ANALYTICAL PROBABILISTIC MODELING OF URBAN DRAINAGE SYSTEMS

B.J. Adams* and F. Papa**

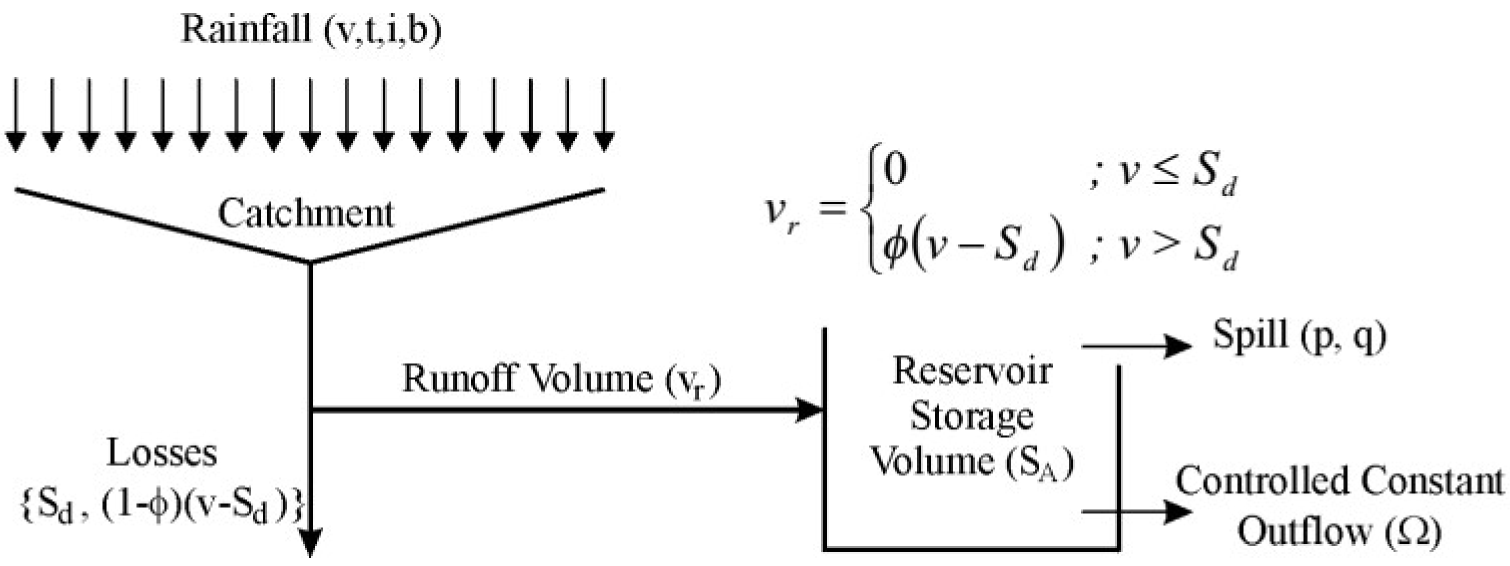
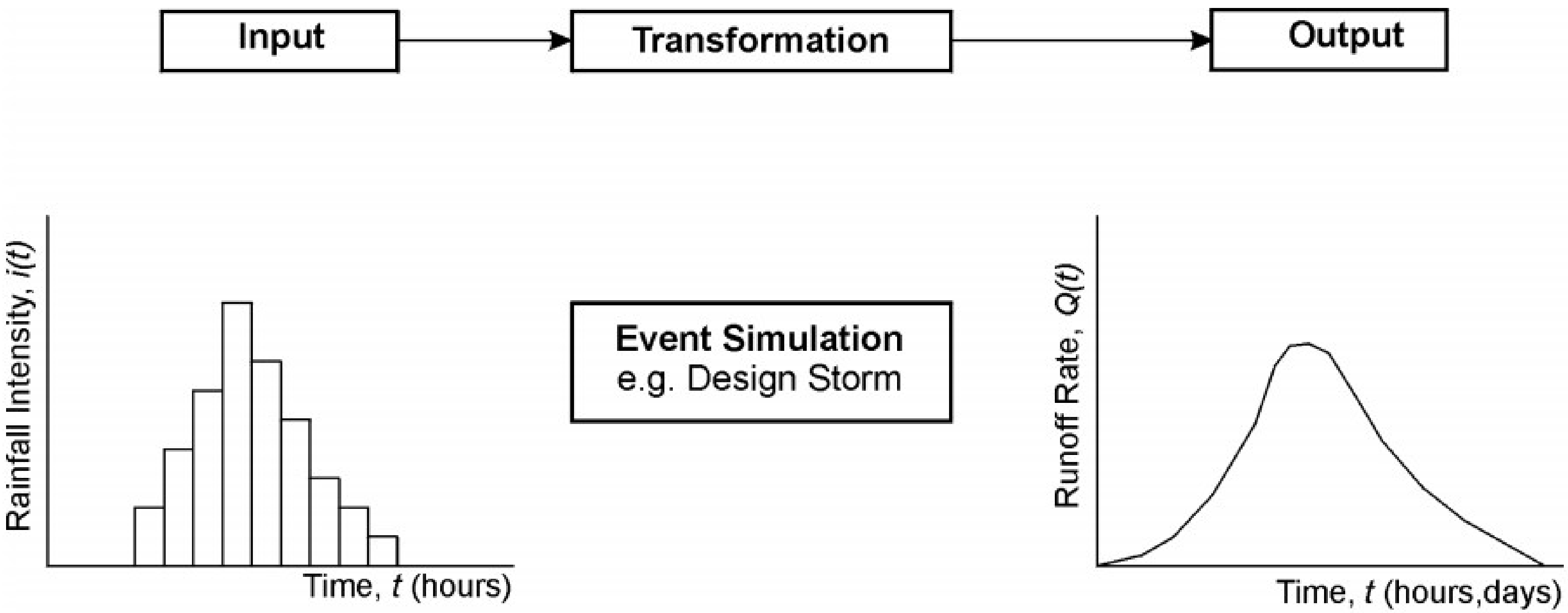
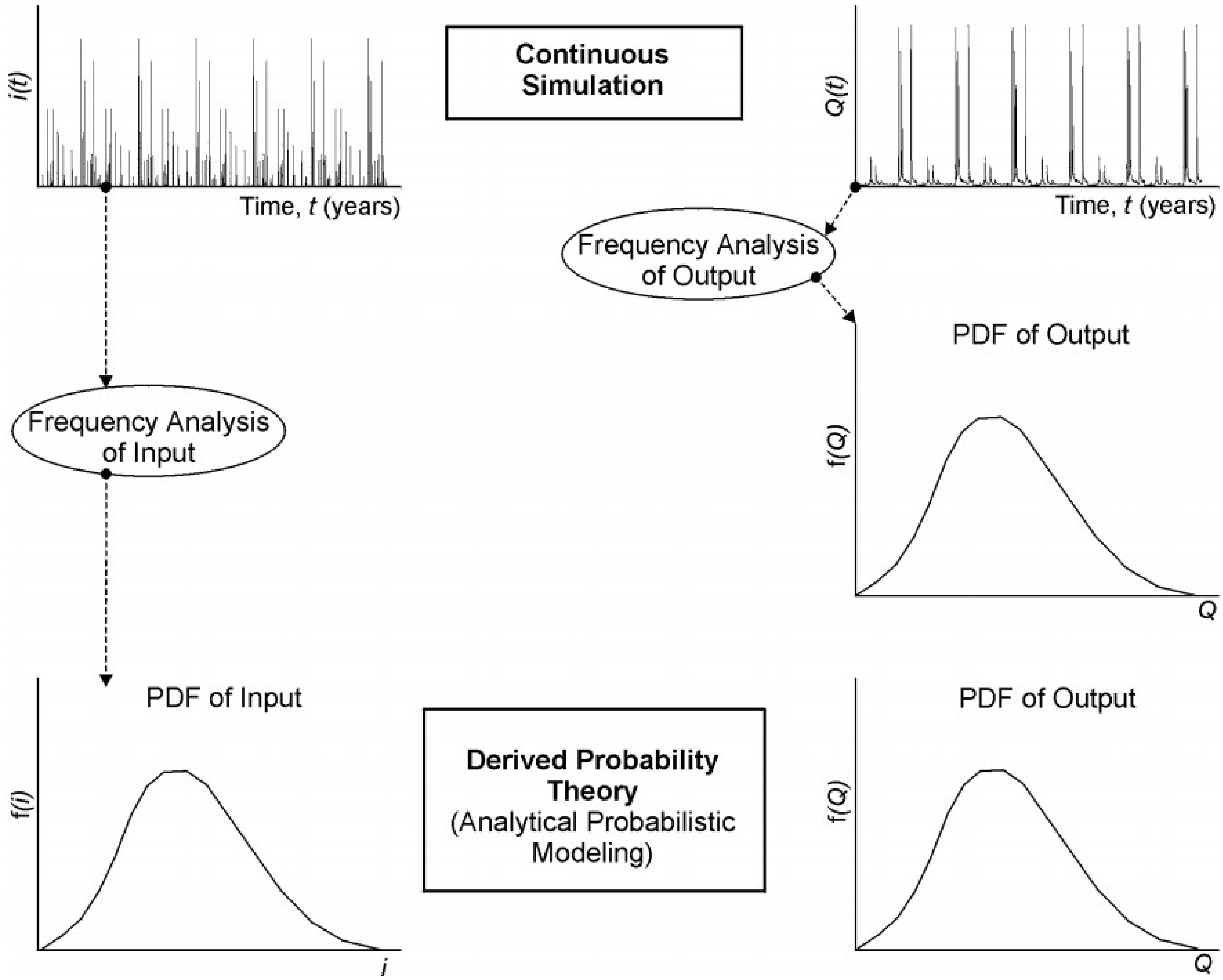


Figure 2. Schematic model of urban drainage systems.


