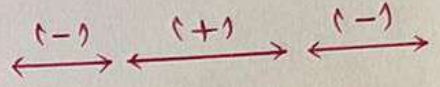
Exercise Lec 4 = VC-dimension of other hypothesis spaces, e.g. intervals in $\mathbb{R}$ =

$$h(x) = \begin{cases} +1 & \text{if } a \leq x \leq b \\ -1 & \text{otherwise} \end{cases}$$

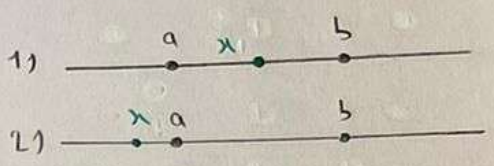


1) If we consider 1 point, we have =

If $h(x) = +1$, then $a \leq x \leq b$,
If $h(x) = -1$, then $(a > x)$ or $(b < x)$

$\Rightarrow$ VC = 1

$(2^1 = 2)$



2) If we consider 2 points, we have =

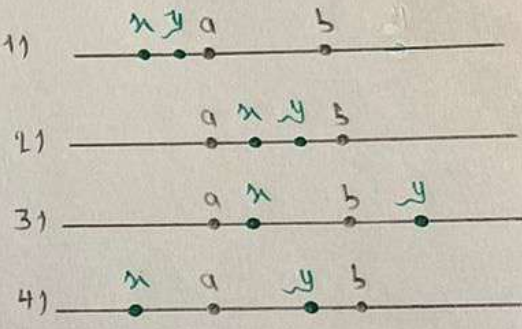
$(-1, -1) \rightarrow (x \leq y < a < b)$ or $(a < b < x \leq y)$ or $(x < a < b < y)$

$(+1, +1) \rightarrow (a \leq x < y \leq b)$

$(+1, -1) \rightarrow (a \leq x \leq b < y)$

$(-1, +1) \rightarrow (x < a \leq y \leq b)$

$(2^2 = 4)$ $\Rightarrow$ VC = 2



3) If we consider 3 points, we have =

$(-1, +1, -1)$ $h(x) = -1, h(y) = +1, h(z) = -1$
$(x < a)(a < y < b)(b < z)$

$(-1, -1, -1)$ $h(x) = -1, h(y) = -1, h(z) = -1$
$(x < a)(y < a)(x \leq y)(b < z)$

$(-1, -1, +1)$ $h(x) = -1, h(y) = -1, h(z) = +1$
$(x < a)(y < a)(x \leq y)(b \leq z)(z \leq a)$

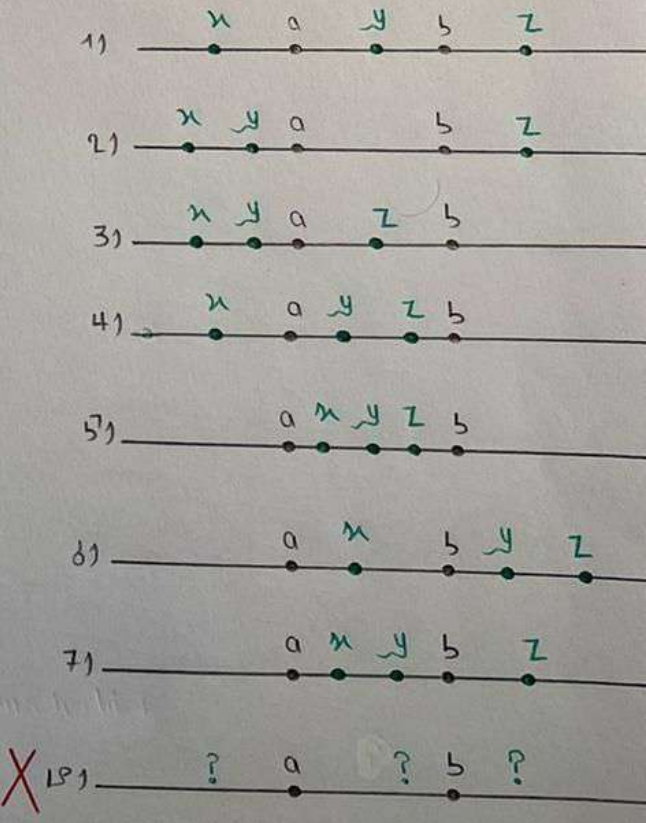
$(-1, +1, +1)$ $h(x) = -1, h(y) = +1, h(z) = +1$
$(x < a)(a \leq y < z \leq b)$

$(+1, +1, +1)$ $h(x) = +1, h(y) = +1, h(z) = +1$
$(a \leq x \leq y < z \leq b)$

$(+1, -1, -1)$ $h(x) = +1, h(y) = +1, h(z) = +1$
$(a \leq x \leq b)(b \leq y)(b \leq y < z)$

$(+1, +1, -1)$ $h(x) = +1, h(y) = +1, h(z) = -1$
$(a \leq x < y \leq b)(b < z)$

$(+1, -1, +1)$ $\Rightarrow$ $h(x) = +1$ $a \leq x \leq b$
$h(y) = -1$ $(y < a) \vee (y > b)$
$h(z) = +1$ $a \leq z \leq b$



$\Rightarrow$ Contradiction $\Rightarrow$ VC = 2