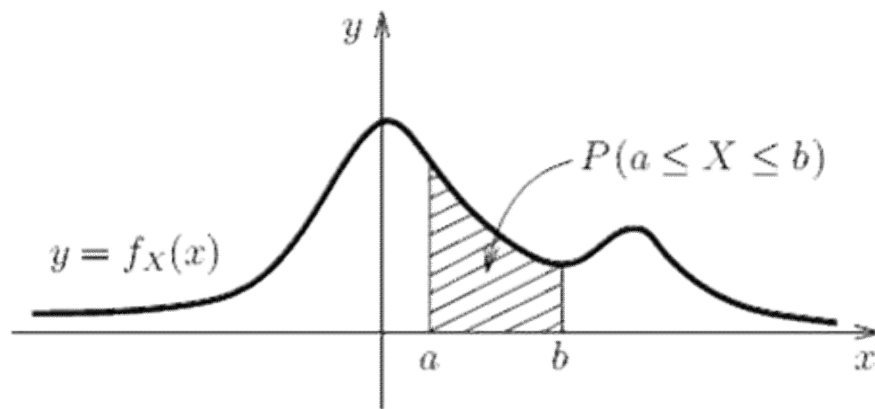
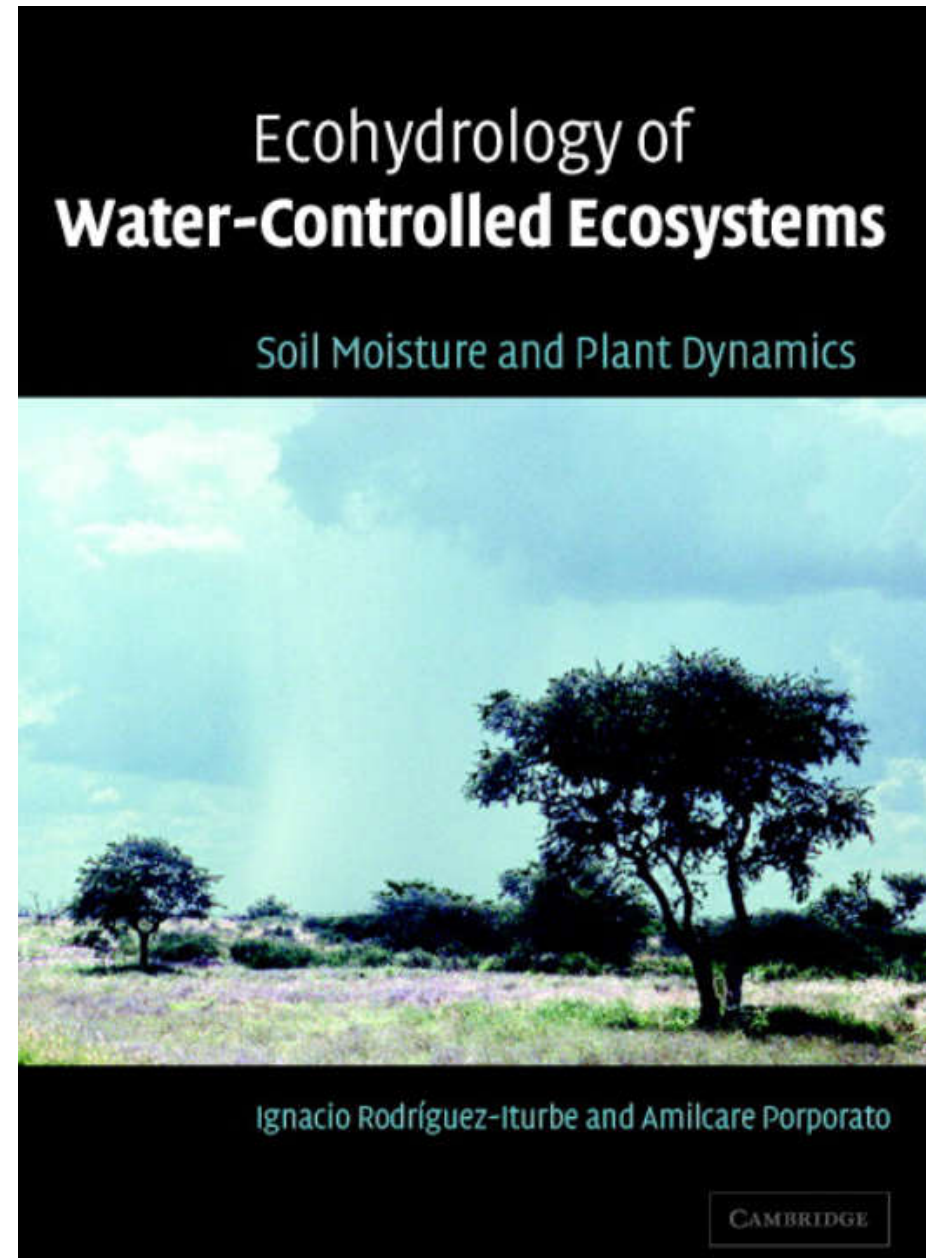


**Rischio idraulico** = Probabilità di fallanza di un opera idraulica (di difesa o di utilizzo) o di un sistema idraulico naturale

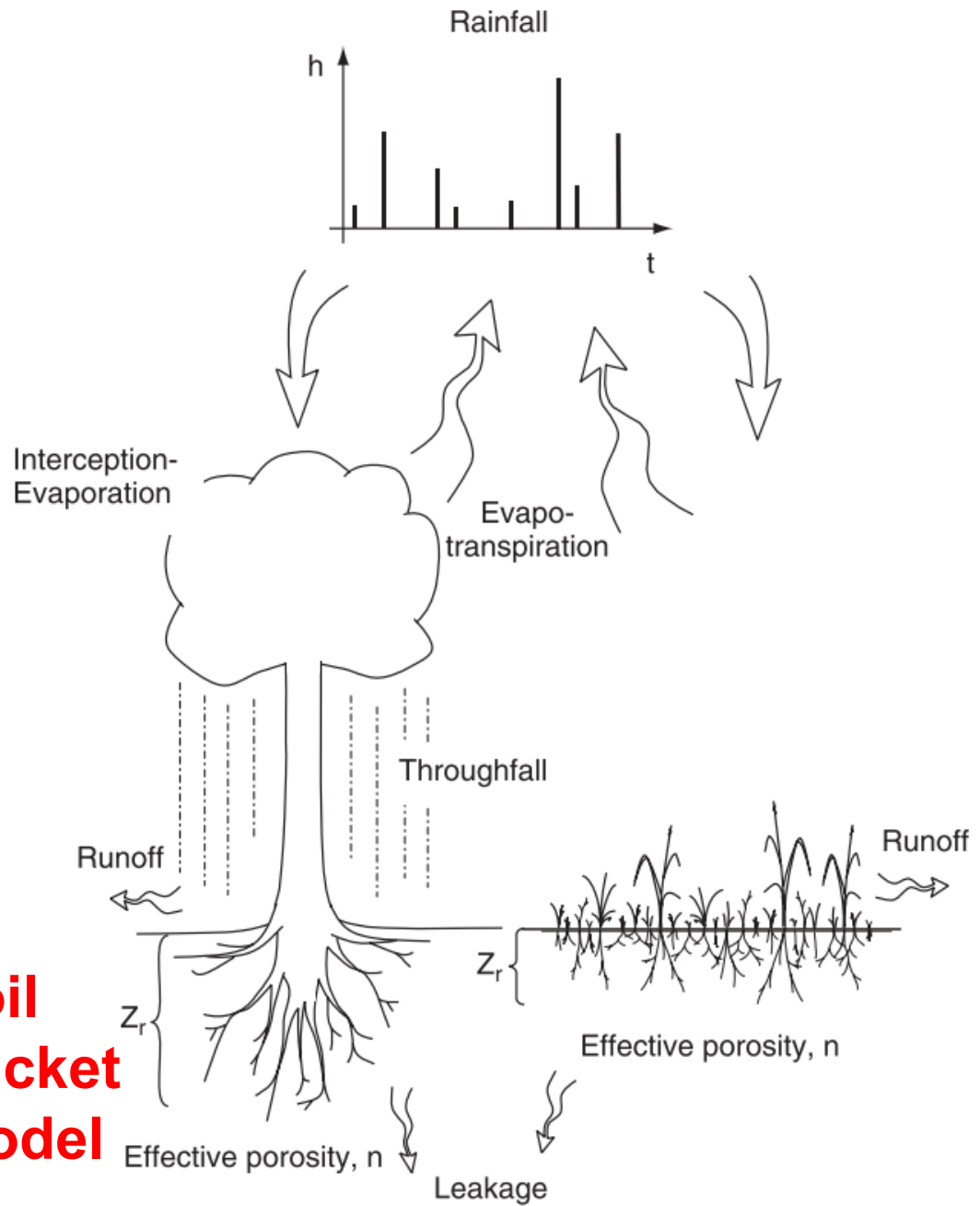


**Andamento aleatorio del  
contenuto d'acqua nel  
terreno**





# Soil bucket model



$$S = \frac{V_w}{V_w + V_a} = \frac{\mathcal{G}}{n}$$

$S$  = saturazione del terreno

$V_w$  = volume d'acqua nel terreno

$V_a$  = volume d'aria nel terreno

$\mathcal{G}$  = contenuto d'acqua nel terreno

$$n = \text{porosità del terreno} = \frac{V_a + V_w}{V_w + V_a + V_s}$$

$V_s$  = volume della matrice solida del terreno



**Soil  
bucket  
model**

$$nZ_r \frac{dS(t)}{dt} = \varphi[S(t), t] - X[S(t), t]$$

$S(t)$  = saturazione del terreno in funzione del tempo

$Z_r$  = profondità del terreno sede degli apparati radicali delle piante

$\varphi[S(t), t]$  = flussi in entrata nel volume di controllo del terreno

(infiltrazione = precipitazione - intercettazione - scorrimento superficiale)

$X[S(t), t]$  = flussi in uscita nel volume di controllo del terreno

(infiltrazione profonda, evapotraspirazione...)