Input
Transformation
Output
Rainfall Intensity, i(t)
Time, t (hours)
Event Simulation
e.g. Design Storm
Runoff Rate, Q(t)
Time, t (hours,days)



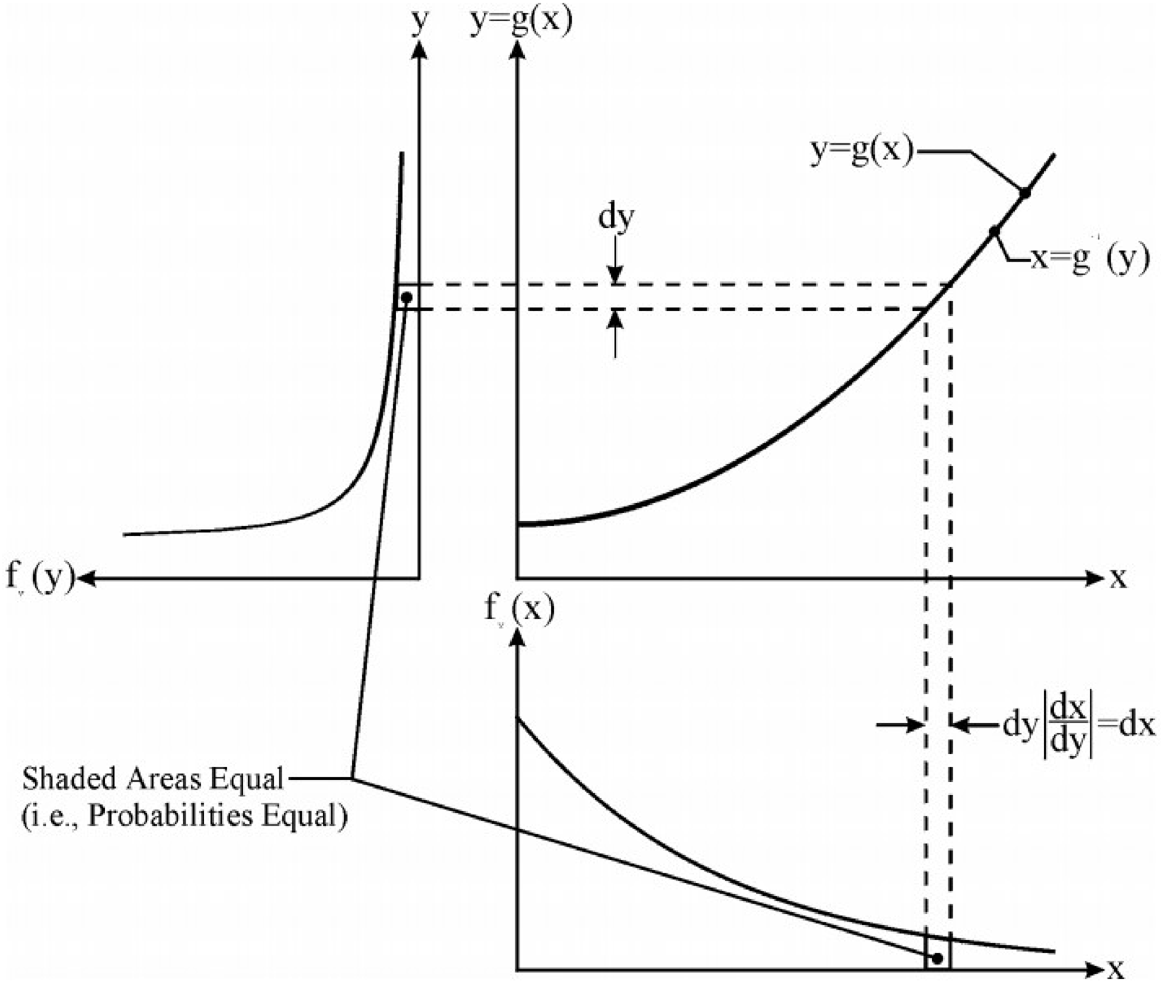
i(t)
Time, t (years)
Continuous Simulation
Q(t)
Time, t (years)
Frequency Analysis of Output
PDF of Output
f(Q)
Q
Frequency Analysis of Input
PDF of Input
f(i)
i
Derived Probability Theory
(Analytical Probabilistic Modeling)
PDF of Output
f(Q)
Q

Table 1. PDFs of rainfall characteristics

Rainfall Characteristics	Exponential PDF		Applicable Range
Volume, v (mm)	$f_V(v) = \zeta\, e^{-\zeta v}$	$\zeta = \frac{1}{\bar{v}}$	$0 \le v \le \infty$
Duration, t (h)	$f_T(t) = \lambda\, e^{-\lambda t}$	$\lambda = \frac{1}{\bar{t}}$	$0 \le t \le \infty$
Average intensity, i (mm/h)	$f_I(i) = \beta\, e^{-\beta t}$	$\beta = \frac{1}{\bar{i}}$	$0 \le i \le \infty$
Interevent time, b (h) Simplified version	$f_B(b) = \psi\, e^{-\psi(b-IETD)}$ $f_B(b) = \psi\, e^{-\psi b}$	$\psi = \frac{1}{\bar{b}\text{-}IETD}$ $\psi = \frac{1}{\bar{b}}$	$IETD \le b \le \infty$ $0 \le b \le \infty$

DERIVED PROBABILITY DISTRIBUTION THEORY

$$f_Y(y) = \frac{dx}{dy} f_X(x)$$



Consider a random variable X with a known probability density function (PDF), $f_X(x)$. Also consider a monotonically increasing function, $y = g(x)$, which transforms values of X to values of Y in one-to-one correspondence (i.e., only one value of Y exists for every value of X and vice versa). Then Y is a random variable whose PDF, $f_Y(y)$, can be derived as follows (see Figure 3).

If $y = g(x)$ [therefore, $x = g^{-1}(y)$] is monotonically increasing and maps $X \rightarrow Y$ one-to-one, then

$$F_Y(y) = \text{Prob}[Y \le y] = \text{Prob}\left[X \le g^{-1}(y)\right] = F_X\left[g^{-1}(y)\right] \tag{2}$$

where $F_Y(y)$ is the cumulative distribution function (CDF) of Y. By definition, the PDF is the first derivative of the CDF, or

$$f_Y(y) = \frac{d}{dy}F_Y(y) = \frac{d}{dy}F_X\left[g^{-1}(y)\right] = \frac{d}{dy}\int_{-\infty}^{g^{-1}(y)} f_X(x)dx \tag{3}$$

Using Leibniz's rule for differentiating an integral, Equation 3 is equivalent to

