Prove that:
If the Perceptron algorithm initialized with the null vector, then coefficients η does not affect learning.

Decision function:
$$sign(z) = \begin{cases} +1 & z > 0 \\ -1 & \text{otherwise} \end{cases}$$

weights vector: w, training examples: X,
learning rate: $\eta \geq 0$, target output: y,
Output generated by perceptron: $\hat{y}$.

$\Rightarrow$ Initialize the weights vector: w_0
$\Rightarrow$ Then we have $\hat{y}_1 = sign(w_0 \cdot x_1)$

and we can validate/update the weigths vectors according to the perceptron training:
$$w_1 = w_0 + \eta(y_1 - \hat{y}_1) \cdot x_1$$

$$\Rightarrow \hat{y}_2 = sign(w_1 \cdot x_2) = sign\left(\left(w_0 + \eta(y_1 - \hat{y}_1)x_1\right) \cdot x_2\right)$$

$$w_2 = w_1 + \eta(y_2 - \hat{y}_2)x_2 = \underbrace{w_0 + \eta(y_1 - \hat{y}_1)x_1}_{w_1} + \eta(y_2 - \hat{y}_2)x_2 =$$

$$= w_0 + \eta\left((y_1 - \hat{y}_1)x_1 + (y_2 - \hat{y}_2)x_2\right) =$$

$$= w_0 + \eta\left[\left(y_1 - \underbrace{sign(w_0 x_1)}_{\hat{y}_1}\right)x_1 + \left(y_2 - \underbrace{sign(w_1 \cdot x_2)}_{\hat{y}_2}\right)x_2\right]$$

$$= w_0 + \eta\left[\left(y_1 - sign(w_0 \cdot x_1)\right)x_1 + \left(y_2 - sign\left(\left(w_0 + \eta(y_1 - \hat{y}_1)x_1\right)x_2\right)x_2\right]\right.$$

$$\overset{\boxed{w_0 = 0}}{=} \eta\left[(y_1 - sign(0))x_1 + \left(y_2 - sign\left(\eta(y_1 - \hat{y}_1) \cdot x_1\right) \cdot x_2\right) \cdot x_2\right]$$

if $\eta = 0 \Rightarrow \omega_2 = \omega_1 = \omega_0 = 0 = \omega_i$

then $\hat{y}_i = sign(0) = -1 \quad \forall i$

$\Rightarrow$ there is not learning because the perceptron classifies all of the training examples with the same label!

if $\eta > 0 \Rightarrow$ focus on $sign(\eta(y_1 - \hat{y}_1) \cdot x_1)$:

since $sign(\eta) = +1$, η does not affect the $sign(\ldots)$, so neither the learning algorithm.

$\square$