**Soil  
bucket  
model**

Profondità effettiva di suolo



With these assumptions, the distribution of the times  $\tau$  between precipitation events is exponential with mean  $1/\lambda$  (e.g., Cox and Miller, 1965), i.e.,

$$f_T(\tau) = \lambda e^{-\lambda\tau}, \text{ for } \tau \geq 0, \quad (2.6)$$

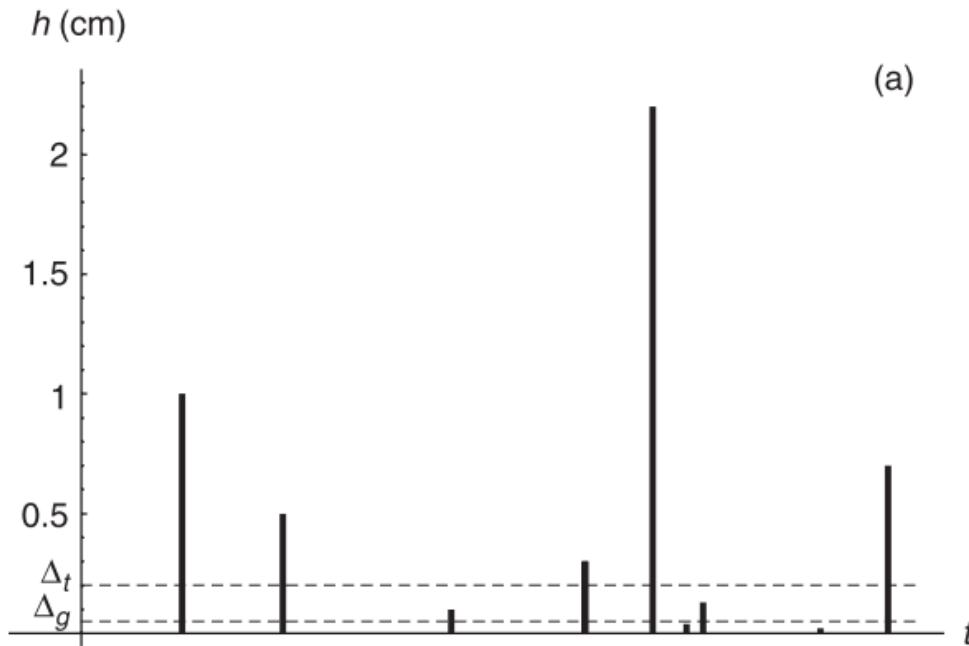
while the depth of rainfall events is assumed to be an independent random variable  $h$ , described by an exponential probability density function

$$f_H(h) = \frac{1}{\alpha} e^{-\frac{1}{\alpha}h}, \text{ for } h \geq 0, \quad (2.7)$$

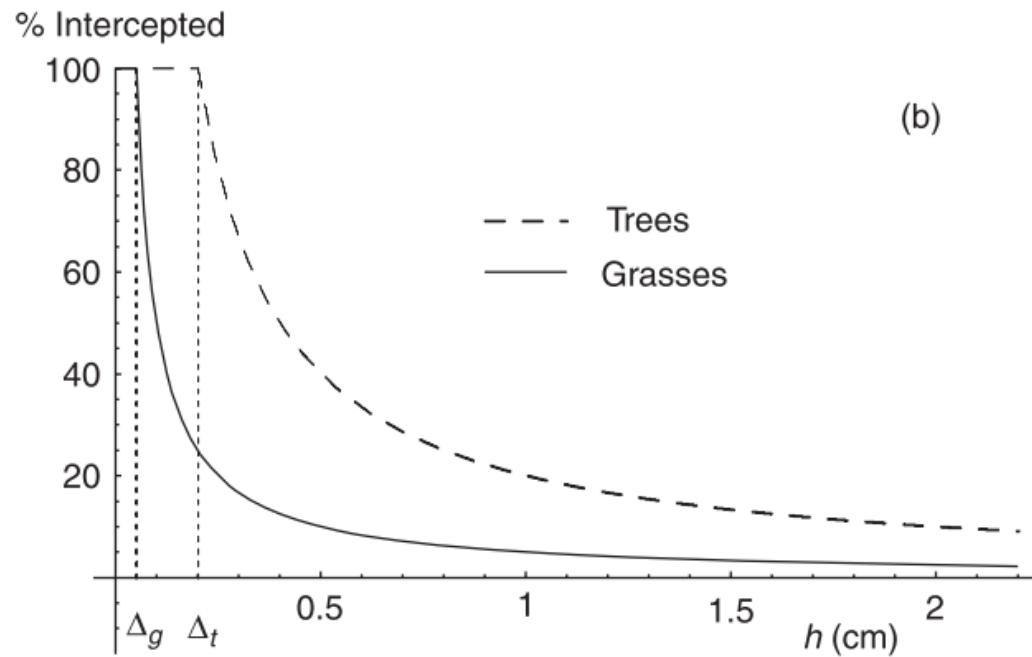
where  $\alpha$  is the mean depth of rainfall events. Since the model is interpreted at the daily time scale,  $\alpha$  may be estimated as the mean daily rainfall in days

$$R(t) = \sum_i h_i \delta(t - t_i) \quad (2.9)$$

where  $\delta(\cdot)$  is the Dirac delta function,  $\{h_i, i = 1, 2, 3, \dots\}$  is the sequence of random rainfall depths with distribution given by Eq. (2.7), and  $\{\tau_i = t_i - t_{i-1}, i = 1, 2, 3, \dots\}$  is the interarrival time sequence of a stationary Poisson process with rate  $\lambda$ .



Intercettazione di parte della precipitazione da parte della vegetazione



From a mathematical viewpoint the consideration of a threshold on the rainfall Poisson process does not complicate its analytical tractability. The rainfall process is in fact transformed into a new marked-Poisson process, called a censored process, where the frequency of rainfall events is now

$$\frac{\lambda'}{\lambda} = \frac{\int_{\Delta}^{\infty} f_h dh}{\int_0^{\infty} f_h dh} = e^{-\frac{\Delta}{\alpha}} \quad \lambda' = \lambda \int_{\Delta}^{\infty} f_H(h) dh = \lambda e^{-\Delta/\alpha} \quad (2.10)$$

and the depths  $h'$  have the same distribution as  $h$ , given by Eq. (2.7). Thus, one can simply write

$$R(t) - I(t) = \sum_i h'_i \delta(t - t'_i) \quad (2.11)$$

where  $\{\tau'_i = t'_i - t'_{i-1}, i = 1, 2, 3, \dots\}$  is the interarrival time sequence of a stationary Poisson process with frequency  $\lambda'$ .



$$\frac{1}{\gamma} = \frac{\alpha}{nZ_r}. \quad (2.8)$$

The probability distribution of the infiltration component may be easily written in terms of the exponential rainfall-depth distribution of Eq. (2.7) and the soil moisture state  $s$ . Referring to its dimensionless counterpart  $y$  (i.e., the infiltrated depth of water normalized by  $nZ_r$ ) one can write

$$f_Y(y, s) = \gamma e^{-\gamma y} + \delta(y - 1 + s) \int_{1-s}^{\infty} \gamma e^{-\gamma u} du, \quad \text{for } 0 \leq y \leq 1 - s, \quad (2.12)$$

where  $\gamma$  is defined in Eq. (2.8). Equation (2.12) is thus the probability distribution of having a jump in soil moisture equal to  $y$ , starting from a level  $s$ .

**y = precipitazione che si infila normalizzata**

**$\gamma^{-1} = \frac{\alpha}{nZ_h}$  = altezza media di precipitazione normalizzata**

**Se  $\frac{h}{nZ_h} \leq 1-s \rightarrow y = \frac{h}{nZ_h}$**

**Se  $\frac{h}{nZ_h} \geq 1-s \rightarrow y = 1 - s$**

**$f_y(y) = f_Y(\gamma)$  Se  $\frac{h}{nZ_h} \leq 1-s$**

**$P_y(y \geq 1-s) = P_Y(\gamma = 1-s)$**

$$f_Y(y, s) = \gamma e^{-\gamma y} + \delta(y - 1 + s) \int_{1-s}^{\infty} \gamma e^{-\gamma u} du, \quad \text{for } 0 \leq y \leq 1 - s$$

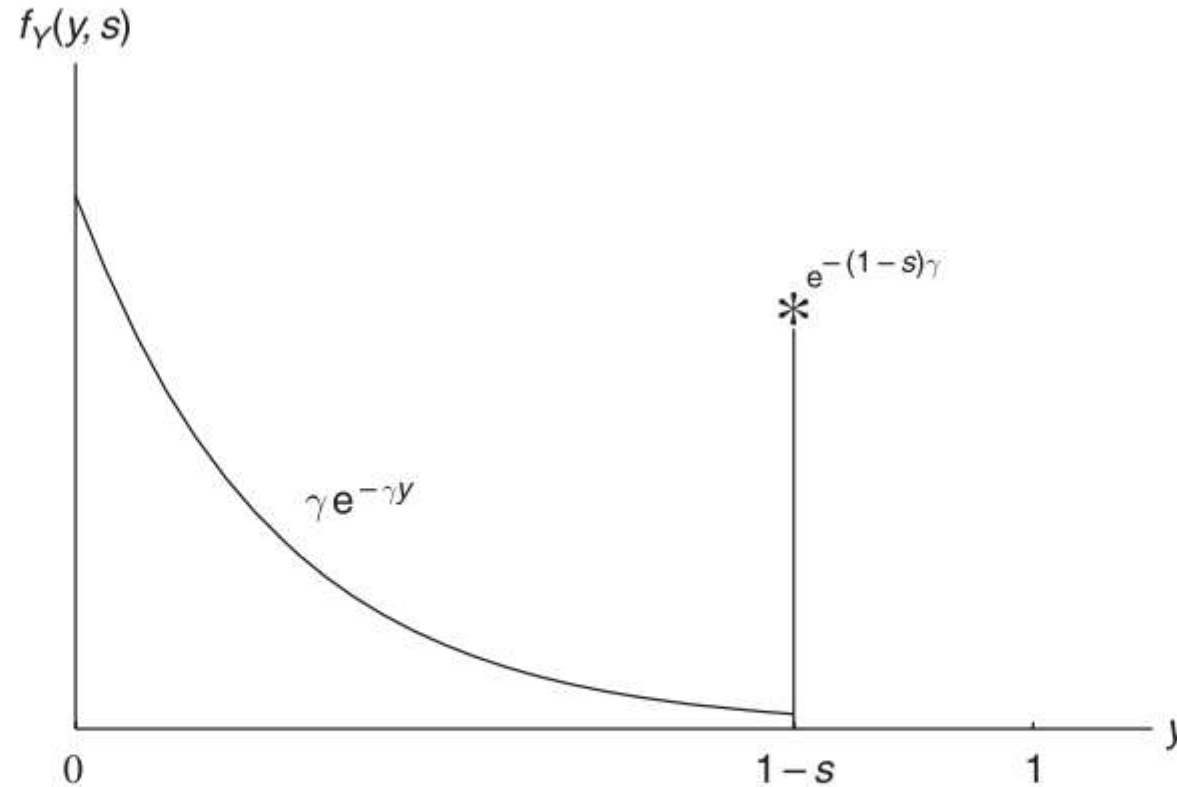


Figure 2.3 Sketch of the probability density function describing infiltration from rainfall  $y$ . The asterisk represents the atom of probability corresponding to soil saturation. After Rodriguez-Iturbe et al. (1999a).

$$R(t) - I(t) = \sum_i h'_i \delta(t - t'_i) \quad (2.11)$$

where  $\{\tau'_i = t'_i - t'_{i-1}, i = 1, 2, 3, \dots\}$  is the interarrival time sequence of a stationary Poisson process with frequency  $\lambda'$ .