$$f_Y(y) = \frac{d}{dy}g^{-1}(y) \cdot f_X\left[g^{-1}(y)\right] \tag{4}$$

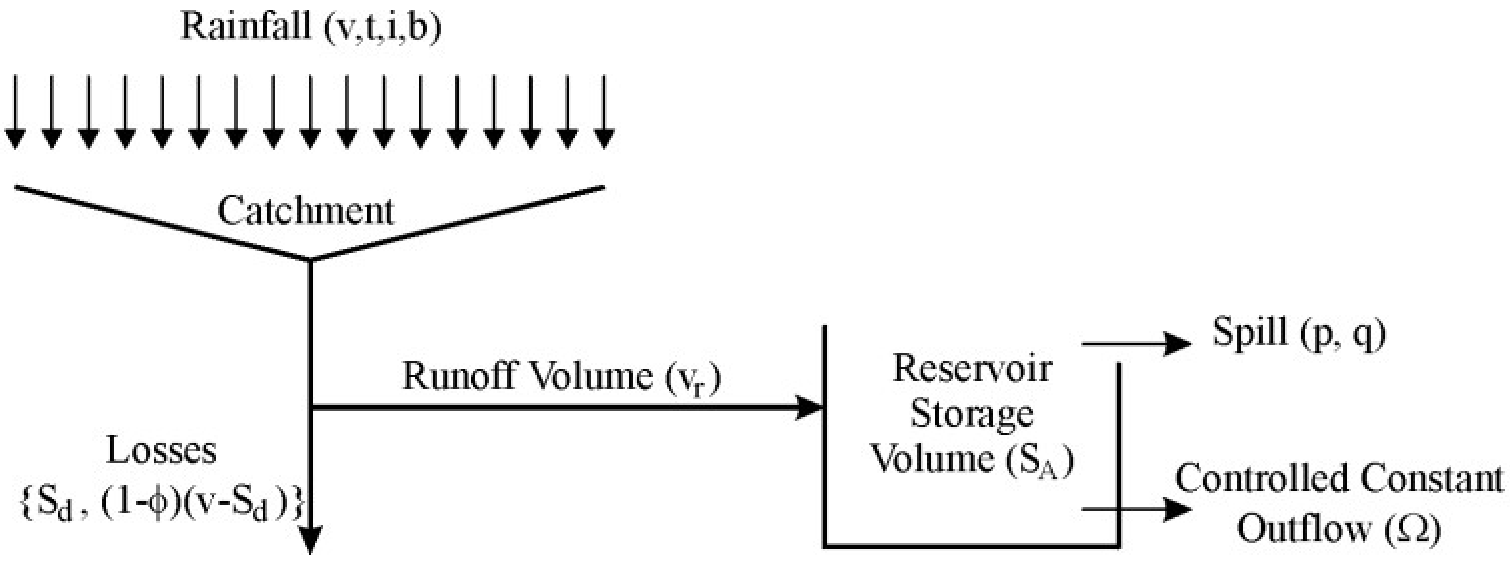


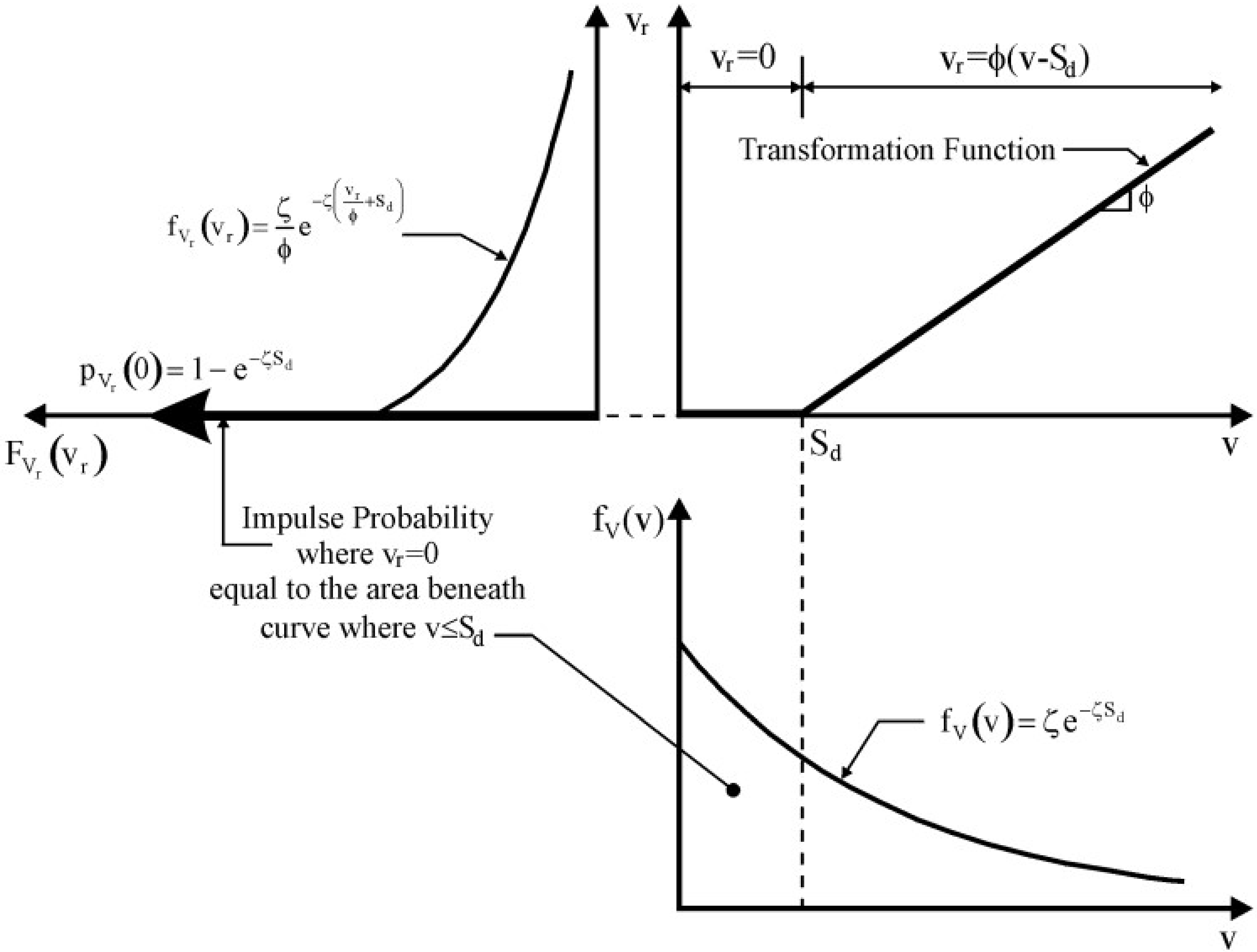
Figure 2. Schematic model of urban drainage systems.

$$f_Y(y) = \frac{dx}{dy} f_X(x)$$

S_d=surface depression

EXAMPLE OF DERIVED PROBABILITY DISTRIBUTION – RUNOFF VOLUME

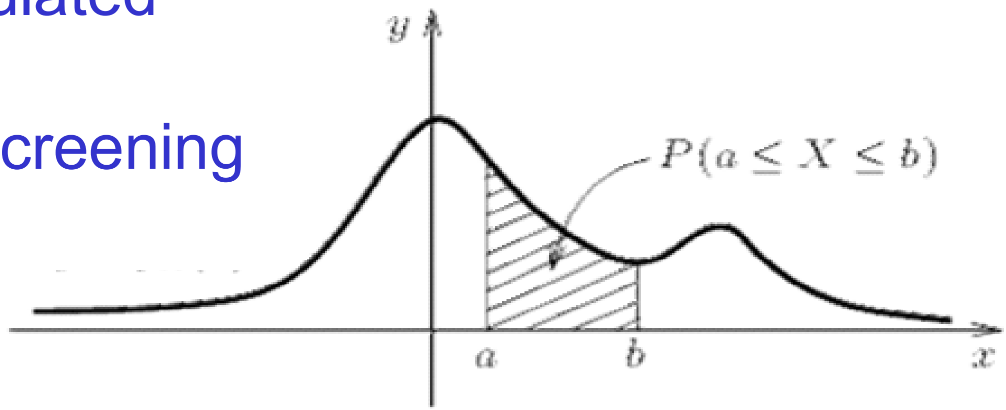
$$p_{V_r}(0) = \mathrm{Prob}[V_r = 0] = \mathrm{Prob}[V \le S_d] = \int_{v=0}^{S_d} f_V(v)\,dv = \int_{v=0}^{S_d} \zeta e^{-\zeta v}\,dv = 1 - e^{-\zeta S_d}$$



v_r
v_r=0
v_r=φ(v-S_d)
Transformation Function
φ
f_{V_r}(v_r) = ζ/φ e^{-ζ(v_r/φ + S_d)}
p_{V_r}(0) = 1 − e^{-ζS_d}
F_{V_r}(v_r)
S_d
v
Impulse Probability
where v_r=0
equal to the area beneath
curve where v≤S_d
f_V(v)
f_V(v) = ζ e^{-ζS_d}
v

Reliability Theories

Analytical models formulated with derived probability distribution theory, for screening level analysis



Reliability of detention systems

The probability density functions of rainfall characteristics are exponential:

$$f_h=\zeta e^{-\zeta h}$$

$$f_t=\lambda e^{-\lambda t}$$

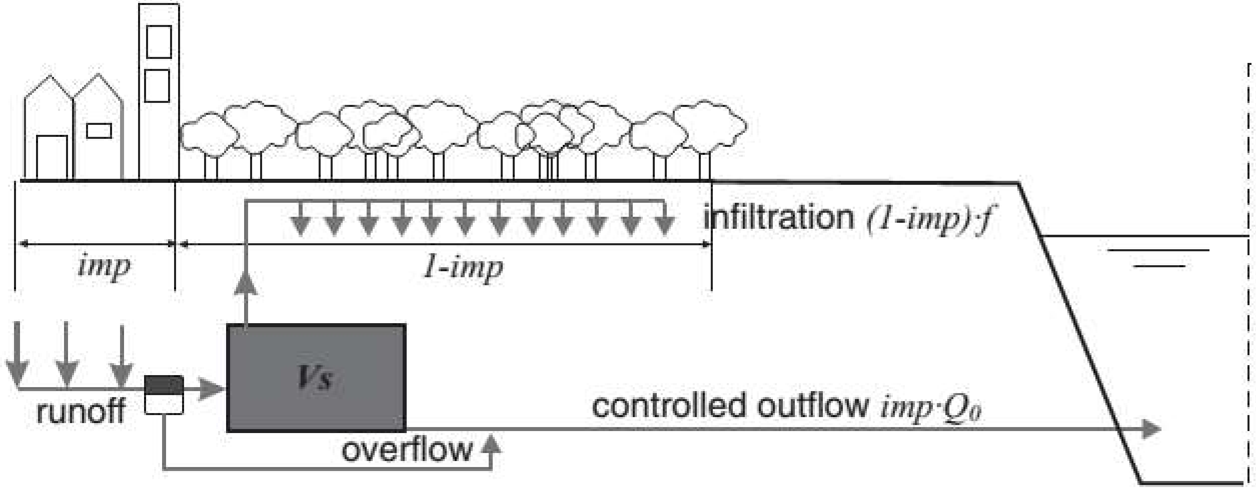
$$f_b=\psi e^{-\psi b}$$

h=rainfall depth
t=rainfall duration
b=interstorm interval

Are independent random variables

The distribution parameters are:

ζ=inverse of expected value of rainfall depth,
λ=inverse of expected value of rainfall duration,
ψ=inverse of expected value of rainfall inter-storm interval.



Detention: f=0

Assuming as certain that the network can transfer water from the impervious sub-catchment to the tank, and that the overflow divider performs correctly, only the detention system is subject to failure.

Initial condition

In applying the analytical probabilistic approach to assess the efficiency of a stormwater detention system, it is important to obtain the **storage facility's initial condition**, e.g., the available stormwater storage capacity of the facility.

Depending on the conditions preceding a random rainfall event, **the storage facility may be completely or partly empty when the rainfall event starts**.

Simplifying assumptions previously adopted in literature

1. The storage facility is completely full at the end of the rainfall event preceding the random rainfall event under analysis. [Howard's conservative assumption (Howard 1976)] .

