Rightarrow m < n < p < a < b$ or $m < n < a < b < p$ or $m < a < b < n < p$ or $a < b < m < n < p$
3. $h(m) = 1$, $h(n) = 0$, $h(p) = 0 \Rightarrow a < m < b < n < p$
4. $h(m) = 1$, $h(n) = 1$, $h(p) = 0 \Rightarrow a < m < n < b < p$
5. $h(m) = 1$, $h(n) = 0$, $h(p) = 1 \Rightarrow$ ✗ It is not possible
6. $h(m) = 0$, $h(n) = 1$, $h(p) = 1 \Rightarrow m < a < n < p < b$
7. $h(m) = 0$, $h(n) = 0$, $h(p) = 1 \Rightarrow m < n < a < p < b$
8. $h(m) = 0$, $h(n) = 1$, $h(p) = 0 \Rightarrow m < a < n < b < p$

⇒ $VC(H)$ in $\mathbb{R}$ is 2