Planning Detention Storage for Stormwater Management

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Abstract: A method for estimating detention storage capacity in stormwater management is presented. A generalized storage-overflow relationship is derived. This relationship defines real available storage (empty space in detention basin) on the positive range and overflow volumes on the negative range. By using exponential probability density functions for the independent hydrologic variables runoff volumes, runoff durations, and interevent times, and the generalized storage relationship, a new probability distribution is derived for the treatment plant overflow volumes. The new distribution provides an easy method for estimating the detention storage and treatment capacity for a design risk level. The methodology has the advantage that it provides easy to use preliminary planning information for stormwater management without the need for extensive simulation.

Introduction

Urban stormwater and combined sewer overflow are major sources of pollution in many receiving streams. Unless the combined sewer overflows are controlled and treated, many receiving streams may become unsuitable for human needs. The methods for planning stormwater control measures (treatment and storage) can be grouped under the following three categories: (1) Design storm approach; (2) simulation modeling; and (3) derived distribution approach.

The design storm is usually obtained from frequency-duration-depth curves. The design storm is coupled with a unit hydrograph and routed through a treatment plant of specified capacity to estimate the needed storage. Because the method ignores the cumulative effect of closely spaced storms, the estimate of the treatment capacity-storage combination is not accurate. The use of this method is severely criticized in the literature. For example, Linsley and Crawford (6) point out that while the cost of storm drainage systems runs to billions of dollars per year, the use of such a crude method is thus hardly justified.

Simulation is the conceptual modeling of a physical system. This approach recognizes not only the properties of a storm but also the effect of successive storm events. For prespecified treatment capacity-storage combinations, performance curves for storm overflows can be developed (4,9,10). In general, these models are extensive, data intensive, and require a large computer memory. Also, a large number of computer runs

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have to be made to develop the performance curves.

The derived distribution approach is based on the statistical distributions of random variables. Using hydrological relationships, new probability distributions are derived for the dependent variables such as runoff and overflow. It may yield closed form solutions that provide quick estimates for the treatment and storage and are easily adaptable to further analytical treatment. In the present work, based on the derived distribution approach, analytical solutions are developed for the treatment-storage planning without resorting to extensive simulation models.

Howard (5) assumed that the storm volumes and the interevent times (time between storms) were exponentially distributed. Analytical expressions for the probability distributions of the overflows and related variables were derived but the durations of storms were not taken into account. Di Toro and Small (3) proposed a derived distribution for the stormwater overflows. The flows were assumed to be uniform over the duration. The flows, durations, and interevent times were assumed to be gamma distributed. In the formulation, several expressions did not have analytical solutions and were numerically evaluated. Chan and Bras (1) proposed a distribution for overflows based on the kinematic routing. This method did not consider the carryover storage. This formulation also required an approximation scheme for end results. Smith (12) took into consideration the duration of storms. The storm volumes, durations, and interevent times were assumed to be exponentially distributed. The storage level in the reservoir was also considered as a random variable. The expression for the distribution of storage level ruled out a strictly analytical solution. Following Smith (12), Schwarz and Adams (11) obtained analytical expressions for the probability distribution of spill volumes from two detention storage reservoirs in series. Many of these algorithms require extensive numerical schemes to obtain end results. Loganathan and Delleur (8) derived an analytical expression for the distributions of combined sewer overflows based on exponential distributions for runoff volumes, durations of events, and interevent times. No numerical scheme was required. Also a probability distribution function for the quality of the pollutant after mixing with the untreated overflows was derived. Both Refs. 5 and 8 assumed that the previous storm completely filled the storage. This assumption leads to an upperbound estimate of treatment and storage because of the consideration of the worst scenerio. However, it is not realistic and leads to "over design." In the present work, this assumption is relaxed, which changes the whole analysis. The problem falls into an analysis of certain stochastic process rather than an analysis of functions of random variables.

Generalized Storage

The urban stormwater management process may be idealized as shown in Fig. 1. A sequence of runoff events with volumes, X_1 (basin inches), durations, X_2 (hours), and interevent times, X_3 (hours) is shown. The stormwater runoff X_1 is treated at treatment rate a (in./hr). Because of the limited treatment capacity, if the runoff volume, X_1, is greater than that which can be treated in X_2 (hours), the untreated excess $(X_1 - aX_2)$

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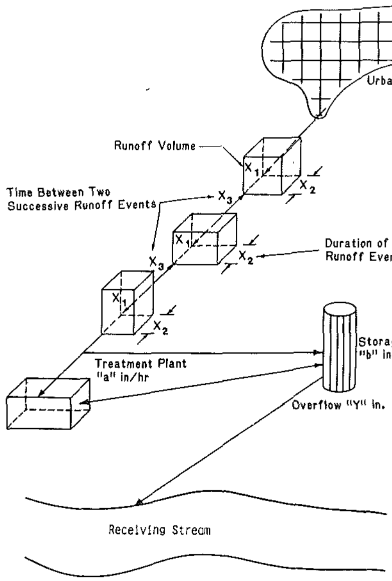