Similarly to Eq. (2.11), infiltration from rainfall can be written in Eq. (2.3) as

$$\varphi[s(t), t] = nZ_r \sum_i y_i \delta(t - t'_i) \quad (2.13)$$

where  $\{y_i, i = 1, 2, 3, \dots\}$  is the sequence of random infiltration events having a distribution as in Eq. (2.12).

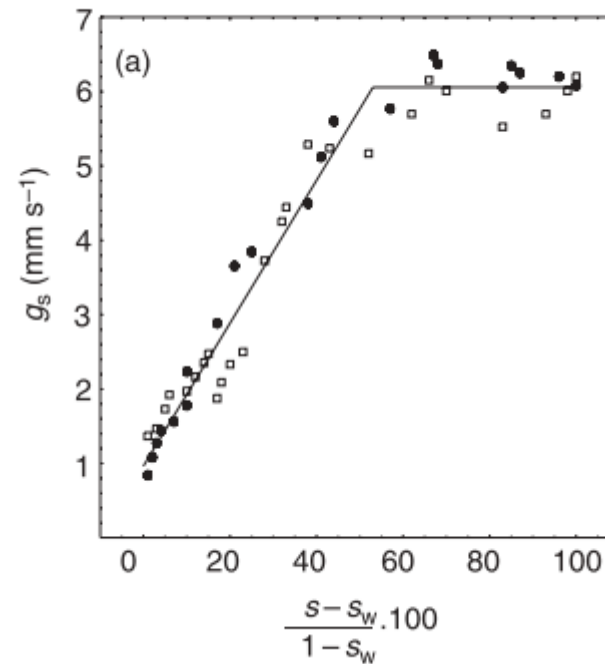
**Le perdite di umidità del suolo, attribuite a deflussi profondi ed evapotraspirazione, sono deterministiche perchè dipendono dal contenuto d'acqua nel terreno e non dalla precipitazione**



The dependence of evapotranspiration losses on soil moisture is thus summarized in the following expression

$$E(s) = \begin{cases} 0 & 0 < s \leq s_h \\ E_w \frac{s-s_h}{s_w-s_h} & s_h < s \leq s_w \\ E_w + (E_{\max} - E_w) \frac{s-s_w}{s^*-s_w} & s_w < s \leq s^* \\ E_{\max} & s^* < s \leq 1, \end{cases} \quad (2.14)$$

whose behavior is shown in Figure 2.5.

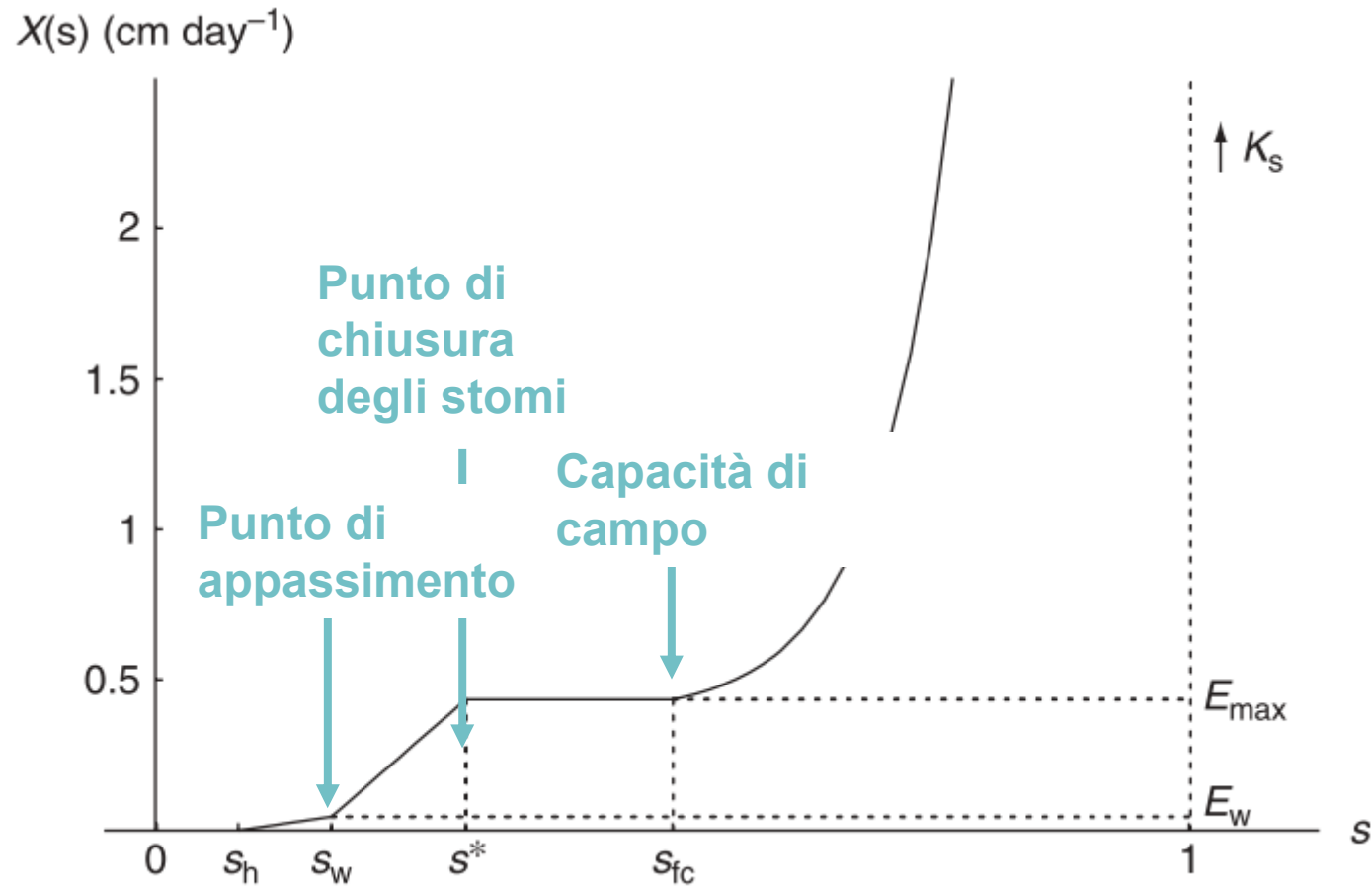


Leakage losses are assumed to happen by gravity

$$L(s) = K(s) = \frac{K_s}{e^{\beta(1-s_{fc})} - 1} \left[ e^{\beta(s-s_{fc})} - 1 \right], \quad s_{fc} < s \leq 1, \quad (2.16)$$

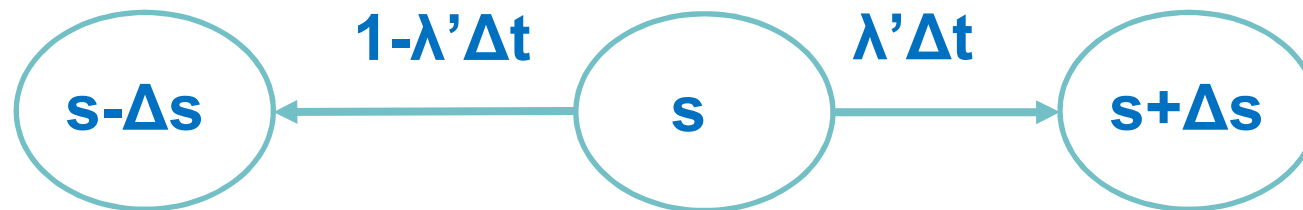
where  $\beta$  is a coefficient that is used to fit the above expression to the power law (e.g., Clapp and Hornberger, 1978; Dingman, 1994; Hillel, 1998)

$$K(s) = K_s s^c, \quad (2.17)$$



Soil water losses (evapotranspiration and leakage)  $X(s)$  as a function of soil moisture  $s$ , for typical climate, soil and vegetation characteristics

## Distribuzione di probabilità di $s$



## Frequenza degli eventi di pioggia

$$\lambda' = \lambda \int_{\Delta}^{\infty} f_H(h) dh = \lambda e^{-\Delta/\alpha} \quad (2.10)$$

Suppose that at time  $t$  the soil moisture level is  $s(t)$  and consider a small time interval from  $t$  to  $t + \Delta t$ . The probability that no positive increment in soil moisture occurs is  $1 - \lambda' \Delta t + o(\Delta t)$ , where  $\lambda'$  is defined in Eq. (2.10). In this case soil moisture evolves deterministically as

$$s(t + \Delta t) = \begin{cases} s(t) - \Delta s & s(t) > \Delta s + s_h \\ s_h & s_h \leq s(t) \leq \Delta s + s_h, \end{cases} \quad (2.24)$$

