

The definition of VC dimension is: if there exists a set of n points that can be shattered by the classifier and there is no set of $n+1$ points that can be shattered by the classifier, then the VC dimension of the classifier is n .

For finding VC dimension of $\begin{cases} h(x) = +1 & \text{if } a \leq x \leq b \\ h(x) = -1 & \text{otherwise} \end{cases}$ we will consider several points, starting from the number of one point. For one point it is possible for classifying. For 2 points we consider 4 group:

1: $h_1=+1, h_2=+1$

2: $h_1=-1, h_2=-1$

3: $h_1=-1, h_2=+1$

4: $h_1=+1, h_2=-1$

All 4 modes are possible. For all 4 modes, you can find a number like x for the output of the function.

For 3 points we consider 8 group:

1: $h_1=+1, h_2=+1, h_3= +1$

2: $h_1=+1, h_2=+1, h_3= -1$

3: $h_1=+1, h_2=-1, h_3= +1$

4: $h_1=+1, h_2=-1, h_3= -1$

5: $h_1=-1, h_2=+1, h_3= +1$

6: $h_1=-1, h_2=+1, h_3= -1$

7: $h_1=-1, h_2=-1, h_3= +1$

8: $h_1=-1, h_2=-1, h_3= -1$

all states is possible except number 3. Due to $h_1=+1$ and $h_3= +1$ so $a \leq X_1 \leq b$ and $a \leq X_3 \leq b$. We have to find X_2 where $X_1 \leq X_2 \leq X_3$. The only number that can be found is either smaller than x_1 or larger than x_3 which is not acceptable. So VC dimension is 2.