FIG. 1.—Schematic Representation of Urban Stormwater Runoff Process

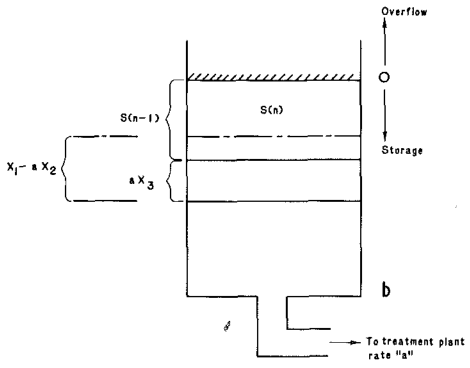

FIG. 2.—Generalized Storage

must be stored for later treatment. Let the storage capacity be b (in.). It is possible that a major storm can exceed both the treatment capacity and the available storage (empty space in the tank). In such a situation an overflow of Y (in.) occurs. Considering the nth runoff event the following notation is used: $S(n)$ = available storage (empty space) at the end of the nth runoff event; $X_1^{(n)}$ = volume of the nth event; $X_2^{(n)}$ = duration of the nth event; $X_3^{(n)}$ = time between $(n-1)$st and nth events; $Y(n)$ = overflow volume at the end of the nth event. Consider $S(n-1)$, the available storage at the end of the $(n-1)$st storm. Because the stormwater can be withdrawn from the storage for treatment during the interevent time, $X_3^{(n)}$, the available storage at the beginning of the nth event can be written as (see Fig. 2) $[S(n-1)+aX_3^{(n)}]$. This storage will be reduced by $[X_1^{(n)}-aX_2^{(n)}]$ at the end of the nth event. Thus, the available storage at the end of the nth event is

$$S(n)=\min\{\min[S^*(n-1)+aX_3^{(n)},b]-[X_1^{(n)}-aX_2^{(n)}],b\} \quad (1)$$

in which

$$\left.\begin{aligned} S^*(n-1) &= S(n-1) \quad \text{if} \quad S(n-1)>0 \\ &= 0 \qquad\qquad \text{if} \quad S(n-1)\le 0 \end{aligned}\right\} \quad (2)$$

Because of the limited storage capacity b, the minima are used in Eq. 1. It is seen that $S(n)$ can be negative in Eq. 1. The negative storage $S(n)$ will occur whenever the runoff volume X_1 is very large. In such a situation, an overflow occurs and is precisely equal to $-S(n)$.

$S(n-1)<0$ means that there is an overflow at the end of $(n-1)$st stage and there is no empty space in the tank at the end of the $(n-1)$st stage (tank is full). This implies physically at the end of the $(n-1)$st storm the actual available storage (empty space) must be zero. Therefore, one defines $S^*(n-1)$ as in Eq. 2 in computing the available storage at the end of the nth storm. S^* is called the generalized storage because $S(n)$ can be negative.

It is of interest to examine Eq. 1 more carefully. In general, the ranges for $X_1^{(n)}$, $X_2^{(n)}$, $X_3^{(n)}$ are $[0,\infty)$, and thus $S(n)$ has a range $(-\infty,\infty)$. There are several cases of interest in analyzing the range of $S(n)$:

1. For $X_1^{(n)}>aX_2^{(n)}$ and $X_1^{(n)}-aX_2^{(n)}<\min[S^*(n-1)+aX_3^{(n)},b]$ there is real available storage (empty space >0).

$$S(n)=\min[S^*(n-1)+aX_3^{(n)},b]-[X_1^{(n)}-aX_2^{(n)}]<b \quad (3)$$

because $X_1^{(n)}-aX_2^{(n)}>0$.

2. For $X_1^{(n)}>aX_2^{(n)}$ and $X_1^{(n)}-aX_2^{(n)}>\min[S^*(n-1)+aX_3^{(n)},b]$ there is an overflow (see Fig. 3).

$$Y(n)=-S(n)=X_1^{(n)}-aX_2^{(n)}-\min[S^*(n-1)+aX_3^{(n)},b] \quad (4)$$

because $b>0$.

3. For $X_1^{(n)}<aX_2^{(n)}$ it is possible to withdraw water from storage for treatment, even during the storm period. Thus, the storage space can be increased by $[aX_2^{(n)}-X_1^{(n)}]>0$ during the storm period.

$$S(n)=\min\{\min[S^*(n-1)+aX_3^{(n)},b]+[aX_2^{(n)}-X_1^{(n)}],b\} \quad (5)$$

Thus, the positive $S(n)$ provides for real storage and negative $S(n)$ represents the overflow.

The probabilities of exceedence of overflow volumes can be used as various risk levels. Suppose an expression for these risk levels is available, then, it is possible to compute the treatment-storage combinations for a design risk level. For mathematical simplicity, Eq. 1 can be written as

$$S(n)=\min\{\min[S(n-1)+aX_3^{(n)},b]-[X_1^{(n)}-aX_2^{(n)}],b\}\,I_{[S(n-1)>0]}$$
$$+\min\{\min[aX_3^{(n)},b]-[X_1^{(n)}-aX_2^{(n)}],b\}\,I_{[S(n-1)\le 0]} \quad (6)$$

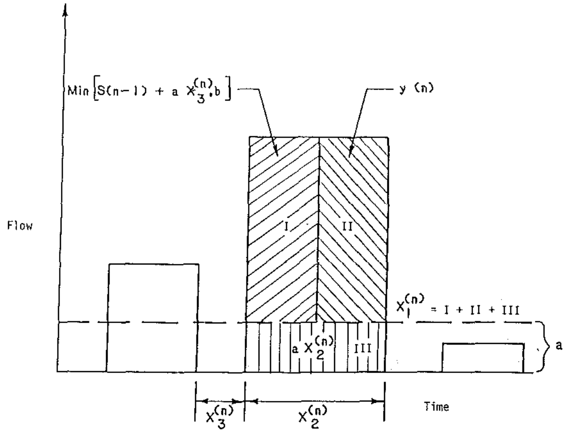

FIG. 3.—Overflow

in which

$$I_{[S(n-1)>0]} = 1 \quad \text{if} \quad S(n-1) > 0 \\ = 0 \quad \text{if} \quad S(n-1) \le 0 \qquad (7)$$

$I_{[\cdot]}$ is called the indicator function; and Eq. 6 is used to derive the distribution of $S(n)$. Based on suitable probability distributions for X_1, X_2, and X_3, and Eq. 6, a new distribution for $S(n)$ can be derived. If $X_1^{(1)}$, ..., $X_1^{(n)}$ are independent, identically distributed (i.i.d.) variables, and $X_2^{(n)}$, $X_3^{(n)}$ are i.i.d. variables, then $S(n)$ forms a time homogeneous Markov chain. It was shown in Refs. 8 and 12 that X_1, X_2, and X_3 could be taken as exponentially distributed random variables.