2. The storage facility is completely empty when the analyzed random rainfall event starts

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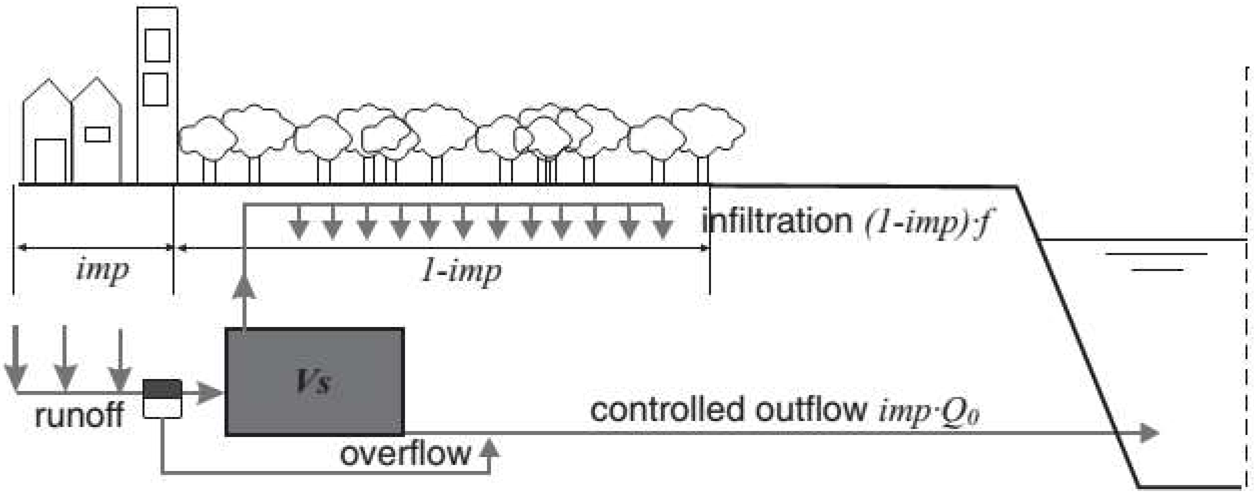
2. The storage facility is completely empty when the analyzed random rainfall event starts

Sd=storage depression =0

VASCHE DI LAMINAZIONE

Risk of overflow R_f is estimated, as the probability that either the rainfall volume is imp h >V_s or that two consecutive rainfall events occur in a short period of time. At the beginning of the second rainfall event, tank storage capacity is impQ_0b=V'_s <V_s and rainfall volume is imp h> V’s



Under the most conservative assumption that the reservoir is full at the end of the first of two consecutive rainfall events,

$$Rf = P[b \geq Vs/Q_0 imp] \cdot P[h > Vs/imp] + P[b < Vs/Q_0 imp] \cdot P[h > Q_0 b]$$

$R_f = P[b \geq V_s/Q_0 imp] \cdot P[h > V_s/imp] + P[b < V_s/Q_0 imp] \cdot P[h > Q_0 b]$

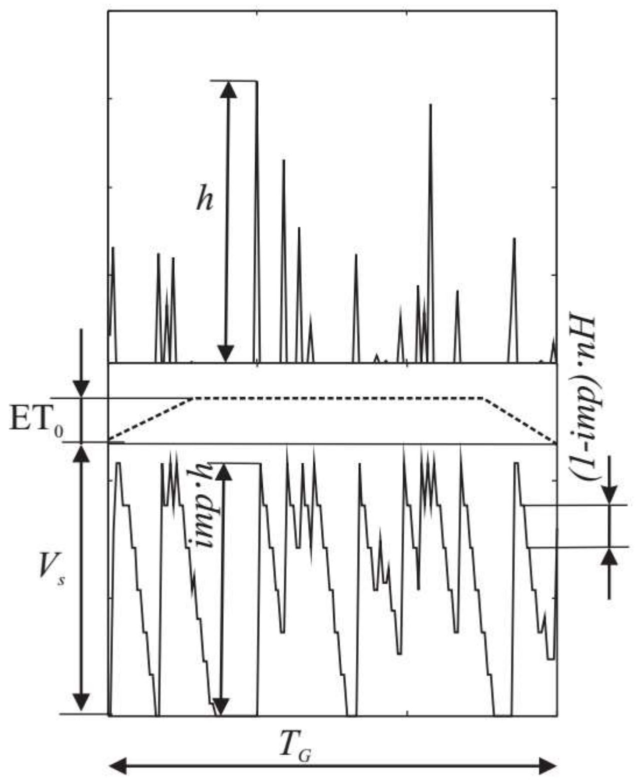
$$p_h(h) = \zeta\, e^{-\zeta h}$$

$$p_b(b) = \psi\, e^{-\psi b}$$

$$R_f = \int_{\frac{V_s}{Q_0 imp}}^{\infty} \psi e^{-\psi b} \int_{\frac{V_s}{imp}}^{\infty} \zeta e^{-\zeta h}\, dh\, db + \int_{0}^{\frac{V_s}{Q_0 imp}} \psi e^{-\psi b} \int_{bQ_0}^{\infty} \zeta e^{-\zeta h}\, dh\, db$$

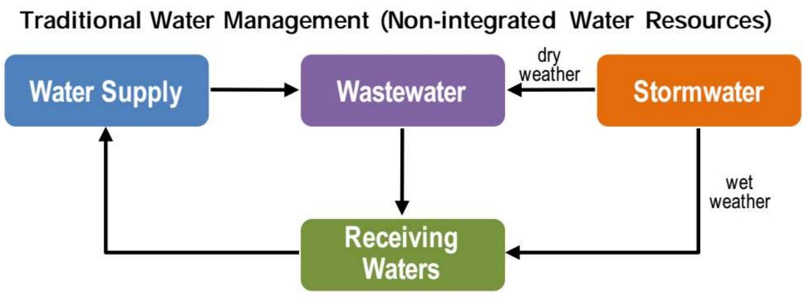
$$R_f = \frac{\psi}{\zeta Q_0 + \psi} + \frac{\zeta Q_0}{\zeta Q_0 + \psi} e^{\frac{-V_s}{imp}\left(\zeta + \frac{\psi}{Q_0}\right)}$$

SE I VOLUMI ACCUMULATI NELLA VASCA DI LAMINAZIONE VENGONO RIUTILIZZATI PER GLI USI CONSENTITI, E' NECESSARIO VALUTARE IL RISCHIO CHE I VOLUMI RISULTINO INSUFFICIENTI ALLA LAMINAZIONE E/O AL RIUTILIZZO