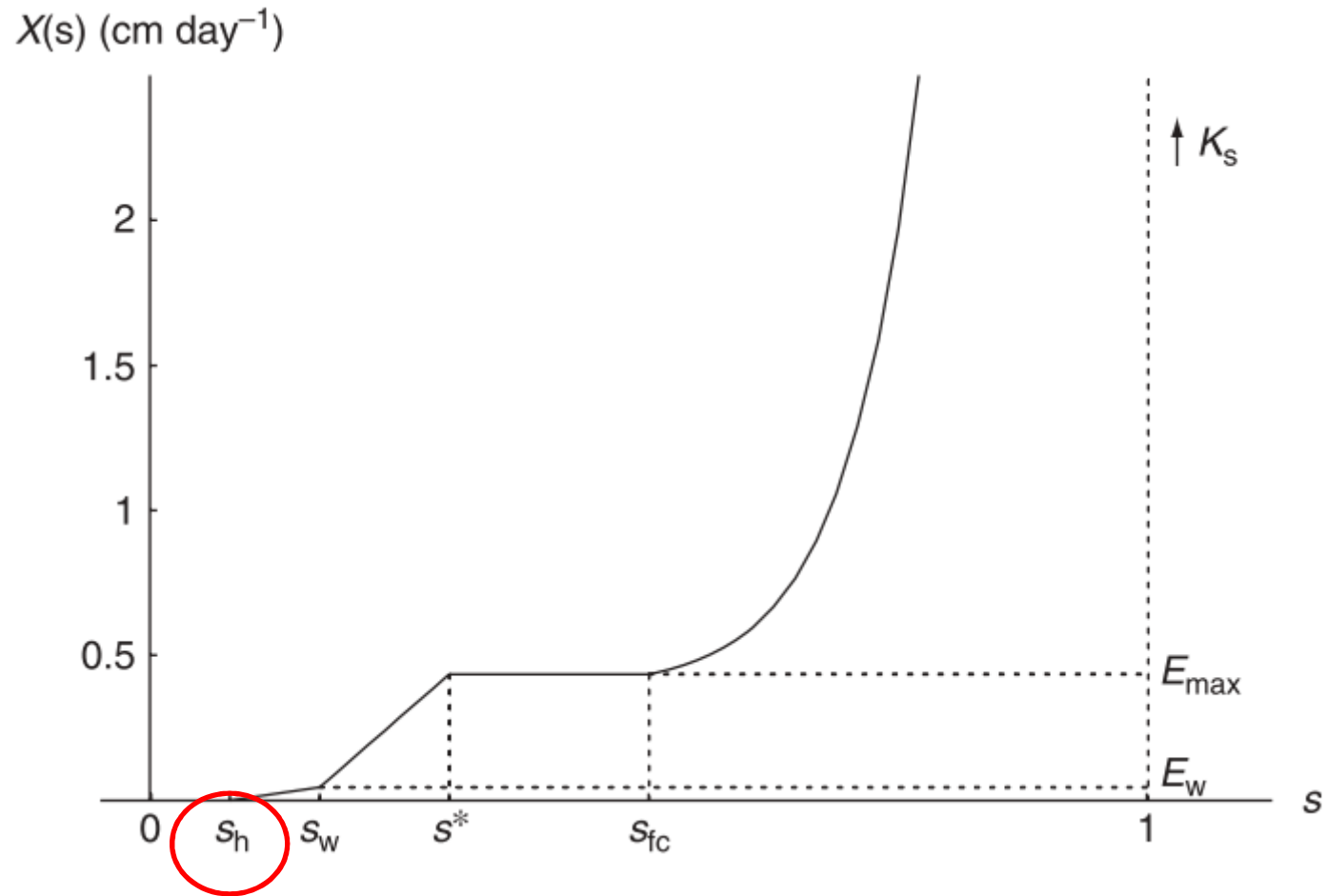
where  $\Delta s = \int_t^{t+\Delta t} \rho[s(\tau)] d\tau = \rho[s(t)] \Delta t + o(\Delta t)$ .



The probability that a positive increment in soil moisture takes place is  $\lambda' \Delta t + o(\Delta t)$ . In this case

$$s(t + \Delta t) = \begin{cases} s(t) + y - \Delta s & s(t) > \Delta s + s_h + y \\ s_h & s_h \leq s(t) \leq \Delta s + s_h + y, \end{cases} \quad (2.25)$$

where  $y$  is the normalized soil moisture increment governed by the probability distribution  $f_Y(y, s)$  described by Eq. (2.12).



In general, at time  $t$  the distribution of  $s(t)$  consists of a discrete atom of probability  $p_0(t)$  that the soil is at  $s = s_h$  and a density  $p(s, t)$  for  $s > s_h$ . The probability that the soil moisture takes a value in  $(s, s + \Delta s)$  at the time  $t + \Delta t$  can therefore be expressed as follows (e.g., Cox and Miller, 1965, page 241)

$$\begin{aligned}
 p(s, t + \Delta t)\Delta s = & (1 - \lambda'\Delta t)p(s + \Delta s, t) d(s + \Delta s) \\
 & + \lambda'\Delta t \int_{s_h}^s p(u + \Delta u, t) f_Y(s - u, u) d(u + \Delta u) ds \quad (2.26) \\
 & + \lambda'\Delta t p_0(t) f_Y(s - s_h, s_h)\Delta s + o(\Delta t).
 \end{aligned}$$

The first term on the r.h.s. of Eq. (2.26) describes the situation when the soil moisture reaches the value  $s$  at time  $t + \Delta t$  given that no rainfall events have occurred in the interval  $\Delta t$ . The second term allows for the case when the soil moisture reaches  $s$  due to a positive increment resulting from the arrival of a rainfall event and the third term corresponds to the case when the process is at  $s = s_h$  at time  $t$  and the arrival of a storm event makes the moisture content jump to  $s$  at time  $t + \Delta t$ .

Similarly, the atom of probability at  $s_h$  satisfies the equation

$$\begin{aligned} p_0(t + \Delta t) &= (1 - \lambda' \Delta t) p_0(t) + (1 - \lambda' \Delta t) \int_{s_h}^{\rho(s_h) \Delta t} p(u, t) \, du + o(\Delta t) \\ &= (1 - \lambda' \Delta t) [p_0(t) + p(s_h, t) \rho(s_h) \Delta t] + o(\Delta t), \end{aligned} \tag{2.27}$$

where the second term on the r.h.s. accounts for the probability of moving to  $s = s_h$  from a value infinitesimally above it, in the absence of rain.

Substituting now for  $\Delta s$  and  $\Delta u$  in the r.h.s. of Eq. (2.26), one obtains

$$\begin{aligned}
p(s, t + \Delta t)\Delta s &= (1 - \lambda'\Delta t) p[s + \rho(s)\Delta t, t] d[s + \rho(s)\Delta t] \\
&\quad + \lambda'\Delta t \int_{s_h}^s p[u + \rho(u)\Delta t, t] f_Y(s - u, u) d[u + \rho(u)\Delta t] ds \\
&\quad + \lambda'\Delta t p_0(t) f_Y(s - s_h, s_h) ds + o(\Delta t) \\
&= (1 - \lambda'\Delta t) \left[ p(s, t) + \rho(s)\Delta t \frac{\partial}{\partial s} p(s, t) \right] \left[ 1 + \frac{\partial}{\partial s} \rho(s)\Delta t \right] ds \\
&\quad + \lambda'\Delta t \Delta s \int_{s_h}^s p(u, t) f_Y(s - u, u) du \\
&\quad + \lambda'\Delta t \Delta s p_0(t) f_Y(s - s_h, s_h) + o(\Delta t). \tag{2.28}
\end{aligned}$$



Finally, dividing by  $\Delta s$ , subtracting  $p(s, t)$  from both sides, dividing by  $\Delta t$ , and taking the limit as  $\Delta t \rightarrow 0$ , one gets

$$\begin{aligned} \frac{\partial}{\partial t} p(s, t) = & \frac{\partial}{\partial s} [p(s, t)\rho(s)] - \lambda' p(s, t) + \lambda' \int_{s_h}^s p(u, t) f_Y(s - u, u) du \\ & + \lambda' p_0(t) f_Y(s - s_h, s_h). \end{aligned} \quad (2.29)$$

The various terms on the r.h.s. of the integro-differential Eq. (2.29) represent the contributions to  $p(s, t)$  of the different mechanisms of the soil moisture process. The first term is related to the gain of probability due to the drift of the pdf in the deterministic decay caused by  $\rho(s)$ , the second term represents the loss of probability due to possible jumps with frequency  $\lambda'$  which cause the process to leave the given trajectory moving to the level  $s$  at time  $t$ , and the last term is the positive contribution to the probability due to jumps to level  $s$  starting from lower soil moisture values.