Derivation of Distribution Functions

The following assumptions are used in the analysis. The validity of these assumptions is examined in the application section:

1. It is assumed that $X_1^{(1)}$, $X_1^{(2)}$, ..., $X_1^{(n)}$ are independent, identically distributed (i.i.d.) variables. Also, $X_2^{(n)}$ and $X_3^{(n)}$ are i.i.d. variables.
2. X_1, X_2, X_3 are statistically independent (8,12).
3. X_1, X_2, X_3 are exponentially distributed with parameters α, β, and γ, respectively (8,12).

Based on Eq. 6 the following probability statement can be made for $S(n-1) > 0$:

$$P[S(n) > s | S(n-1) = c] = P\{\min[\min(c + aX_3, b) - (X_1 - aX_2), b] > s\}; \quad S(n-1) > 0, \ c \ge 0 \qquad (8)$$

and for $S(n-1) \le 0$:

$$P[S(n) > s | S(n-1) = c] = P\{\min[\min(aX_3, b) - (X_1 - aX_2), b] > s\}; \quad S(n-1) \le 0, \ c = 0 \qquad (9)$$

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By letting $T = \min[c + aX_3, b]$ and $W = X_1 - aX_2$ .... (10)

then Eq. 8 is rewritten for $s \geq b$, as

$$P[S(n) > s|S(n-1) = c] = P[T - W > s]\,P[b > s] = 0 \quad (11)$$

since $P(b > s) = 0$, and for $s < b$, Eq. 8 is rewritten as

$$P[S(n) > s|S(n-1) = c] = P[T - W > s] \quad (12)$$

since $P(b \geq s) = 1$. The evaluation of Eq. 12 requires the calculation of the distributions of T and W.

Distribution of T.—The exceedence probability function of T can be derived as follows. From the definition of T in Eq. 10, it may be written as

$$\left.\begin{aligned} P[T > t] &= P[c + aX_3 > t]\,P[b > t] \\ &= 0 \qquad\qquad\qquad \text{if} \quad t \geq b \\ &= P[c + ax_3 > t] \quad \text{if} \quad t < b \end{aligned}\right\} \quad (13)$$

In particular for $t < b$, and making use of assumption 3

$$\left.\begin{aligned} P[c + aX_3 > t] &= \exp\left[-\frac{\gamma}{a}(t - c)\right] \quad \text{for} \quad t \geq c \\ &= 1 \qquad\qquad\qquad\qquad \text{for} \quad t < c \end{aligned}\right\} \quad (14)$$

Thus, the probability density function of T is obtained from Eq. 14 as

$$\left.\begin{aligned} f_T(t) &= \frac{\gamma}{a}\exp\left[-\frac{\gamma}{a}(t - c)\right] \quad \text{for} \quad b > t \geq c \\ &= 0 \qquad\qquad\qquad\qquad\quad \text{for} \quad t < c \\ &= \exp\left[-\frac{\gamma}{a}(b - c)\right] \qquad \text{for} \quad t = b \end{aligned}\right\} \quad (15)$$

Distribution of W.—The probability distribution function of W is derived as follows from the definition Eq. 10 and assumption 3:

$$\left.\begin{aligned} P(W \leq w) &= \alpha\beta \int_{x_2=0}^{\infty}\int_{x_1=0}^{w+ax_2} \exp(-\alpha x_1 - \beta x_2)\,dx_1\,dx_2, \quad \text{for} \quad w > 0 \\ &= \beta \int_{-w/a}^{\infty} \{1 - \exp[-\alpha(w + ax_2)]\}\exp(-\beta x_2)\,dx_2, \quad \text{for} \quad w \leq 0 \end{aligned}\right\} \quad (16)$$

The probability density function of W is obtained from Eq. 16

$$\begin{aligned} f_W(w) &= \frac{\alpha\beta}{\alpha a + \beta}\exp(-\alpha w) \quad \text{for} \quad w > 0 \\ &= \frac{\alpha\beta}{\alpha a + \beta}\exp\left(\frac{\beta}{a}w\right) \quad \text{for} \quad w \leq 0 \end{aligned} \quad (17)$$

Conditional Distribution of $S(n)$.—Considering Eq. 12, the probability of storage at the end of the nth storm to be greater than some threshold value, s, given that the storage at the end of the $(n-1)$st storm equals c is calculated for the two cases $c \leq s < b$ and $s \leq c < b$. For the first case namely, $c \le s < b$

$$P[S(n) > s|S(n-1) = c] = P(T - W > s) \qquad (18)$$

$$P[S(n) > s|S(n-1) = c] = \int_c^s P(W < t - s)\, f_T(t)\, dt$$

$$+ \int_s^{b-} P(W < t - s)\, f_T(t)\, dt$$

$$+ \exp\left[-\frac{\gamma}{a}(b - c)\right]\left\{1 - \frac{\beta}{(\alpha a + \beta)}\exp\left[-\alpha(b - s)\right]\right\} \qquad (19)$$

in which the first integral corresponds to $t - s < 0$, the second integral to $t - s > 0$ and the last term is due to the point mass of T at b. The conditional cumulative distribution of $S(n)$ can be written as