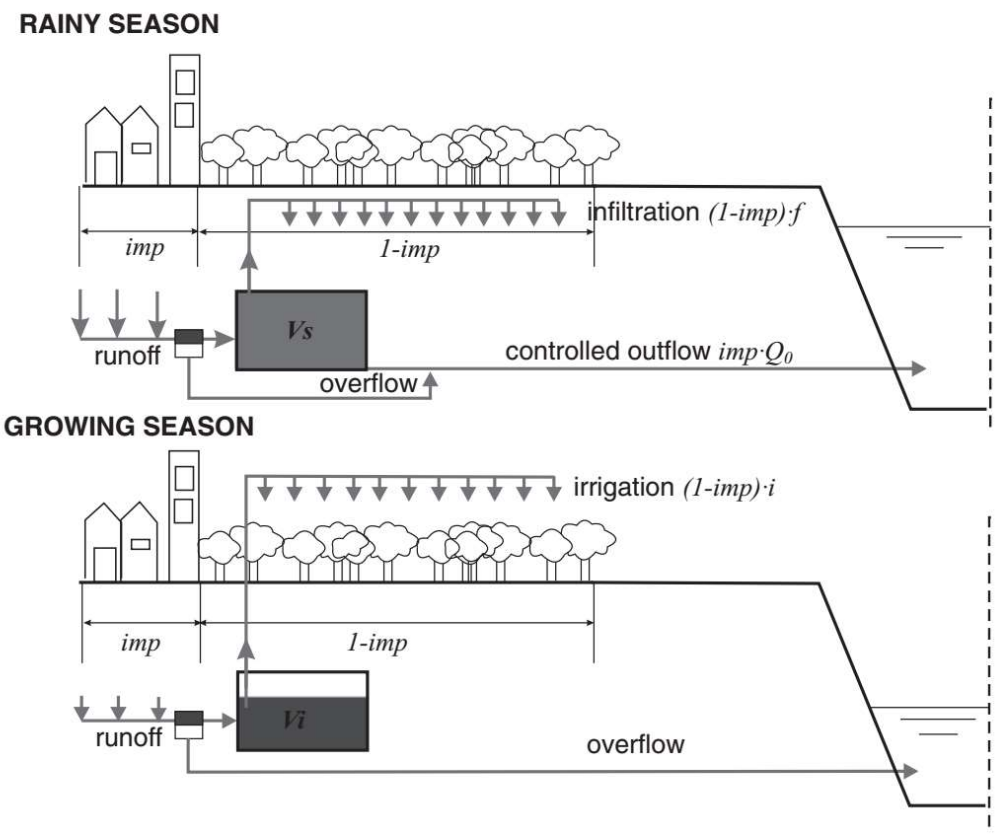


h
ET0
Vs
imp·h
(1-imp)·Hu
TG

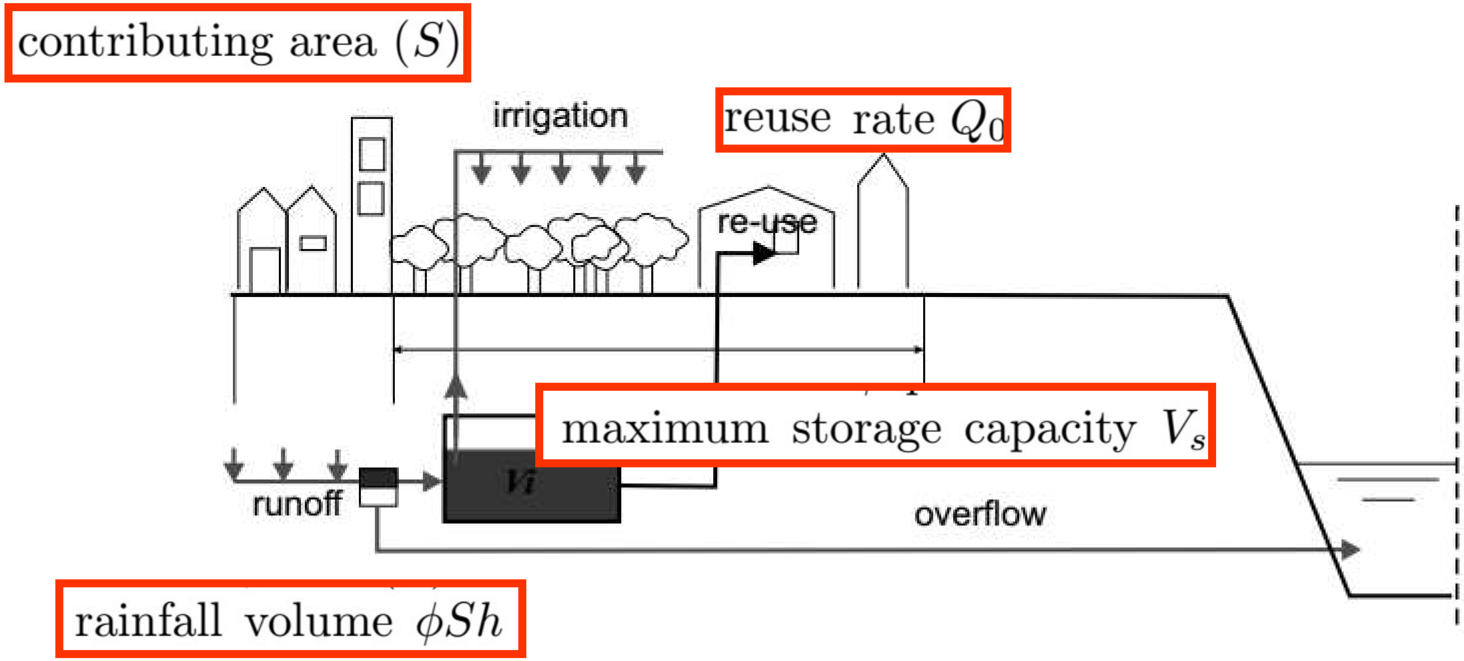
Ursino, 2015



Traditional Water Management (Non-integrated Water Resources)
Water Supply
Wastewater
dry weather
Stormwater
wet weather
Receiving Waters



RAINY SEASON
infiltration (1-imp)·f
imp
1-imp
Vs
runoff
controlled outflow imp·Q0
overflow
GROWING SEASON
irrigation (1-imp)·i
imp
1-imp
Vi
runoff
overflow



Dimensionless parameters

$$a = \frac{\zeta V_s}{\phi S}$$

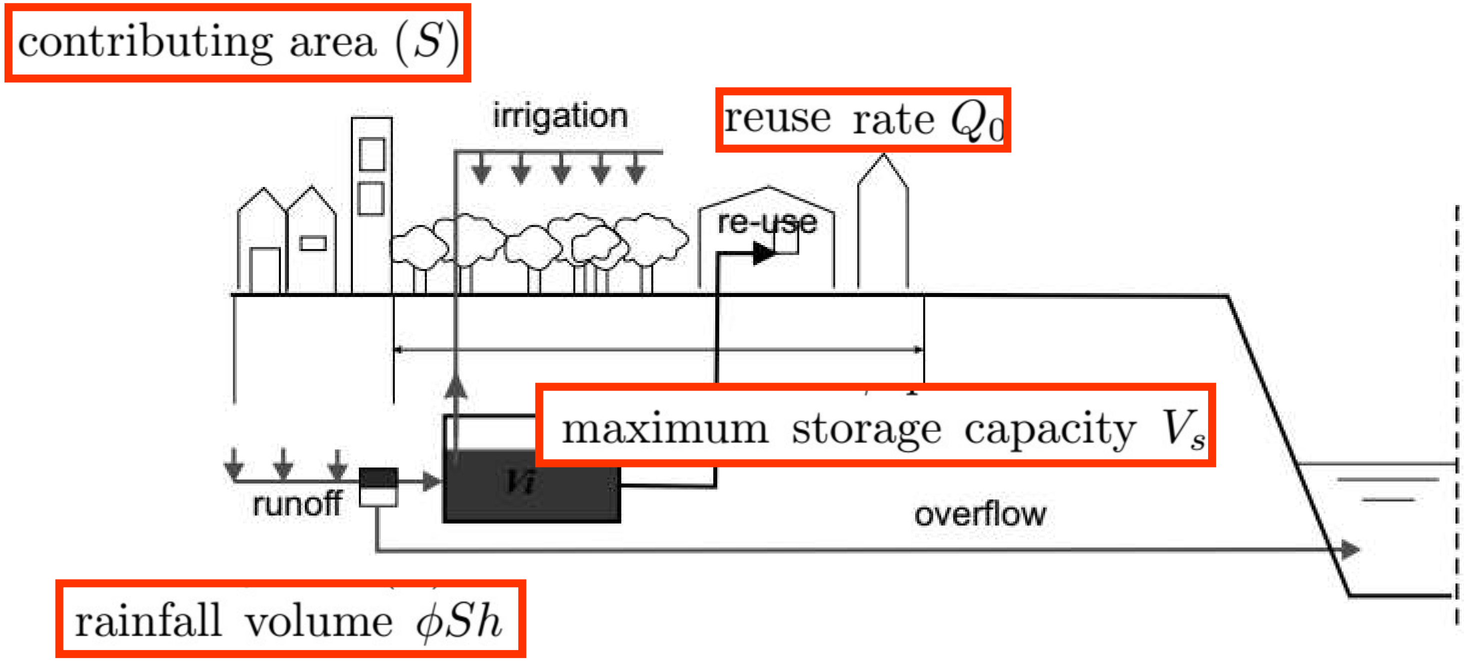
$$b = \frac{\lambda V_s}{Q_0}$$

Rainfall model

$$f_h = \zeta e^{-\zeta h}$$

$$f_t = \lambda e^{-\lambda t}$$

ζ = inverse of expected value of rainfall depth and
λ = inverse of expected inter-storm interval



Dimensionless parameters

$$a = \frac{\zeta V_s}{\phi S}$$

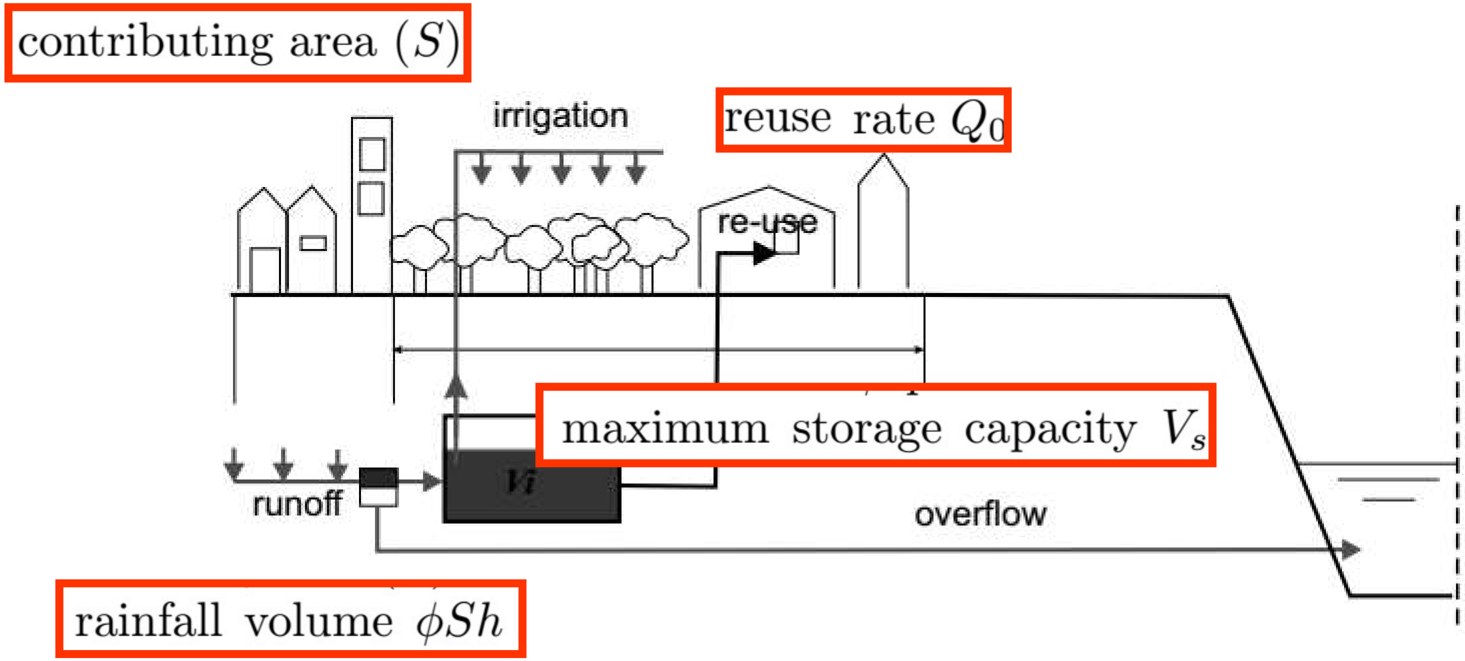
$$b = \frac{\lambda V_s}{Q_0}$$

The storage facility is completely full at the end of the rainfall event preceding the random rainfall event under analysis. [Howard's conservative assumption