Similarly, taking the limit as  $\Delta t \rightarrow 0$ , Eq. (2.27) yields

$$\frac{d}{dt} p_0(t) = -\lambda' p_0(t) + \rho(s_h) p(s_h, t). \quad (2.30)$$

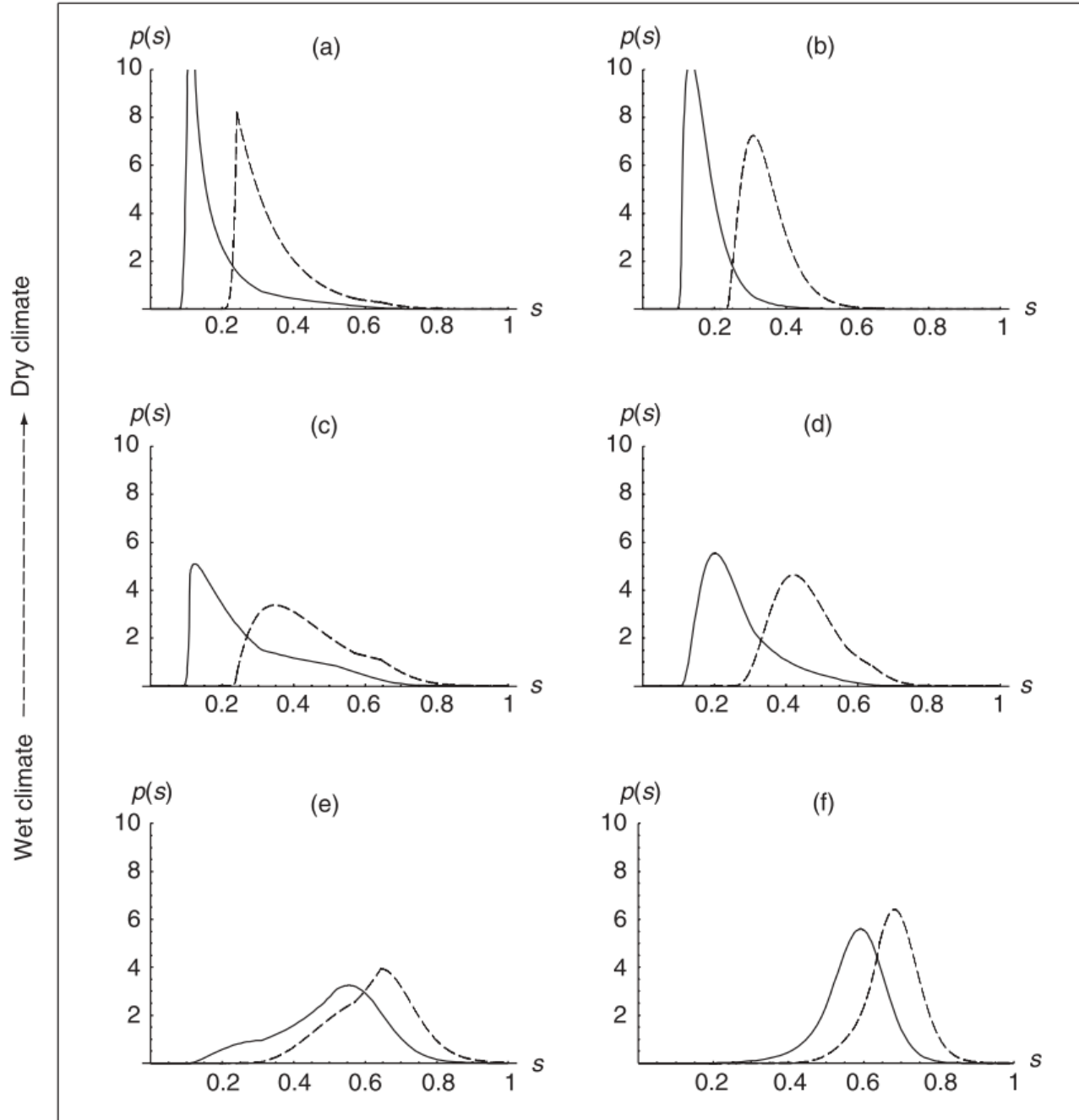
**...steady state  
solution...**

$$p(s) = \frac{C}{\rho(s)} e^{-\gamma s + \lambda' \int \frac{du}{\rho(u)}} \quad \text{for } s_h < s \leq 1,$$

where  $C$  is the normalization constant such that

$$\int_{s_h}^1 p(s) ds = 1.$$

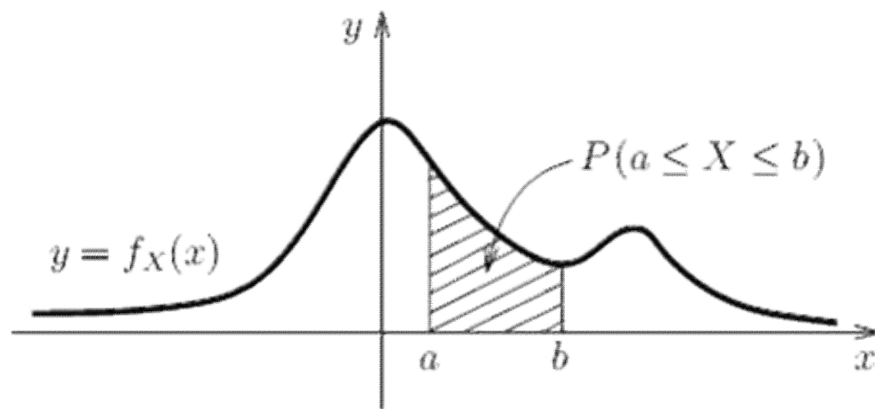
Shallow soil -----> deep soil



...steady state solution...

Continuous lines → loamy sand  
Dashed lines → loam

**Rischio idraulico** = Probabilità di fallanza di un opera idraulica (di difesa o di utilizzo) o di un sistema idraulico naturale



**Rischio idraulico** = Probabilità di fallanza di un opera idraulica (di difesa o di utilizzo) o di un sistema idraulico naturale







# Fire as a global 'herbivore': the ecology and evolution of flammable ecosystems

William J. Bond<sup>1</sup> and Jon E. Keeley<sup>2,3</sup>

<sup>1</sup>Department of Botany, University of Cape Town, Rondebosch, South Africa

<sup>2</sup>U.S. Geological Survey, Western Ecological Research Center, Sequoia-Kings Canyon National Parks, Three Rivers, CA 93271-9651, USA

<sup>3</sup>Department of Ecology and Evolutionary Biology, University of California, Los Angeles, CA 90095, USA

**It is difficult to find references to fire in general textbooks on ecology, conservation biology or biogeography, in spite of the fact that large parts of the world burn on a regular basis, and that there is a considerable literature on the ecology of fire and its use for managing ecosystems. Fire has been burning ecosystems for hundreds of millions of years, helping to shape global biome distribution and to maintain the structure and function of fire-prone communities. Fire is also a significant evolutionary force, and is one of the first tools that humans used to re-shape their world. Here, we review the recent literature, drawing parallels between fire and herbivores as alternative consumers of vegetation. We point to the common questions, and some surprisingly different answers, that emerge from viewing fire as a globally significant consumer that is analogous to herbivory.**

**Modelli per popolazioni  
in interazione  
(prede-predatori)**

**Mathematical Biology:  
I. An Introduction,  
Third Edition**

*J.D. Murray*

**Springer**

### 3.1 Predator–Prey Models: Lotka–Volterra Systems

Volterra (1926) first proposed a simple model for the predation of one species by another to explain the oscillatory levels of certain fish catches in the Adriatic. If  $N(t)$  is the prey population and  $P(t)$  that of the predator at time  $t$  then Volterra's model is

$$\frac{dN}{dt} = N(a - bP), \quad (3.1)$$

$$\frac{dP}{dt} = P(cN - d), \quad (3.2)$$

where  $a$ ,  $b$ ,  $c$  and  $d$  are positive constants.

The assumptions in the model are: (i) The prey in the absence of any predation grows unboundedly in a Malthusian way; this is the  $aN$  term in (3.1). (ii) The effect of the predation is to reduce the prey's per capita growth rate by a term proportional to the prey and predator populations; this is the  $-bNP$  term. (iii) In the absence of any prey for sustenance the predator's death rate results in exponential decay, that is, the  $-dP$  term in (3.2). (iv) The prey's contribution to the predators' growth rate is  $cNP$ ; that is, it is proportional to the available prey as well as to the size of the predator population.



As a first step in analysing the Lotka–Volterra model we nondimensionalise the system by writing

$$u(\tau) = \frac{cN(t)}{d}, \quad v(\tau) = \frac{bP(t)}{a}, \quad \tau = at, \quad \alpha = d/a, \quad (3.3)$$

and it becomes

$$\frac{du}{d\tau} = u(1 - v), \quad \frac{dv}{d\tau} = \alpha v(u - 1). \quad (3.4)$$

In the  $u, v$  phase plane (a brief summary of basic phase plane methods is given in Appendix A) these give

$$\frac{dv}{du} = \alpha \frac{v(u-1)}{u(1-v)}, \quad (3.5)$$

In the  $u, v$  phase plane (a brief summary of basic phase plane methods is given in Appendix A) these give

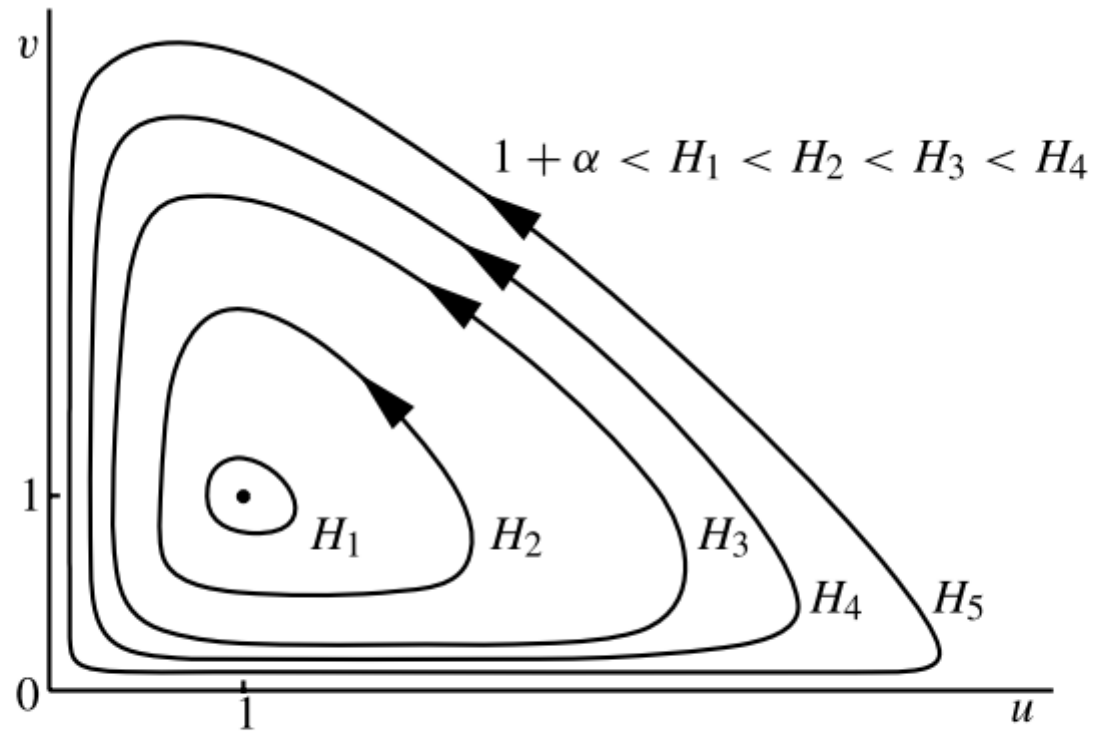
$$\frac{dv}{du} = \alpha \frac{v(u-1)}{u(1-v)}, \quad (3.5)$$

which has singular points at  $u = v = 0$  and  $u = v = 1$ . We can integrate (3.5) exactly to get the phase trajectories

$$\alpha u + v - \ln u^\alpha v = H, \quad (3.6)$$

where  $H > H_{\min}$  is a constant:  $H_{\min} = 1 + \alpha$  is the minimum of  $H$  over all  $(u, v)$  and it occurs at  $u = v = 1$ . For a given  $H > 1 + \alpha$ , the trajectories (3.6) in the phase plane are closed as illustrated in Figure 3.1.