$$P[S(n) \le s|S(n-1) = c] = 1 - P[S(n) > s|S(n-1) = c] \qquad (20)$$

By using Eqs. 15, 19, and 20 the conditional distribution of $S(n)$ for $c \le s < b$ can be written as

$$P[S(n) \le s|S(n-1) = c] = 1 - m\left\{\exp\left[-\frac{\beta}{a}(s - c)\right] - \exp\left[-\frac{\gamma}{a}(s - c)\right]\right\}$$

$$- (1 - k)\exp\left[-\frac{\gamma}{a}(s - c)\right]$$

$$+ k\frac{\alpha a}{\gamma}\exp\left[-\alpha(b - s) - \frac{\gamma}{a}(b - c)\right] \quad \text{for} \quad 0 \le c \le s < b \qquad (21)$$

in which $m = \dfrac{\alpha\gamma a}{(\alpha a + \beta)(\gamma - \beta)}$; $k = \dfrac{\beta\gamma}{(\alpha a + \beta)(\alpha a + \gamma)}$ (22)

Considering the second case, namely $s \le c < b$

$$P[S(n) \le s|S(n-1) = c] = \int_c^{b-} P(W < t - s)\, f_T(t)\, dt$$

$$+ \exp\left[-\frac{\gamma}{a}(b - c)\right]\left\{1 - \frac{\beta}{(\alpha a + \beta)}\exp\left[-\alpha(b - s)\right]\right\} \qquad (23)$$

Thus, the conditional distribution of $S(n)$ is

$$P[S(n) \le s|S(n-1) = c] = k\left\{\exp\left[-\alpha(c - s)\right] + \frac{\alpha a}{\gamma}\right.$$

$$\left.\exp\left[-\alpha(b - s) - \frac{\gamma}{a}(b - c)\right]\right\} \quad \text{for} \quad s \le c < b \qquad (24)$$

The point probability mass of $S(n)$ at b for a given $S(n - 1)$ is

$$P[S(n) = b|S(n-1) = c] = m\left\{\exp\left[-\frac{\beta}{a}(b - c)\right]\right.$$

$$\left.- \frac{\beta}{\gamma}\exp\left[-\frac{\gamma}{a}(b - c)\right]\right\} \qquad (25)$$

Special Case.—Consider the special case wherein previous storm completely filled the storage. It is seen that $c = 0$. By using Eq. 24 and recalling $Y(n) = -S(n)$ there results

$$P[Y(n) \geq y] = K \exp(-\alpha y) \quad \text{for} \quad y > 0 \qquad (26)$$

in which $$K = k\left\{1 + \frac{\alpha a}{\gamma} \exp\left[-b\left(\alpha + \frac{\gamma}{a}\right)\right]\right\} \qquad (27)$$

$$\left.\begin{aligned} &\text{Thus,} \quad P(0 < Y(n) \leq y) = K[1 - \exp(-\alpha y)] \quad &&\text{for} \quad y > 0 \\ &\text{and} \quad P(Y = 0) = 1 - K &&\text{for} \quad y = 0 \end{aligned}\right\} \qquad (28)$$

This result was previously obtained by Loganathan and Delleur (8) and is a particular case of the present analysis.

Storage Estimation

In many urban drainage problems, estimation of storage capacity for a fixed treatment rate and a given risk level is required. The risk is defined as the probability of overflows into the receiving stream. Let ϵ be the risk level. Two cases are of interest:

Case a.—Initially the tank is empty, $c = b$. This is the most favorable scenario. From Eq. 24

$$P[S(n) \leq 0 | S(n-1) = b] = \frac{\beta}{\alpha a + \beta} \exp(-\alpha b) \leq \epsilon \qquad (29)$$

For a given treatment rate a

$$b \geq \frac{1}{\alpha} \ln\left[\frac{\beta}{(\alpha a + \beta)\epsilon}\right] \qquad (30)$$

Using Eq. 30, the storage capacity for a given treatment rate can be computed. This will also guarantee that the probability of overflows will be less than or equal to the design risk level, ϵ. In Eq. 30, it is seen that the parameter γ is not present. That is, the interevent time does not play any role in computing the probability. It is because of the assumption that the entire storage is available as empty space at the end of the previous storm. Since the whole tank is empty, no water is withdrawn during the interevent time and thus γ does not appear in Eq. 30.

It can be shown analytically that for a given storage capacity b, the overflow probability is a minimum when the tank is kept empty; also the overflow probability is a maximum when the tank is kept full. By letting $c = \delta b$, $0 \leq \delta \leq 1$, in Eq. 24 the overflow probability is given as

$$P[S(n) < 0 | S(n-1) = \delta b] = k\left\{\frac{\alpha a}{\gamma} \exp\left[-b\left(\alpha + \frac{\gamma}{a}(1 - \delta)\right)\right] + \exp(-\alpha \delta b)\right\} \qquad (31)$$

which is a function of δ, $$f(\delta) = P[S(n) < 0 | S(n-1) = \delta b] \qquad (32)$$

The minimum is found at the stationary point given by $\delta = 1$. So, the minimum overflow probability is obtained when the tank is empty. Because $f(\delta)$ is a convex function on $0 \le \delta \le 1$, the maximum is attained at $\delta = 0$, which corresponds to the full tank situation.