Risk of overflow

$$R_f = P[t \geq V_s/Q_0] \cdot P[\phi Sh > V_s] + P[t < V_s/Q_0] \cdot P[\phi Sh > Q_0\, t]$$

$$R_f = \frac{a}{a+b}\, e^{-(a+b)} + \frac{b}{a+b}$$



Dimensionless parameters

$$a = \frac{\zeta V_s}{\phi S}$$

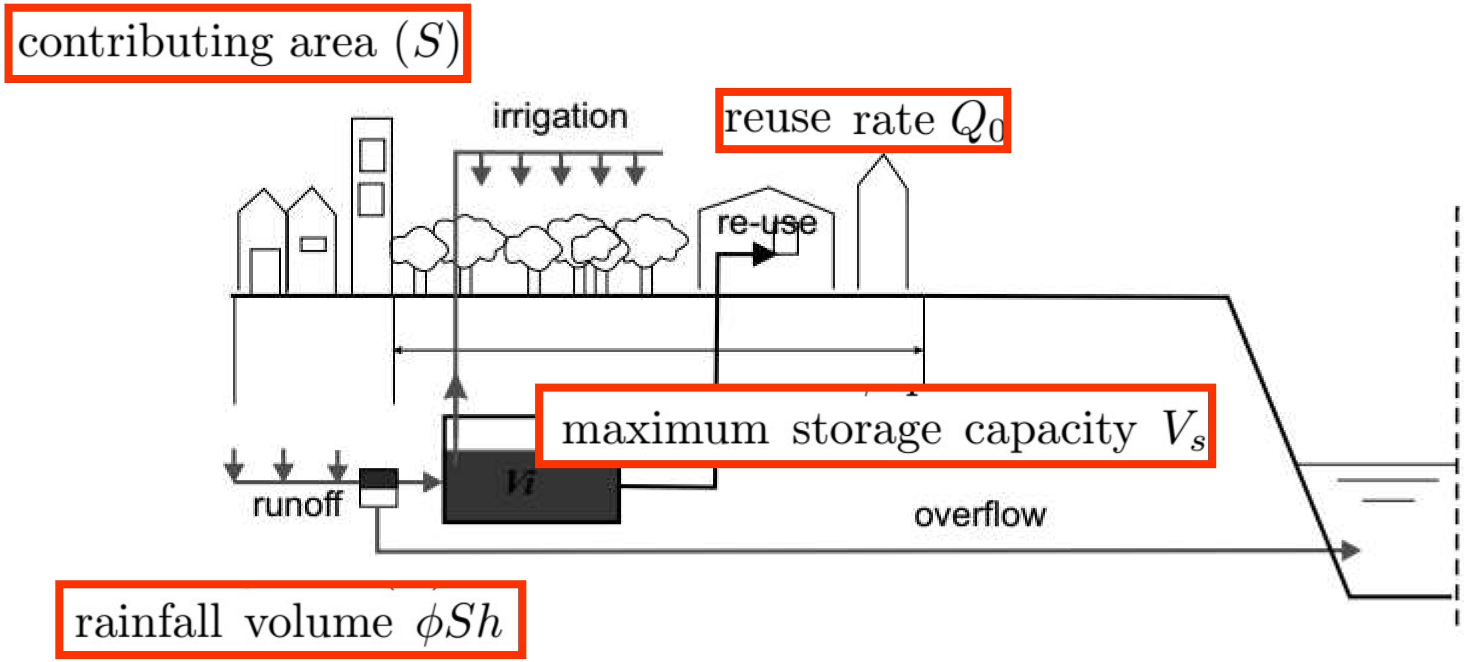
$$b = \frac{\lambda V_s}{Q_0}$$

The storage facility is empty at the end of the dry period preceding the random rainfall event under analysis. [Howard's conservative assumption

Risk of water scarcity

$$R_i = P[h < V_s/(\phi S)] \cdot P[t > \phi S h/Q_0] + P[h \geq V_s/(\phi S)] P[t > V_s/Q_0]$$

$$R_i = \frac{b}{a+b}\, e^{-(a+b)} + \frac{a}{a+b}$$



Dimensionless parameters

$$a = \frac{\zeta \, V_s}{\phi \, S}$$

$$b = \frac{\lambda \, V_s}{Q_0}$$

$$f_h = \zeta \, e^{-\zeta h}$$

$$f_t = \lambda \, e^{-\lambda t}$$

$$R_f = \frac{a}{a+b} \, e^{-(a+b)} + \frac{b}{a+b}$$

$$R_i = \frac{b}{a+b} \, e^{-(a+b)} + \frac{a}{a+b}$$

Il volume della vasca V_s, non influenza i rapporti $\frac{a}{a+b}$ e $\frac{b}{a+b}$

$$a = \frac{\zeta \, V_s}{\phi \, S}$$

$$b = \frac{\lambda \, V_s}{Q_0}$$

Il volume della vasca V_s, influenza influenza piuttosto il peso relativo dei due addendi che compaiono nelle espressioni di rischio

$$R_f = \frac{a}{a+b} \, e^{-(a+b)} + \frac{b}{a+b}$$

$$R_i = \frac{b}{a+b} \, e^{-(a+b)} + \frac{a}{a+b}$$

$$a = \frac{\zeta V_s}{\phi S}$$

$$b = \frac{\lambda V_s}{Q_0}$$

All'aumentare di V_s, $e^{-(a+b)}$ risulterà sempre iù piccolo e quindi I rischi di fallanza tenderanno ad un valore limite che dipende solo dal clima e dall'utilizzo $R_f \rightarrow \frac{b}{a+b}$ e $R_i \rightarrow \frac{a}{a+b}$

$$R_f = \frac{a}{a+b} e^{-(a+b)} + \frac{b}{a+b}$$

$$R_i = \frac{b}{a+b} e^{-(a+b)} + \frac{a}{a+b}$$

RISK ANALYSIS OF RAINWATER HARVESTING SYSTEMS AROUND THE WORLD

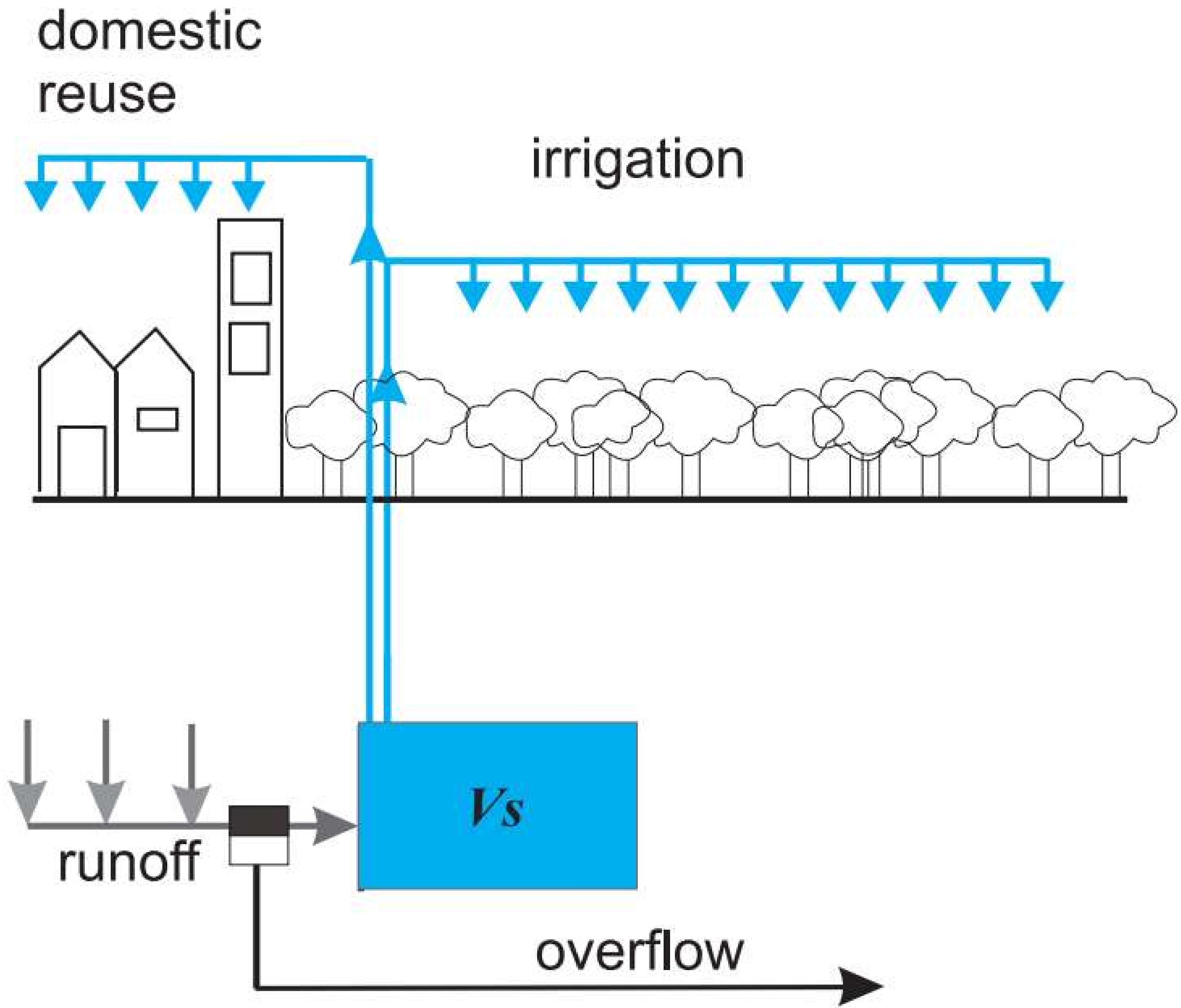
water

MDPI

Article

Risk Analysis Approach to Rainwater Harvesting Systems

Nadia Ursino


domestic
reuse
irrigation
Vs
runoff
overflow

Simplifying assumptions previously adopted in literature

1. The storage facility is completely full at the end of the rainfall event preceding the random rainfall event under analysis. [Howard's conservative assumption (Howard 1976)] .

