

SIGMOIDAL ACTIVATION

• Show that  $\sigma'(y) = \sigma(y)(1 - \sigma(y))$  knowing that:

$$\sigma(y) = \frac{1}{1+e^{-y}}$$

so we have:

$$\begin{aligned}\sigma'(y) &= \frac{d\sigma(y)}{dy} = \left( \frac{1}{1+e^{-y}} \right)' = \frac{d}{dy} (1+e^{-y})^{-1} = \\ &= -\frac{1}{(1+e^{-y})^2} (-e^{-y}) = \frac{e^{-y}}{(1+e^{-y})^2} = \overset{\text{"1"}}{=} \sigma(y) \left[ \frac{e^{-y}+1}{e^{-y}+1} - \frac{1}{1+e^{-y}} \right] = \\ &= \frac{e^{-y}}{1+e^{-y}} \cdot \underbrace{\frac{1}{1+e^{-y}}}_{\sigma(y)} = \sigma(y) \left[ \frac{e^{-y}+1}{e^{-y}+1} - \frac{1}{1+e^{-y}} \right] = \\ &= \sigma(y) (1 - \sigma(y))\end{aligned}$$

Therefore, it has been demonstrated that  $\sigma'(y) = \sigma(y)(1 - \sigma(y))$