Case b.—Initially the tank is full, $c = 0$. This is the most critical scenario. From Eqs. 24 and 27

$$P[S(n) < 0 | S(n-1) = 0] = K \quad (33)$$

From Eqs. 33 and 27 it can be seen that no matter how large a storage is provided, there is a finite probability of overflow, k. This probability is exactly equal to $P[X_1 > a(X_2 + X_3)]$. There is a finite probability of occurrence of runoff volumes that can exceed the volume that can be treated during the interevent time and the duration of the event. This result agrees with the intuition that initially full storage would lead to more overflows. For a given risk level ϵ (ϵ must be greater than the finite overflow probability), and fixed treatment rate a, the storage capacity can be computed from Eq. 33 as

$$b \ge \frac{a}{\alpha a + \gamma} \ln \left[\frac{\alpha \beta a}{\epsilon(\alpha a + \beta)(\alpha a + \gamma) - \beta \gamma} \right] \quad (34)$$

Eqs. 30 and 34 can be used as design aids for estimating detention storage capacity.

Treatment Rate Estimation

Using Eq. 24 for various risk levels the storage-treatment combination can be found. It is of interest to consider the case when storage capacity is zero. From Eq. 29 for $b = 0$ it is seen

$$\frac{\beta}{\alpha a + \beta} = \epsilon \quad (35)$$

which yields $$a = \frac{\beta(1 - \epsilon)}{\alpha \epsilon} \quad (36)$$

It is to be noticed when the storage capacity $b = 0$, it does not matter whether the initial storage is full or empty. Thus, Eq. 34 also yields the same value for the treatment rate a as in Eq. 36. In Eq. 34 a quadratic expression is to be solved for a.

Flow Capture Efficiency

The fraction of the runoff volume captured by the storage is defined as the flow capture efficiency, which can be expressed as $[1 - E(Y)/E(X_1)]$. This parameter is used in Refs. 3 and 4 for determining the storage capacity for a given treatment rate. In the following, a relationship between the flow capture efficiency and the storage estimators given in Eqs. 30 and 34 is derived. From Eq. 24 by using $Y(n) = -S(n)$, the conditional cumulative distribution function for $Y(n)$ can be obtained. It is given as

$$P[Y(n) \le y | S(n-1) = c] = 1 - k \left\{ \exp[-\alpha(c + y)] + \frac{\alpha a}{\gamma} \exp\left[-\alpha(b + y) - \frac{\gamma}{a}(b - c) \right] \right\} \quad \text{for} \quad y > 0 \quad (37)$$

Also, by letting $P[Y(n) \le y|S(n-1) = c] = G(y)$ .... (38)

and because only positive overflows need be considered

$$E[Y(n)|S(n-1) = c] = \int_{0^+}^{\infty} y dG(y) \qquad (39)$$

$$E[Y(n)|S(n-1) = c] = \frac{1}{\alpha} P[S(n) \le 0|S(n-1) = c] \qquad (40)$$

Recalling $E(X_1) = 1/\alpha$ there results

$$\frac{1}{E(X_1)} E[Y(n)|S(n-1) = c] = P[S(n) \le 0|S(n-1) = c] \qquad (41)$$

Based on Eq. 40, upper and lower bounds on storage capacity, b, for a given treatment rate, a, can be derived. Considering the initial storage is full (critical case), $S(n-1) = 0$

$$\frac{1}{E(X_1)} [E(Y(n)|S(n-1) = 0] = P[(S(n) \le 0|S(n-1) = 0] \qquad (42)$$

which is exactly the same as Eq. 33 and yields the upper bound storage given by Eq. 34. Considering the most favorable situation that the initial storage is empty, $S(n-1) = b$

$$\frac{1}{E(X_1)} [E(Y(n)|S(n-1) = b] = P[(S(n) \le 0|S(n-1) = b] \qquad (43)$$

leads to the lower bound storage given in Eq. 30. Thus, it may be concluded that Eqs. 30 and 34 provide analytical expressions for storage capacity for a specified flow capture efficiency. The flow capture efficiency and the risk level ϵ used in Eqs. 30 and 34 are related by

$\epsilon = 1 -$ flow capture efficiency .... (44)

Transition Probabilities

It is also of interest to compute the probabilities of storage changes. To fix the value of c, the following mid point approximation will be used.

The event $[a_1 \le S(n) \le a_2] \simeq \left[S(n) = \frac{a_1 + a_2}{2} \right]$ .... (45)

Transition to Real Storage.—Suppose $(a_1 + a_2)/2 = c \ge 0$ and $b > a_3$, $a_4 > c$. The transition probability $P[a_3 < S(n) < a_4|S(n-1) = c]$ can be computed from Eq. 21 as follows:

$$P[a_3 < S(n) < a_4|S(n-1) = c] = P[S(n) < a_4|S(n-1) = c] - P[S(n) < a_3|S(n-1) = c] \qquad (46)$$

For $c > a_4 > a_3 > 0$, Eq. 24 can be used to compute $P[a_3 < S(n) < a_4|S(n-1) = c]$.

Transition from an Overflow.—Whenever $S(n-1) < 0$, $c = 0$. In such a case, the transition probability is given as, $P[a_3 < S(n) < a_4|c = 0]$. It is to be noted that $c = 0$, regardless of the numerical value of $S(n-1)$ as long as $S(n-1) < 0$. It is also physically conceivable because the nth storage depends on the available empty space (in this case zero) and not on the magnitude of the overflow.

Transition to an Overflow.—For an overflow $a_3, a_4 < 0 \leq c$. Eq. 24, is used to compute the transition probabilities $P[a_3 < S(n) < a_4 | S(n-1) = c]$.

Transition to Entire Storage.—Full capacity becomes available. This probability $P[S(n) = b | S(n-1) = c]$ is computed from Eq. 25.

