Methods and Models for Combinatorial Optimization

Introduction

Luigi De Giovanni

Dipartimento di Matematica “Tullio Levi-Civita”, Università di Padova
Contacts

Luigi De Giovanni
Dipartimento di Matematica, office 423
luigi@math.unipd.it
tel. 049 827 1349, Zoom meeting
office hours: Thursday, h 10:30 - 12:30 (or others, please, book via email)

Moodle of the course unit

https://stem.elearning.unipd.it/course/view.php?id=3570
http://www.math.unipd.it/~luigi/courses/metmodoc.html
Course unit goals

- Introduction to advanced modelling and solution techniques for **combinatorial optimization problems** in decision supporting, where an optimal solution has to be determined among a number of alternatives that *combinatorially explodes*

- The course aims at providing **mathematical and algorithmic tools** to solve optimization problems of practical interest, also with the use of the most popular **software packages or libraries**

- Ability to search for, find, understand, **adapt and implement** state-of-the-art approaches to solve real-world combinatorial optimization problems
Combinatorial Optimization: some examples

- **Logistic and transportation network**: optimal origin-destination paths, optimal pickup/delivery routes, line configuration, driver scheduling
- **Production management**: production and resource planning, job shop scheduling, optimal cutting patterns
- **Machine learning**: neural network configuration, optimal structure and weight of neural networks
- **Data-driven decision making**: cooling schedule based on massive simulation, air traffic management based on trajectory repositories
- **Optimization on graphs and networks**: coloring, cliques, quickest paths, multicommodity flows
- **Telecommunication networks**: telecom-facility location, virtual network configuration, optimal routing
- **Social network analysis**: community detection, influence maximization
- **.. and many others**
Combinatorial optimization problem: TOY example 1
“Young Money Makers”

Goal:

Decisions: how many T, B?
The space of feasible combinations

- "Easy" to find a feasible solution
- "Easy" to find the optimal solution if all the feasible combinations can be explored
- but, what if the number of product models and/or resources is large?

How to manage the combinatorial explosion of the size of the solution space using a unifying approach?

↓

Methods and Models for Combinatorial Optimization
A farmer owns 11 hectares of land where he can grow potatoes or tomatoes. Beyond the land, the available resources are: 70 kg of tomato seeds, 18 tons of potato tubers, 145 tons of fertilizer. The farmer knows that all his production can be sold with a profit of 6000 Euros per hectare of tomatoes and 7000 Euros per hectare of potatoes. Each hectare of tomatoes needs 7 kg seeds and 10 tons fertilizer. Each hectare of potatoes needs 3 tons tubers and 20 tons fertilizer. How does the farmer divide his land in order to gain as much as possible from the available resources?
Using a mathematical model: formulation

- Declare “what” is the solution, instead of stating “how” it is found
- What should we decide? **Decision variables**
  \[ x_T \geq 0, \ x_P \geq 0 \]
- What should be optimized? **Objective** as a function of the decision variables
  \[
  \max 6000 \ x_T + 7000 \ x_P \quad \text{(optimal total profit)}
  \]
- What are the characteristics of the feasible combinations of values for the decisions variables? **Constraints** as mathematical relations among decision variables
  \[
  x_T + x_P \leq 11 \quad \text{(land)}
  \\
  7 \ x_T \leq 70 \quad \text{(tomato seeds)}
  \\
  3 \ x_P \leq 18 \quad \text{(potato tubers)}
  \\
  10 \ x_T + 20 \ x_P \leq 145 \quad \text{(fertilizer)}
  \]
Using a mathematical model: solution!

**Linear relations: Linear Programming (LP) models**

- Gradient of the objective function:
  \[ 7x_T = 70 \]
- Level curves (orthogonal):
  \[ 6000x_L + 7000x_P = K \]
- Equations:
  \[ 10x_T + 20x_P = 145 \]
  \[ x_T + x_P = 11 \]

Diagram showing the feasible region with constraints and objective function.
Example: integer variables - exact method

Cutting planes [improved geometry], branch-and-bound [implicit enumeration] (computational resources!)
Example: integer variables - heuristic method

neighborhood search, evolutionary computation etc. to explore a “smart” subset of solutions (limited computational resources required)
Example: a more general combinatorial optimization problem

exact methods may be theoretically and computationally critical, heuristics still work
From decision problem to solution: the Operations Research approach