2. The storage facility is completely empty when the analyzed random rainfall event starts

$$f_h = \zeta \, e^{-\zeta h}$$

$$f_t = \lambda \, e^{-\lambda t}$$

ζ is the inverse of expected value of rainfall depth

λ the inverse of expected inter-storm interval.

Under the most conservative assumption,

$$R_i = P[h < V_s/(\phi S)] \cdot P[t > \phi S h/Q_0] + P[h \geq V_s/(\phi S)] \cdot P[t > V_s/Q_0]$$

$$R_i = \frac{b}{a+b} e^{-(a+b)} + \frac{a}{a+b}$$

Under the least conservative assumption,

$$R_i' = P[t > V_s/Q_0] = e^{-b}$$

$a = \frac{\zeta V_s}{\phi S}$ and $b = \frac{\lambda V_s}{Q_0}$

Efficiency may be estimated as the probability that RWH tank provides water when needed, which is the complementary probability of the Risk of water scarcity.

Under the assumption that the reservoir is empty at the beginning of the first of two rain events,

$$E = 1 - \left[\frac{b}{a+b}\,e^{-(a+b)} + \frac{a}{a+b}\right] = \frac{b}{a+b}\left[1 - e^{-(a+b)}\right]$$

under the assumption that the tank is empty at the beginning of any rain event,

$$E' = 1 - e^{-b}$$

The Demand Ratio is defined as the ratio between the average inter-storm demand and the average stored rainwater at the end of a rain event.

the most conservative assumption is that the tank is full at the end of the first of two consecutive rainfall events.

the average collected volume V_c may be estimated as follows:

$$V_c = \int_0^{\frac{V_s}{Q_0}} \lambda e^{-\lambda t} \left(\int_0^{\frac{Q_0 t}{\phi S}} \phi S \, h \zeta e^{-\zeta h} dh + Q_0 \, t \int_{\frac{Q_0 t}{\phi S}}^{\infty} \zeta e^{-\zeta h} dh \right) dt$$

$$+ \int_{\frac{V_s}{Q_0}}^{\infty} \lambda e^{-\lambda t} \left(\int_0^{\frac{V_s}{\phi S}} \phi S \, h \zeta e^{-\zeta h} dh + \frac{V_s}{\phi S} \int_{\frac{V_s}{\phi S}}^{\infty} \zeta e^{-\zeta h} dh \right) dt$$

$$= \frac{V_s}{a+b} \left[1 - e^{-(a+b)} \right]$$

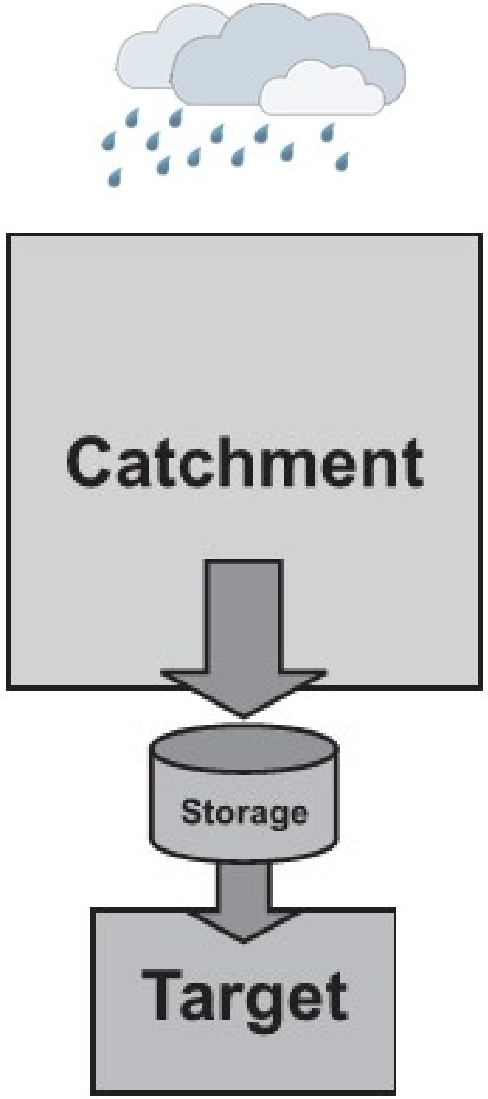
The least conservative assumption is that the tank is empty at the beginning of any rain events.

$$V_c' = \int_0^{\frac{V_s}{\phi S}} \phi S \, h \, \zeta e^{-\zeta h} dh \int_{\frac{V_s}{\phi S}}^{\infty} V_s \, \zeta e^{-\zeta h} dh = \frac{V_s}{a} \left(1 - e^{-a} \right)$$

Demand Ratio,

$$DR = \frac{Q_0}{V_c \lambda} = \frac{(a+b)}{b\,a}\left[1 - e^{-(a+b)}\right]^{-1}$$

$$DR' = \frac{Q_0}{V_c' \lambda} = \frac{a}{b}\left(1 - e^{-a}\right)^{-1}$$



ESEMPI DI SISTEMI DI RIUTILIZZO

Journal of Environmental Management 91 (2009) 222–226

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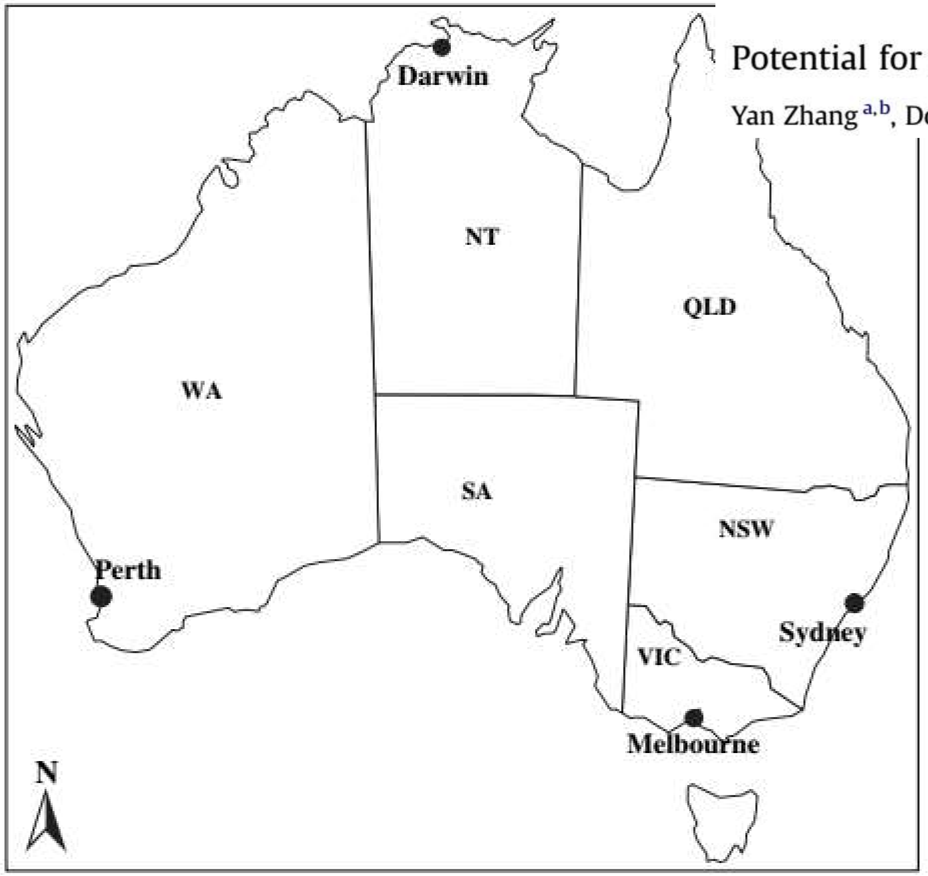
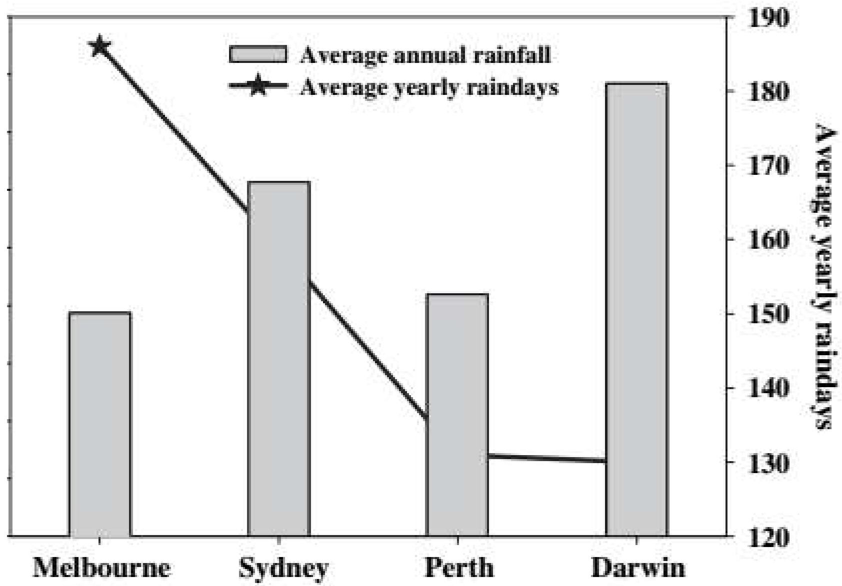
Potential for rainwater use in high-rise buildings in Australian cities