Application

The transition probabilities were computed for the West Lafayette data (7). The West Lafayette area is 3,052 acres (12.35 km^2). The interevent time is defined as the time between the beginning of the current runoff event and the end of the previous runoff event. The runoff events are generated from the observed rainfall using a runoff coefficient 0.21 and a maximum depression storage of 0.18 in. (4.6 mm). The statistical independence and exponential probability distributions for X_1, X_2, and X_3 are verified in Ref. 8. The statistical independence is verified by comparing the expected value of the products with the product of the expected values of X_1, X_2, and X_3, which must be equal. The exponential distribution is verified by plotting the log exceedence probabilities with the corresponding cutoff values, which must be a straight line. The statistics of X_1, X_2, and X_3 are given in Table 1. In computing the transition probabilities (see Table 2), the following ranges are considered for $S(n)$ as eleven states, i: $(-\infty, -0.18)$, $(-0.18, -0.16)$, $(-0.16, -0.14)$, $(-0.14, -0.12)$, $(-0.12, -0.10)$, $(-0.10, -0.08)$, $(-0.08, -0.06)$, $(-0.06, -0.04)$, $(-0.04, -0.02)$, $(-0.02, 0)$, (0).

Case a.—In this case, the existing situation in West Lafayette is analyzed. The special case, Eq. 28 is considered. For a treatment rate $a = 0.006$ and no storage $b = 0.0$, the transition probabilities are given in Table 2. The transition is from zero available storage to an overflow state. From Eq. 25

TABLE 1.—Runoff Data (1953–1974)

Parameter (1)	Value (2)
Mean runoff volume, $E(X_1)$	0.06 in. (1.5 mm) = 16.7 1/in.
Mean duration, $E(X_2)$	2.1 hr = 0.4761 1/hr
Mean interevent time, $E(X_3)$	70 hr = 0.0141 1/hr

TABLE 2.—Probability of Overflow

	State, i										
Initial state, c (1)	1 (2)	2 (3)	3 (4)	4 (5)	5 (6)	6 (7)	7 (8)	8 (9)	9 (10)	10 (11)	11 (12)
0	0.040	0.016	0.023	0.032	0.044	0.062	0.086	0.120	0.168	0.235	0.174

$$P[S(n) = 0|c = 0] = \frac{\alpha a}{(\alpha a + \beta)} = 0.174 \qquad (47)$$

This is the probability of no overflow which is the same as that obtained in Ref. 8. In Table 2, there is a probability of 0.235 that the overflow will be in state 10, that is $(-0.02, 0)$ in., also, it is seen that there is only 4% probability that an overflow will exceed 0.18 in.

Case b.—In this case no treatment and no storage are assumed, i.e., $a = b = 0$. Thus, the probabilities of overflow must coincide with those of runoff. From Eq. 6, for $a = b = 0$, $S(n) = -X_1^{(n)}$

$$P(X_1 \leq x_1) = P(-X_1 \geq -x_1) = P(S \geq s)$$

$$\text{in which} \quad S = -X_1, x_1 > 0 \qquad (48)$$

From Eq. 24, for $a = b = c = 0$

$$P(S \geq s) = 1 - \exp(\alpha s) = 1 - \exp(-\alpha x_1) = P(X_1 \leq x_1), \quad s < 0 \qquad (49)$$

Case c.—Consider the case of infinite treatment and large storage, then, there can be no overflows. Because of infinite treatment the storage should be empty always. That is, total capacity of storage is available all the time. This can be verified by Eqs. 21, 24, and 25. From Eq. 21 as $a \uparrow \infty$

$$P[S(n) \leq s|S(n-1) = c] = 0 \quad 0 \leq c < s < b \qquad (50)$$

From Eq. 24

$$P[S(n) \leq s)|S(n-1) = c] = 0 \quad s \leq c \qquad (51)$$

From Eq. 25

$$P[S(n) = b|S(n-1) = c] = 1 \qquad (52)$$

Case d.—In this case, a storage capacity $b = 0.09$ and a treatment rate $a = 0.04$ are used. The transition probabilities from state i, to state j are given in Table 3. States i and j represent the available storage (empty space) in storage. From the transition matrix, it is observed if the initial available storage (empty space) is large, then the probability of overflows is low. Also, it agrees with the intuition that initially large available empty space has higher probability of reaching the whole capacity. Using the transition probability matrix **P**, it is possible to compute the unconditional probabilities for each of the states. The steady state probabilities λ, are given by

$$\lambda = \lambda[\mathbf{P}] \qquad (53)$$

For case d the steady state probabilities are computed as in Table 4.

Because the overflow probability is 0.0931, a small value, the storage capacity 0.09 and treatment rate 0.04 can be considered as acceptable control measures. Eqs. 30 and 34 can also be used in this regard. For $\epsilon = 0.1$, and $a = 0.04$, the lower and upper bound storage values are 0.085 and 0.088 in., respectively. It is noted that the relatively close values of upper and lower bounds are due to the fact that the mean interevent time is large. For large interevent times the parameter $\gamma \rightarrow 0$. From Eq. 34 as $\gamma \rightarrow 0$, $b \rightarrow 1/\alpha \ln[\beta/(\alpha a + \beta)\epsilon$, which is the same as Eq. 30. The

J. Water Resour. Plann. Manage. 1985.111:382-398.

TABLE 3.—Transition Probabilities

i (1)	j: 0 (2)	0.009 (3)	0.027 (4)	0.045 (5)	0.063 (6)	0.081 (7)	0.09 (8)
0	0.0964	0.0337	0.0452	0.0603	0.0803	0.1073	0.5768
0.009	0.0955	0.0335	0.0450	0.0602	0.0804	0.1074	0.5780
0.027	0.0941	0.0330	0.0446	0.0600	0.0804	0.1078	0.5801
0.045	0.0932	0.0327	0.0442	0.0597	0.0804	0.1080	0.5818
0.063	0.0927	0.0325	0.0440	0.0594	0.0802	0.1081	0.5831
0.081	0.0925	0.0325	0.0439	0.0592	0.0800	0.1081	0.5838
0.09	0.0925	0.0325	0.0438	0.0592	0.0800	0.1080	0.5840

TABLE 4.—Steady State Probabilities

State (1)	Probability (2)
0	0.0931
0.009	0.0327
0.027	0.0441
0.045	0.0594
0.063	0.0801
0.081	0.1079
0.09	0.5827

treatment rate is computed for no storage case. From Eq. 36 for $\epsilon = 0.1$, treatment rate, $a = 0.26$ in./hr.