A closed trajectory in the  $u, v$  plane implies periodic solutions in  $\tau$  for  $u$  and  $v$  in (3.4). The initial conditions,  $u(0)$  and  $v(0)$ , determine the constant  $H$  in (3.6)



**Figure 3.1.** Closed  $(u, v)$  phase plane trajectories, from (3.6) with various  $H$ , for the Lotka–Volterra system (3.4):  $H_1 = 2.1$ ,  $H_2 = 2.4$ ,  $H_3 = 3.0$ ,  $H_4 = 4$ . The arrows denote the direction of change with increasing time  $\tau$ .

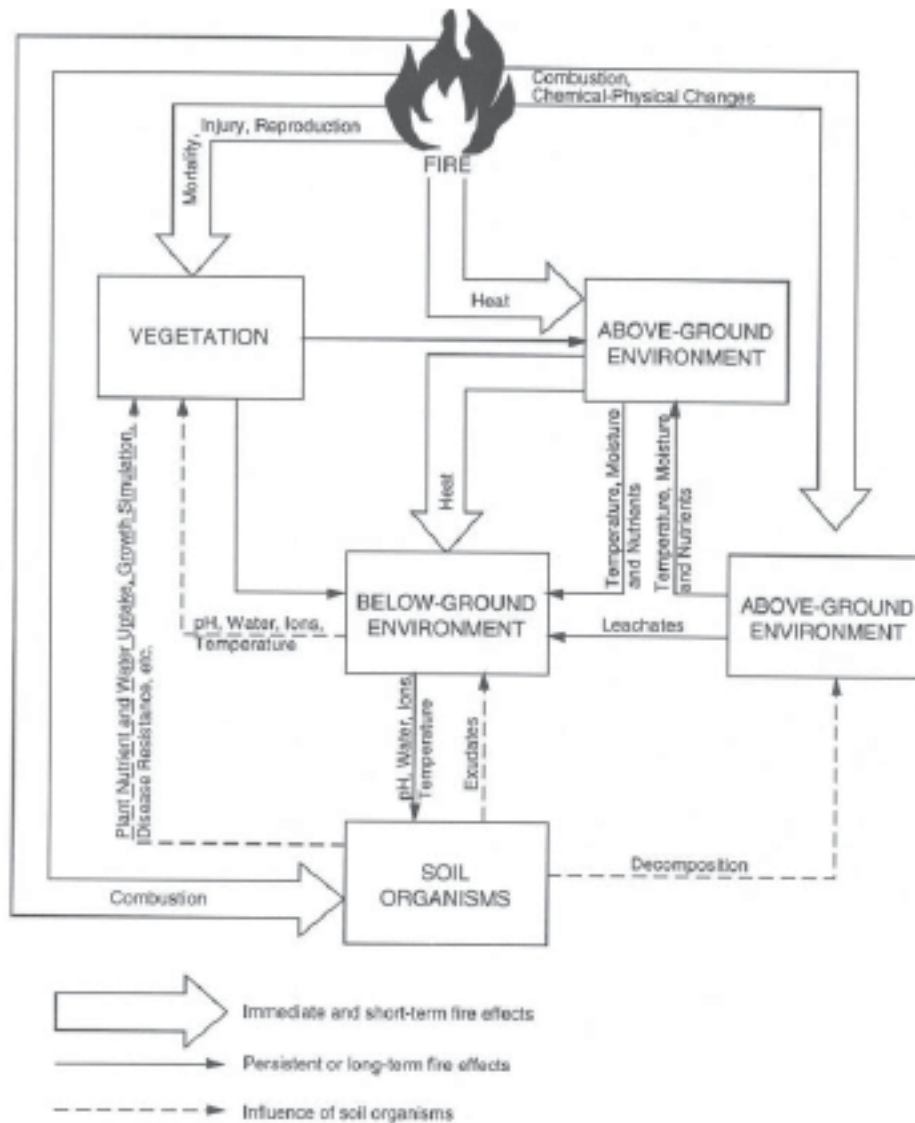
**MODELLI  
ECOIDROLOGICI PRL  
PA PREVISIONE DEL  
RISCHIO INCENDI**

## LE CONDIZIONI DI INNESCO DI UN INCENDIO FORESTALE

- La presenza di **carburante** (vegetazione) e il suo stato di umidità determinano il regime regionale degli incendi (p.e. Archibald et al., 2009).
- Gli incendi sono limitati dalla **disponibilità di carburante** nei climi aridi e dall'**umidità del carburante** nei climi più umidi (Nelson e Hu, 2008).

L'ecosistema mediterraneo è caratterizzato da prolungata siccità estiva.

Il fuoco di solito si verifica a seguito di siccità stagionali (Pyne et al, 1996.).



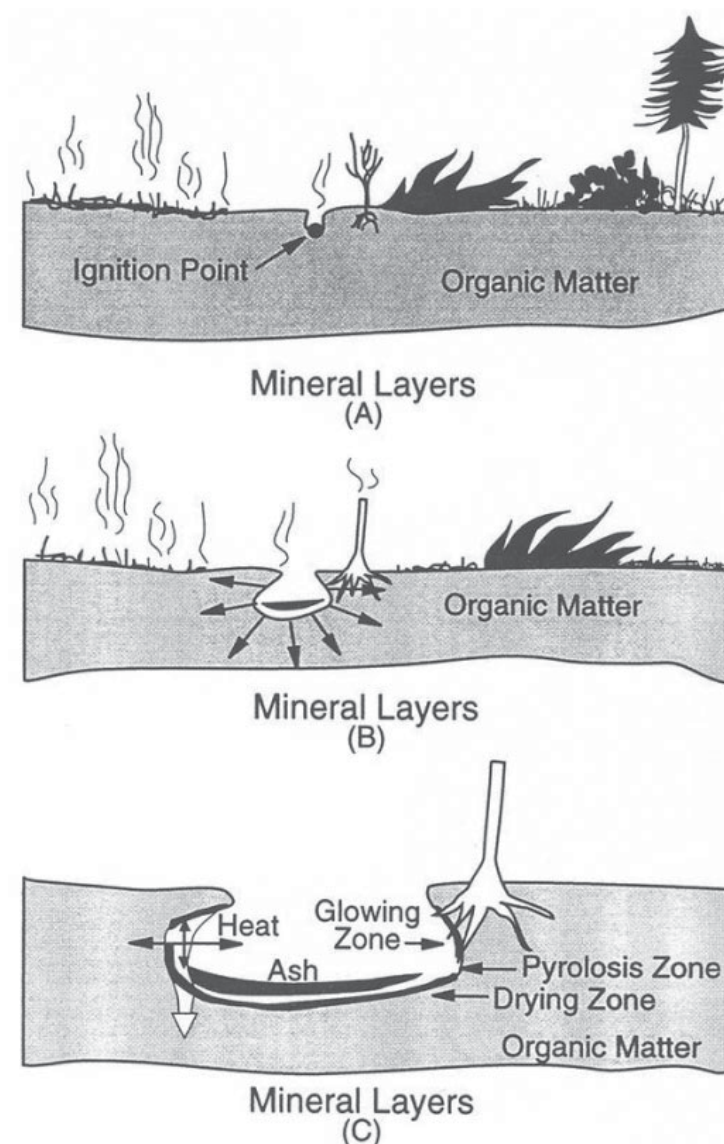
## FUOCO, CLIMA ED USO DEL TERRITORIO

Aumento della frequenza e intensità degli incendi boschivi nel bacino del mediterraneo

Cause possibili:  
 riscaldamento globale,  
 spopolamento rurale,  
 soppressione degli incendi controllati,  
 abbandono delle terre e rimboschimento con specie infiammabili (Shakesby, 2011).

## INTERCONNESSIONE TRA FUOCO, ECOSISTEMA E BILANCIO IDROLOGICO

Il fuoco può modificare (spesso aumentare) le caratteristiche di idrofobicità di un terreno concorrendo alla creazione di uno strato idrorepellente posto pochi centimetri al di sotto della superficie del suolo limitando la capacità di infiltrazione del suolo





## RIPERCUSSIONI SULLA CRESCITA DELLAVEGETAZIONE

Tali trasformazioni subite dal terreno influenzano anche la dinamica della che essendo legata alla disponibilità di risorse idriche, risente della mutata ripartizione degli afflussi in deflussi profondi e superficiali.



