Subject: Lecture 4.

Year. Month. Date. ( )

Exercise

Find the VC-Dimension of:

$$h(x)=\begin{cases}+1 & \text{if } a\le x\le b\\ -1 & \text{otherwise}\end{cases}$$

Lets assume 1 point((A)). if $h(A)=+1$ then $a\le A\le b$ otherwise $h(x)$ is either smaller than $a$ or bigger than $b$.
So VC-Dimension $\ge 1$.

For 2 points we have these cases: $(-1,-1)$, $(+1,+1)$, $(-1,+1)$, $(-1,+1)$. ((we assume these ~~outputs~~ outputs are in incremental order of the inputs.
for the first two, it's like the previous assumption.

$$\begin{cases}(-1,+1)=(h(A),h(B)): & A<a \text{ and } a\le B\le b\\ (+1,-1)=(h(C),h(D)): & a\le C\le b \text{ and } D>b\end{cases}$$

So VC-Dimension $\ge 2$.

For 3 points we have these cases:

$(-1,-1,-1)$ $(-1,+1,+1)$ $(+1,+1,+1)$ $(+1,-1,-1)$
$(-1,+1,+1)$ $(-1,+1,-1)$ $(+1,-1,+1)$ $(+1,+1,-1)$

for those two cases with red under line:

$(-1,+1,-1)=(h(A),h(B),h(C))$: since they are in order of their input so $B$ should be esither smaller than $a$ or bigger than $b$, so $h(B)$ can't be $+1$.

$(+1,-1,+1)=(h(D),h(E),h(F))$: since they are in order of their input, so $a\le E\le b$, so $h(E)$ can't be $-1$.

So VC-Dimension $=2$

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