Application of the Storage Estimation Equations

The results obtained from Eqs. 30 and 34 for storage-treatment combinations are compared with the simulation results obtained by Goforth et al. for the data of Atlanta, Georgia. The estimation of the storage capacity required for a desired combination of treatment rate and risk level is of interest as an aid in the design of facilities. This can be conveniently carried out with Eqs. 30 and 34. It must be noted that the equations are conditioned on the value of $S(n - 1)$. This value cannot be known with certainty because of the dynamic nature of the system. However, safety and operational considerations usually dictate that the most critical condition be considered. This condition corresponds to an initially full tank condition. On the other hand an empty tank condition would be the most favorable situation. The two solutions provide upper and lower boundaries of feasible capacities of the system that would maintain the desired risk level. In areas with long interevent times it may be desirable to choose a storage capacity or treatment rate close to the empty tank condition, while in areas of short interevent times the values may be chosen to the full tank condition. In any case, judgment must be exercised.

The results of the analytical model are compared with the results of simulation studies. The results for the simulation model were obtained

TABLE 5.—Runoff Parameters (from Ref. 4)

Parameter (1)	Description (2)
Year of record	1953
Number of events	71
Mean runoff volume, $E(X_1)$	0.223 in. (5.7 mm); coefficient of variation (c.v.) 1.102
Mean runoff duration, $E(X_2)$	6.887 hr; c.v. 1.121
Mean interevent time, $E(X_3)$	124.3 hr; c.v. 0.937

by Goforth et al. using the EPA's Stormwater Management Model (SWMM). The rainfall data were obtained, from the National Weather Service, for the city of Atlanta, Georgia. The catchment has an area of 24.7 acres (0.1 km^2) with 37% imperviousness. The model transformed the rainfall to runoff and the runoff is collected at a detention facility. For the simulation only one year of hourly rainfall data were utilized because the runoff parameters were very similar to those obtained with the full record of 24.6 yr. The parameters are given Table 5. According to Ref. 4 a minimum interevent time of 8 hr was applied in defining the runoff events, which was used to obtain a value close to unity for the coefficient of variation associated with the interevent time. The coefficient of variation of these parameters is close to one and therefore the assumption of exponential distributions is justified. The results are presented as curves of flow capture efficiency, or the complement of the risk level, for values of the storage volume and treatment rate. For ease of presentation the storage is rescaled with respect to the mean runoff. Also, the treatment rate is rescaled by

$$Q_{ro} = \frac{E(X_1)}{E(X_3)} \quad (54)$$

and the rescaled treatment rate is (a/Q_{ro}). The results are shown in Figs. 4 and 5.

Fig. 4 shows that lower values of the flow capture efficiency provide a close match with simulation results. For higher values of the efficiency the curves fall below those of the simulation, although for higher values of (a/Q_{ro}) the curves tend to converge. The results are as expected because lower risk levels imply larger storage volumes, especially when the treatment rate is small. For higher risk levels a smaller storage is required for a given treatment rate. Thus, the assumption of initial emptiness would provide a better match with the lower efficiencies. For lower risk levels, the larger storage volumes reduce the likelihood of initial emptiness. However, the results are consistent in that the analytical model provides a lower bound on the storage capacity at these levels.

Similar arguments are used for the results shown in Fig. 5. The initial full tank condition from Eq. 34 provides storage levels above those obtained from the simulation for higher efficiencies, while a close match is obtained for lower efficiencies. For low risk levels the storage requirement will increase. The fact that, for these low risk levels, the simulation curve falls below the analytical curves is a reflection of the likelihood

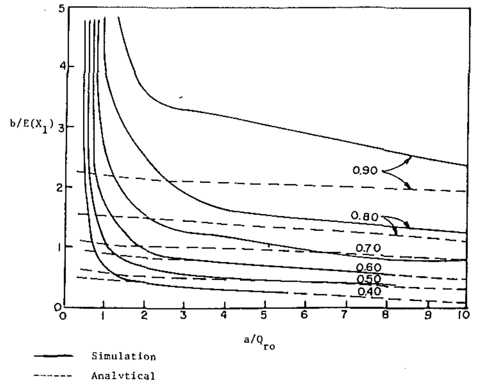

FIG. 4.—Comparison of Flow Capture Efficiency Estimates (Lower Bound)

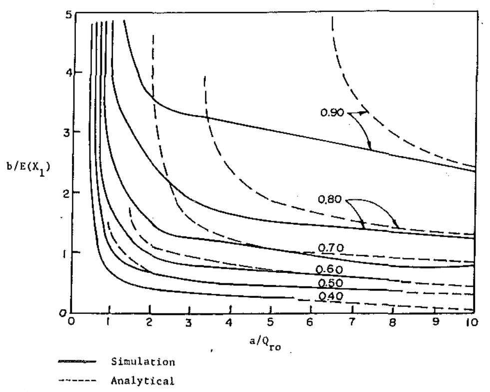

FIG. 5.—Comparison of Flow Capture Efficiency Estimates (Upper Bound)

that the tank may not have been initially full in view of the magnitude of the mean interevent time. However, as the treatment rate is increased the results tend to converge because only less storage is needed. For low storage values, the initial condition of storage does not affect the results significantly.