**MODELLI ECOIDROLOGICI SEMPLICI** → Il fuoco è modellato come un **processo di Poisson** (van Kampen, 1992).

Al verificarsi di un incendio, la vegetazione distrutta è proporzionale alla biomassa viva prima dell'incendio

### **PROBLEMA:**

Un processo di poisson non riproduce l'evidenza sperimentale secondo la quale i **tempi di attesa tra due incendi consecutivi seguono una legge di potenza** ed il regime degli incendi che per la foresta mediterranea è **caotico** (non ciclico)

**L'innesco (sia esso di natura naturale o antropica), dipende dal carico di carburante dall'umidità dell'ecosistema.**

# MODELLO

$$Hn \frac{\partial S}{\partial t} = P \cdot g(R_u, R_l) - ET \cdot \left[ \varpi \cdot \frac{B_u}{k_u} + (1 - \varpi) \cdot \frac{B_l}{k_l} \right] \cdot S^\beta - K_S S^\gamma$$

Bilancio  
idrico

$$\frac{\partial B_u}{\partial t} = G_u(S, B_u) - F_u(S, B_u, R_u, R_l)$$

$$\frac{\partial R_u}{\partial t} = F_u(S, B_u, R_u, R_l) - D_u(R_u)$$

$$\frac{\partial B_l}{\partial t} = G_l(S, B_l, B_u) - F_l(S, B_l, R_u, R_l)$$

$$\frac{\partial R_l}{\partial t} = F_l(S, B_l, R_u, R_l) - D_l(R_l)$$

Bilanci di biomassa viva e in  
fiamme  
(due strati)

## MODELLO

$$nZ_r \frac{ds(t)}{dt} = \varphi[s(t), t] - \chi[s(t)],$$

### Bilancio idrico

$$\underbrace{Hn \frac{\partial S}{\partial t}}_{\substack{\text{derivata} \\ \text{temporale} \\ \text{del volume} \\ \text{d'acqua} \\ \text{immagazzinato} \\ \text{nel suolo}}} = \underbrace{P \cdot g(R_u, R_l)}_{\substack{\text{precipitazione che} \\ \text{si infila, in misura} \\ \text{ridotta se ci sono stati} \\ \text{incendi nell'anno} \\ \text{in corso}}} - \underbrace{ET \cdot \left[ \varpi \cdot \frac{B_u}{k_u} + (1 - \varpi) \cdot \frac{B_l}{k_l} \right]}_{\substack{\text{evapotraspirazione proporzionale alla densità} \\ \text{di biomassa viva ed alla saturazione del terreno}}} \cdot S^\beta - \underbrace{K_S S^\gamma}_{\substack{\text{infiltrazione} \\ \text{profonda}}}$$

# MODELLO

## Bilanci di biomassa viva (alberi e sottobosco)

$$\underbrace{\frac{\partial B_u}{\partial t}}_{\text{derivata temporale della densità arborea}} = \underbrace{G_u(S, B_u)}_{\text{crescita dello strato arboreo}} - \underbrace{F_u(S, B_u, R_u, R_l)}_{\text{mortalità degli alberi dovuta all'occorrenza di un incendio in uno dei due strati}}$$

$$\underbrace{\frac{\partial B_l}{\partial t}}_{\text{derivata temporale della densità del sottobosco}} = \underbrace{G_l(S, B_l, B_u)}_{\text{crescita dello strato di sottobosco}} - \underbrace{F_l(S, B_l, R_u, R_l)}_{\text{mortalità del sottobosco dovuta all'occorrenza di un incendio in uno dei due strati}}$$

# MODELLO

## Bilanci di biomassa in fiamme (due strati)

$$\underbrace{\frac{\partial R_u}{\partial t}}_{\text{derivata temporale della densità di biomassa arborea in fiamme}} = \underbrace{F_u(S, B_u, R_u, R_l)}_{\text{crescita della densità di biomassa arborea in fiamme per la propagazione di un incendio in funzione della biomassa viva presente e dell'umidità}} - \underbrace{D_u(R_u)}_{\text{estinzione dell'incendio nelle corone arboree (la biomassa arborea in fiamme scompare)}}$$

$$\underbrace{\frac{\partial R_l}{\partial t}}_{\text{derivata temporale della densità di biomassa in fiamme nel sottobosco}} = \underbrace{F_l(S, B_l, R_u, R_l)}_{\text{crescita della densità di biomassa in fiamme nel sottobosco per la propagazione di un incendio in funzione della biomassa viva presente e dell'umidità}} - \underbrace{D_l(R_l)}_{\text{estinzione dell'incendio nel sottobosco (la biomassa in fiamme nel sottobosco scompare)}}$$

$$\frac{dN}{dt} = N(a - bP),$$

$$\frac{dP}{dt} = P(cN - d),$$

## MODELLO

Bilanci di biomassa viva e in fiamme  
(due strati)

→ PREDATOR-PREY MODEL

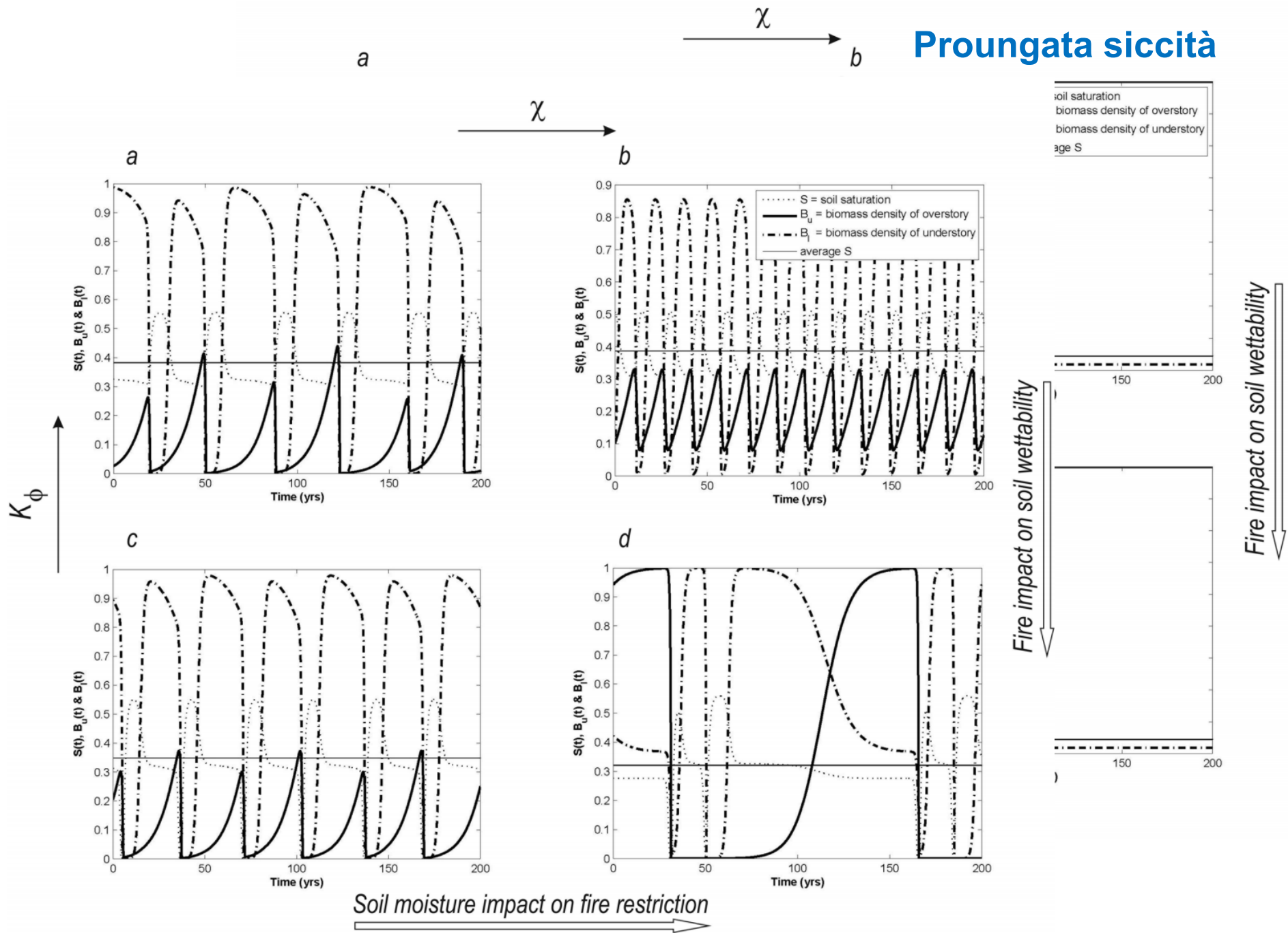
$$\frac{\partial B_u}{\partial t} = G_u(S, B_u) - F_u(S, B_u, R_u, R_l)$$

$$\frac{\partial R_u}{\partial t} = F_u(S, B_u, R_u, R_l) - D_u(R_u)$$

$$\frac{\partial B_l}{\partial t} = G_l(S, B_l, B_u) - F_l(S, B_l, R_u, R_l)$$

$$\frac{\partial R_l}{\partial t} = F_l(S, B_l, R_u, R_l) - D_l(R_l)$$

# Prongata siccità





# Modelli preda-predatore per simulare il regime degli incendi



## Geophysical Research Letters

### RESEARCH LETTER

10.1002/2014GL061560

#### Key Points:

- A new fire model is based on the feedback between fuel abundance and fire
- Land abandonment drives the Mediterranean forest to different fire dynamics
- Wildfire regime sequence is robust along the land development gradient

#### Correspondence to:

N. Ursino,  
nadia.ursino@unipd.it

#### Citation:

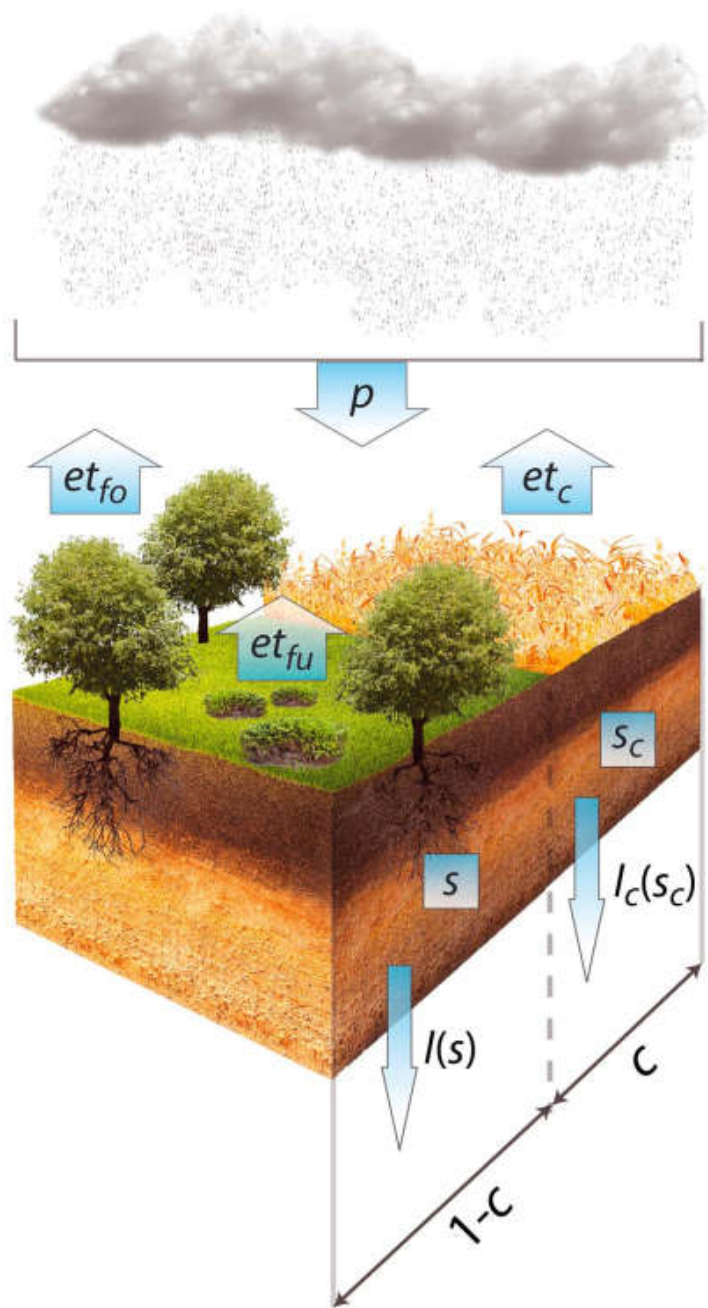
Ursino, N., and N. Romano (2014), Wild forest fire regime following land abandonment in the Mediterranean region, *Geophys. Res. Lett.*, 41, doi:10.1002/2014GL061560.