- **Formulation**: models (mathematical, graph, simulation, game theory), solution representation/perturbation, data driven ...
- **Inference**: quantitative methods, artificial intelligence, efficient algorithms
MeMoCO: Preliminary Programme (i)

- **Review, advanced topics and application of Linear Programming and Duality**
  - Linear Programming models, simplex method, basic notions of duality theory
  - Column generation technique for large size linear programming models
  - Examples: production planning optimization, network flows

- **Advanced methods for Mixed Integer Linear Programming (MILP)**
  - Branch & Bound and relaxation techniques
  - Alternative and strengthened formulations of MILP models
  - Cutting plane methods and Branch & Cut techniques
  - Examples: Travelling Salesman Problem, Facility Location, Set Covering etc.
MeMoCO: Preliminary Programme (ii)

**Meta-heuristics** for Combinatorial Optimization
- Neighbourhood search and variants
- Genetic Algorithms
- Introduction to hybrid methods and Matheuristics

**Sample applications and case studies among:**
- Network Optimization: modelling optimization problems on graphs
- Optimal routing in express freight delivery
- Data driven optimization in Air Traffic Management
- ..

**Labs**
- On-line optimization servers (e.g., NEOS)
- Optimization software and Algebraic modelling languages (e.g. AMPL, IBM-OPL)
- Optimization libraries (e.g. IBM Cplex, Coin-OR, Scip, Google OR-Tools, python, Matlab etc.)
Sample case studies


Beccaro 2018, Tabu search approach, Master Thesis
Sample case studies

**Stanley et al. 2002,** *Evolving Neural Networks through augmenting topologies,* Evol. Comp. Journal

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**De Giovanni et al. 2019,** *A two-level local search heuristic for pickup and delivery problems in express freight trucking,* Networks

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**Shirokikh et al. 2013,** *Comb. Opt. Techniques for Network-Based Data Mining,* Handbook of C.O.
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De Giovanni et al. 2022, *Master Thesis Machine Learning and matheuristics for Air Traffic Management*
Peculiarities and relations to other course units\(^1\)

- Integrated presentation of **diverse optimization techniques**
- Subjects presented with **specific emphasis**. Focus on:
  - **combinatorial** ("discrete", linear) optimization with deterministic settings
  - **engineering aspects**: design and implementation of models and algorithms suitable for real-world applications
  - **comparison and choice** between different approaches

- Insight into several **metaheuristic** optimization techniques and their relation to exact methods

- Introduction to **hybrid approaches**:
  - mixing paradigms (hybrid metaheuristics)
  - metaheuristics supporting exact methods or exploiting them (matheuristics)
  - including machine learning techniques (how data science helps optimization)

\(^1\) e.g., “Operations Research”, “Optimization”, “Optimization for Data Science”, “Stochastic Optimization”
Practical info (i)

- 48 hours (36 lectures + 12 labs, **6 CFU**). First Semester
- **Teaching mode:** classroom or lab + recorded videos (+ streaming?)
- **Moodle:** lecture notes, papers, lab materials, recordings, notices etc. [https://stem.elearning.unipd.it/course/view.php?id=3570](https://stem.elearning.unipd.it/course/view.php?id=3570)
- **Schedule:** Thursday and Friday, 2:30 – 4:30 pm
  - room LuF1 or LabTA: **always check!**
- **Learning activities:** Classes, Discussion about case studies, Labs (implementation of mathematical programming models and basic optimization algorithms).
Practical info (ii)

- **Textbooks and learning supports**
  - Lecture notes provided by the teacher + articles from scientific journals (available **before** the class: reading in advance is recommended!)
  - Optimization software packages available on line and in labs (or free student editions).

- **Examination method**
  - **Two lab exercises**: implementation of 1) a MILP model and 2) a metaheuristic (or alternative) algorithm, to be delivered some days before the oral examination (**no** due date during the classes).
    - Mandatory [1-10 /30, minimum 5]
  - **Oral examination** on course unit contents.
    - Mandatory [1-20 /30, minimum 10]
  - **Lab exercises + Oral examination ≥ 18**
  - Short project. Optional [+2 to +6 /30] (e.g., after the oral to improve the score if necessary): modeling and solving a specific problem, even suggested by you, implementing a component of an optimization method etc. (to agree with the teacher)