The analytical model is thus found to be a useful tool for studying the interactions of the elements of storage and treatment. Furthermore, it provides for various alternative treatment/storage combinations, and thus allows for greater flexibility in design. The relative performance of the analytical solution is compared with the result of the simulation by Goforth et al. because of the nonavailability of the actual observed overflow data.

Example

Estimate the detention storage-treatment capacity required to prevent untreated overflows getting into the receiving stream with a reliability (flow capture efficiency) of 90% for the Atlanta data in Table 5.

Solution:
1. Estimate the mean runoff volume, $E(X_1) = 0.223$ in.; $\alpha = 4.484$.
2. Estimate the mean runoff duration, $E(X_2) = 6.887$ hr; $\beta = 0.145$.
3. Estimate the mean interevent time, $E(X_3) = 124.3$ hr; $\gamma = 0.008$.
4. Given reliability 90% and therefore, the risk $\epsilon = 0.10$.

By using Eq. 30, the lower bound storage

$$b \geq 0.223 \ln \left[\frac{0.145}{(0.4484a + 0.0145)}\right] \quad (55)$$

By using Eq. 34, the upper bound storage

$$b \geq \frac{a}{(4.484a + 0.008)} \ln \left\{\frac{0.65a}{[(0.4484a + 0.0145)(4.484a + 0.008) - 0.00116]}\right\} \quad (56)$$

For a treatment capacity $a = 0.02$ in./hr, the storage limits are 0.41 and 0.5 in.

Conclusion

The generalized storage model is very useful in computing the probabilities of real storage and overflow volumes. Because the model considers carryover storage, it is expected to yield realistic solutions in computing storage capacity and treatment rate. The method provides quick estimates for storage capacity and treatment rate. It may be viewed as a better table top technique for stormwater management planning.

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Appendix I.—References

1. Chan, S. O., and Bras, R. L., "Urban Stormwater Management: Distribution of Flood Volumes," *Water Resources Research*, Vol. 15, No. 2, Apr., 1979, pp. 371–382.
2. Delleur, J. W., "Recent Advances in Urban Hydrology in the USA," *International Symposium on Urban Hydrology, Hydraulics and Sediment Control*, University of Kentucky, July, 1983, pp. 1–10.
3. Di Toro, D. M., and Small, M. J., "Stormwater Interception and Storage," *Journal of the Environmental Engineering Division*, ASCE, Vol. 105, No. EE1, Feb., 1979, pp. 43–54.
4. Goforth, G. F., Heaney, J. P., and Huber, W. C., "Comparison of Basin Performance Modeling Techniques," *Journal of Environmental Engineering*, ASCE,

Vol. 109, No. 5, Oct., 1983, pp. 1082–1098.
5. Howard, C. D., "Theory of Storage and Treatment-Plant Overflows," *Journal of Environmental Engineering Division*, ASCE, Vol. 102, No. EE4, Aug., 1976, pp. 709–722.
6. Linsley, R. K., and Crawford, N. J., "Continuous Simulation Models in Urban Hydrology," *Geophysical Research Letters*, Vol. 1, 1974, pp. 59–62.
7. Loganathan, G. V., and Delleur, J. W., "Multiple Objective Planning of Land/Water Interface in Medium Sizes Cities," *TR #144*, Purdue University Water Resources Research Center, Mar., 1982.
8. Loganathan, G. V., and Delleur, J. W., "Effects of Urbanization on Frequencies of Overflows and Pollutant Loadings from Storm Sewer Overflows: A Derived Distribution Approach," *Water Resources Research*, Vol. 20, No. 7, July, 1984, pp. 857–865.
9. Padmanabhan, G., and Delleur, J. W., "Statistical and Stochastic Analyses of Synthetically Generated Urban Storm Drainage Quantity and Quality Data," *TR 108*, Purdue University Water Resources Research Center, July, 1978.
10. Roesner, L. A., et al., "A Model for Evaluating Runoff Quality in Metropolitan Master Planning," ASCE Urban Water Resources Research Program, TM No. 23, New York, N.Y., 1974.
11. Schwarz, R. B., and Adams, B. J., "Distributed Storage for Urban Runoff Control: An Analytical Model," *Proceedings of the Second International Conference on Urban Storm Drainage*, B. C. Yen, Ed., University of Illinois, June, 1981, pp. 360–369.
12. Smith, D. I., *Probability of Storage Overflow for Stormwater Management*, thesis presented to the University of Toronto, at Toronto, Canada, in 1980, in partial fulfillment of the requirements for the degree of Master of Applied Science.

Appendix II.—Notation

The following symbols are used in this paper:

a = treatment rate (in./hr);
b = detention storage capacity (in.);
c = given storage level due to previous event $(n - 1)$;
$E(\,)$ = expected value of random variable;
G = conditional cumulative distribution function of overflow Y;
I = indicator function;
k = $\beta\gamma/(\alpha a + \beta)(\alpha a + \gamma)$;
m = $\alpha\gamma a/(\alpha a + \beta)(\gamma - \beta)$;
n = index for current event;
$\mathbf{P}$ = transition probability matrix;
S = empty space in detention tank;
T = min $[c + aX_3, b]$;
W = $X_1 - aX_2$;
X_1 = runoff volume;
X_2 = duration of runoff event;
X_3 = interevent time;
Y = overflow volume;
α = $1/E(X_1)$;
β = $1/E(X_2)$;
γ = $1/E(X_3)$;
δ = fraction of total storage b;
ϵ = risk of overflow; and
λ = steady state probabilities for storage levels.