## Wild forest fire regime following land abandonment in the Mediterranean region

Nadia Ursino<sup>1</sup> and Nunzio Romano<sup>2</sup>

<sup>1</sup>Department ICEA, University of Padova, Padua, Italy, <sup>2</sup>Department of Agriculture, AFBE Division, University of Napoli Federico II, Naples, Italy

**Abstract** Land use, climate, and fire have markedly shaped Mediterranean ecosystems. While climate and land use are external forcing, wildfire is an integral component of ecosystem functioning which inevitably poses a threat to humans. With a view to gaining an insight into the mechanisms underlying fire dynamics, fire control, and prevention, we formulated a model that predicts the wildfire regime in fire-prone Mediterranean ecoregions. The model is based on the positive feedback between forest expansion following cropland abandonment, fuel abundance, and fire. Our results demonstrate that progressive land abandonment leads to different fire dynamics in the Mediterranean forest ecosystem. Starting at a no-fire regime when the land is almost completely cultivated, the ecosystem reaches a chaotic fire regime, passing through intermediate land development stages characterized by limit cycle fire dynamics. Wildfires are more devastating, albeit more predictable, in these intermediate stages when fire frequency is higher.



**Figure 1.** Schematic representation of the conceptual model, showing the main variables used in this study.

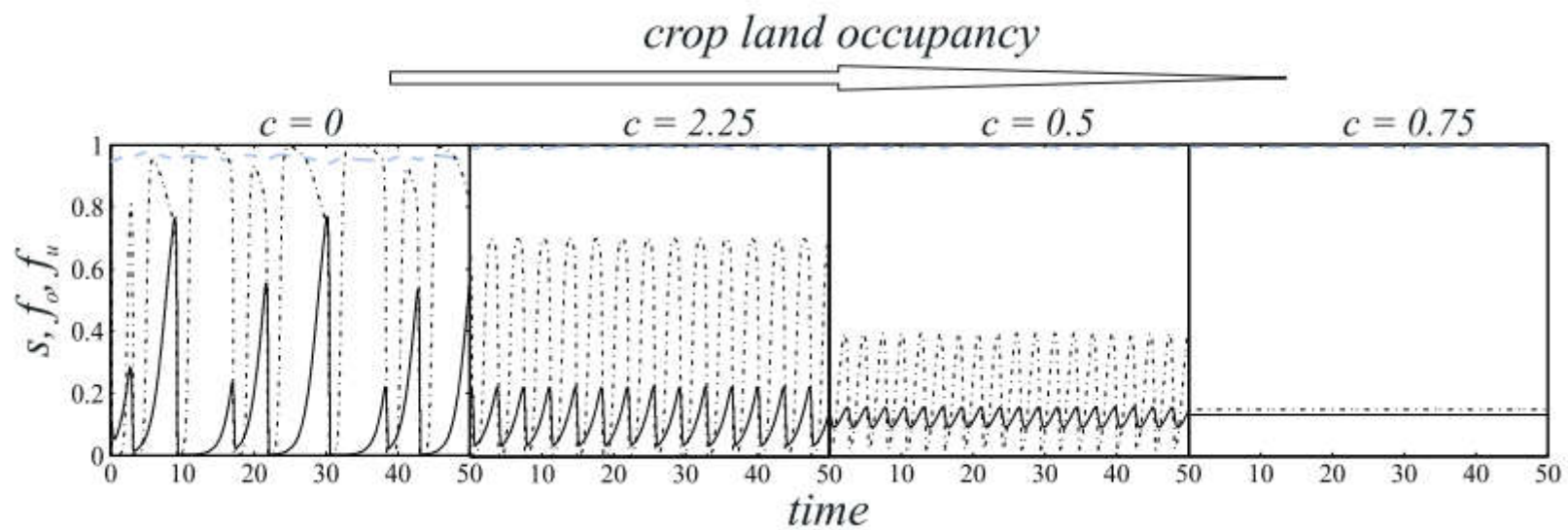
$$\frac{\partial s}{\partial t} = (p - q(p, b_o, b_u) - et - l(s)) \quad (2)$$

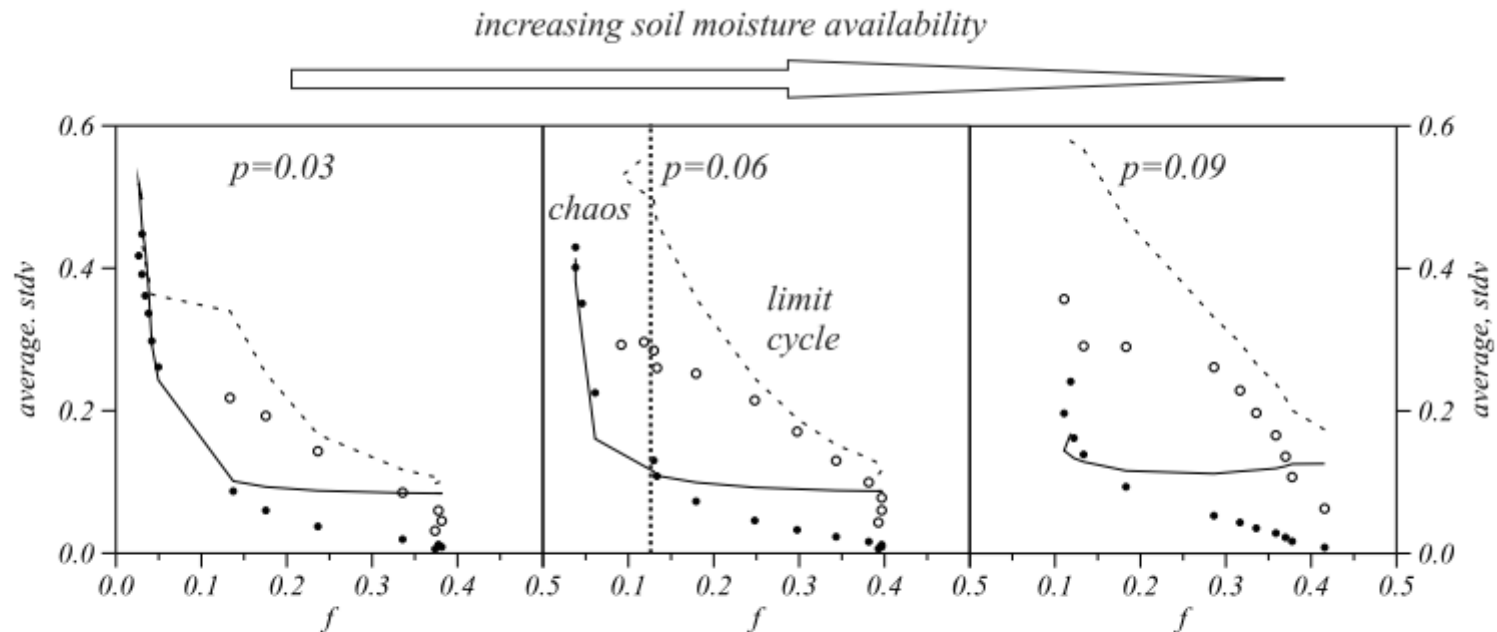
$$\frac{\partial f_o}{\partial t} = f_o (1 - c - f_o) - \left( \beta_o \frac{f_o}{f_o + h} \cdot b_o - \gamma_o \frac{f_o}{f_o + h} \cdot b_u \right) f(s) \quad (3)$$

$$\frac{\partial f_u}{\partial t} = gr \cdot s \cdot f_u (1 - c - f_u) - \alpha f_o \cdot f_u - \left( \beta_u \frac{f_u}{f_u + h} \cdot b_u - \gamma_u \frac{f_u}{f_u + h} \cdot b_o \right) f(s) \quad (4)$$

$$\frac{\partial b_o}{\partial t} = \left( \beta_o \frac{f_o}{f_o + h} \cdot b_o - \gamma_o \frac{f_o}{f_o + h} \cdot b_u \right) f(s) - \delta_o b_o \quad (5)$$

$$\frac{\partial b_u}{\partial t} = \left( \beta_u \frac{f_u}{f_u + h} \cdot b_u - \gamma_u \frac{f_u}{f_u + h} \cdot b_o \right) f(s) - \delta_u b_u \quad (6)$$





**Figure 5.** Standard deviation and average  $f_o$  and  $f_u$  versus fire frequency, for different stages of land development, and three average annual precipitation values  $p$ . Continuous line: average  $f_o$ ; dashed line: average  $f_u$ ; bold circles: standard deviation of  $f_o$ ; open circles standard deviation of  $f_u$ .  $p =$  (left) 0.03, (middle) 0.06, (right) 0.09.