Yan Zhang [a,b], Donghui Chen [c,*], Liang Chen [b], Stephanie Ashbolt [d]





	Tank size (m^3)		Tank water use (m^3/yr)	
	3A	5A	3A	5A
Melbourne	25	35	334.4	334.5
Sydney	80	80	496.5	432.3
Perth	80	80	363.0	322.0
Darwin	150	150	454.9	390.4

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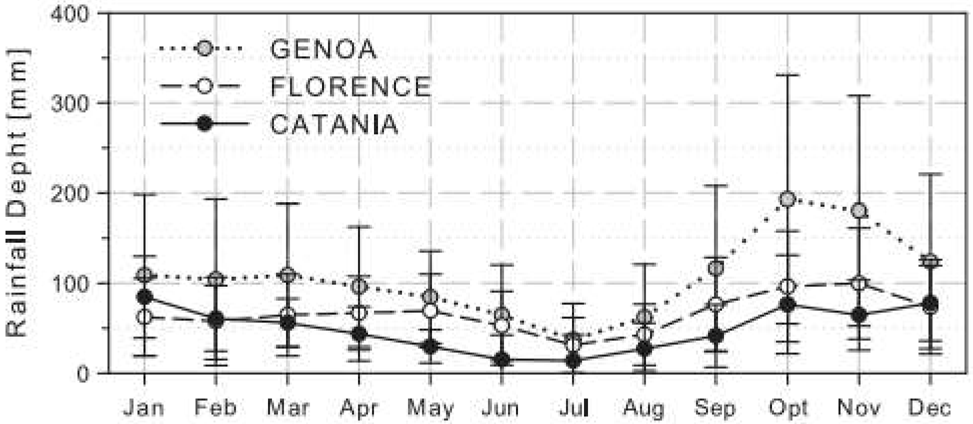
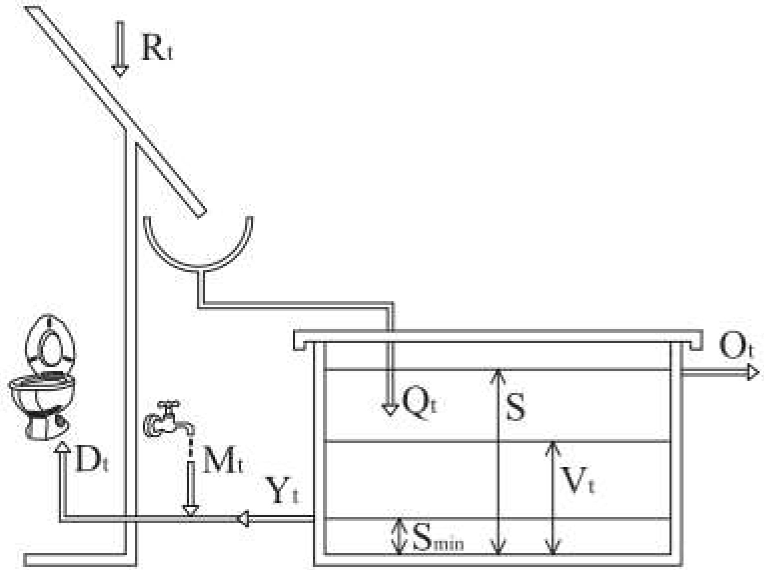
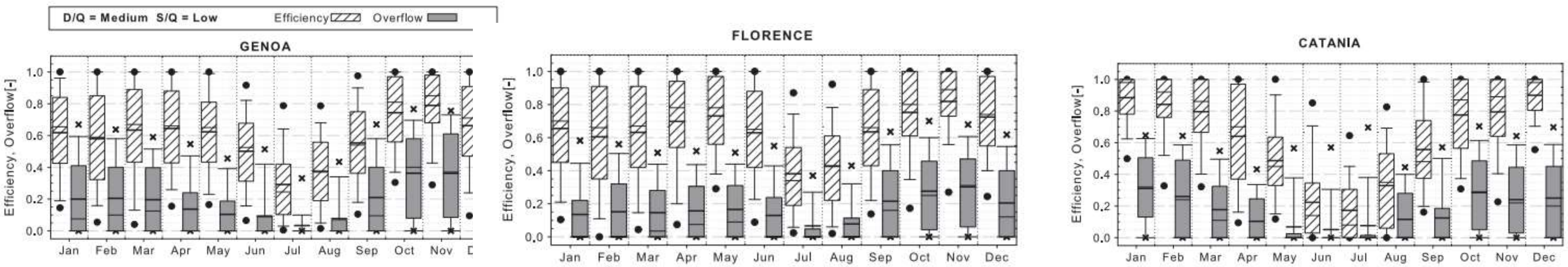
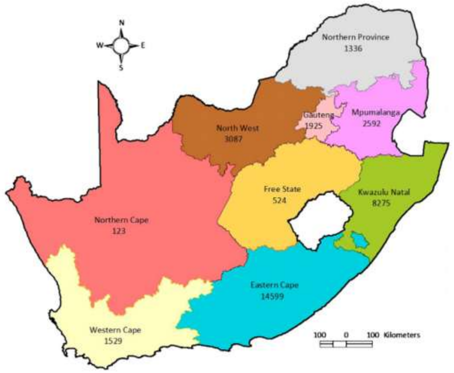
Non-dimensional design parameters and performance assessment of rainwater harvesting systems

A. Palla *, I. Gnecco, L.G. Lanza









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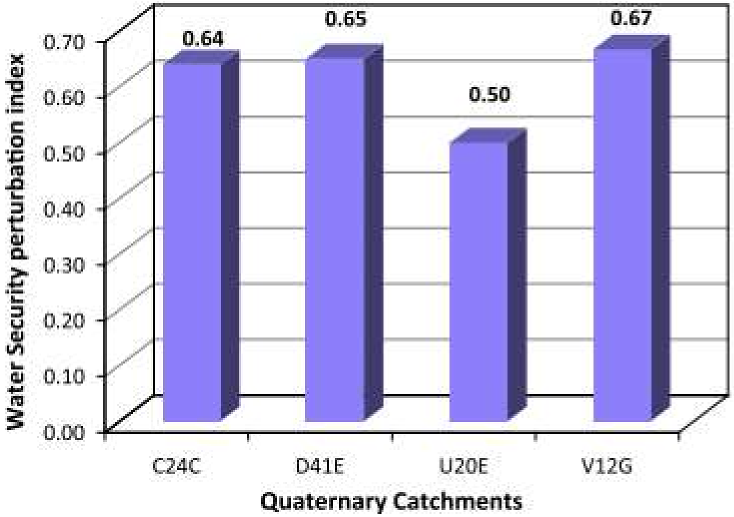
Domestic rainwater harvesting as an adaptation measure to climate change in South Africa

J. Mwenge Kahinda [a,*], A.E. Taigbenu [a], R.J. Boroto [b]



Resources, Conservation and Recycling 54 (2009) 79–85

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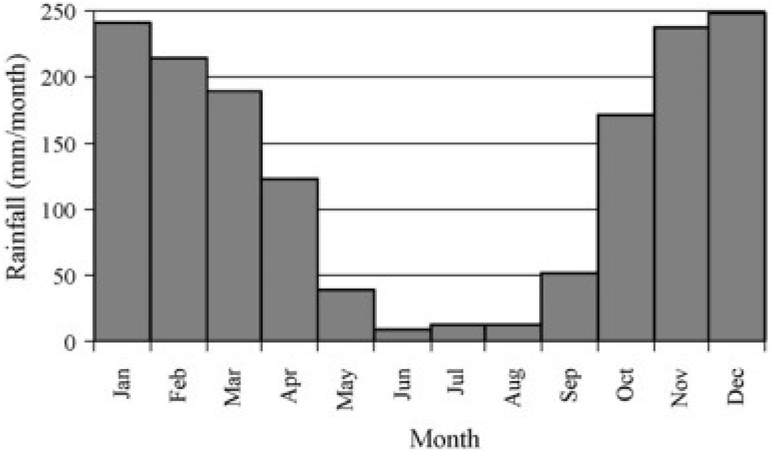
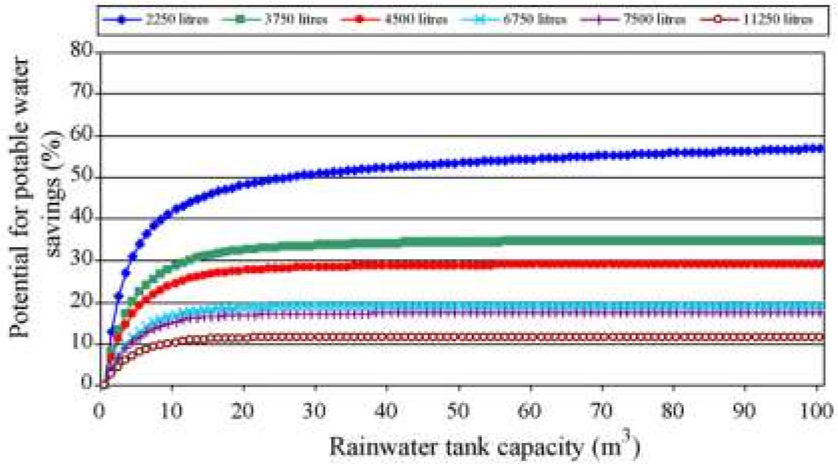
Rainwater harvesting in petrol stations in Brasília: Potential for potable water savings and investment feasibility analysis

Enedir Ghisi*, Davi da Fonseca Tavares, Vinicius Luis Rocha





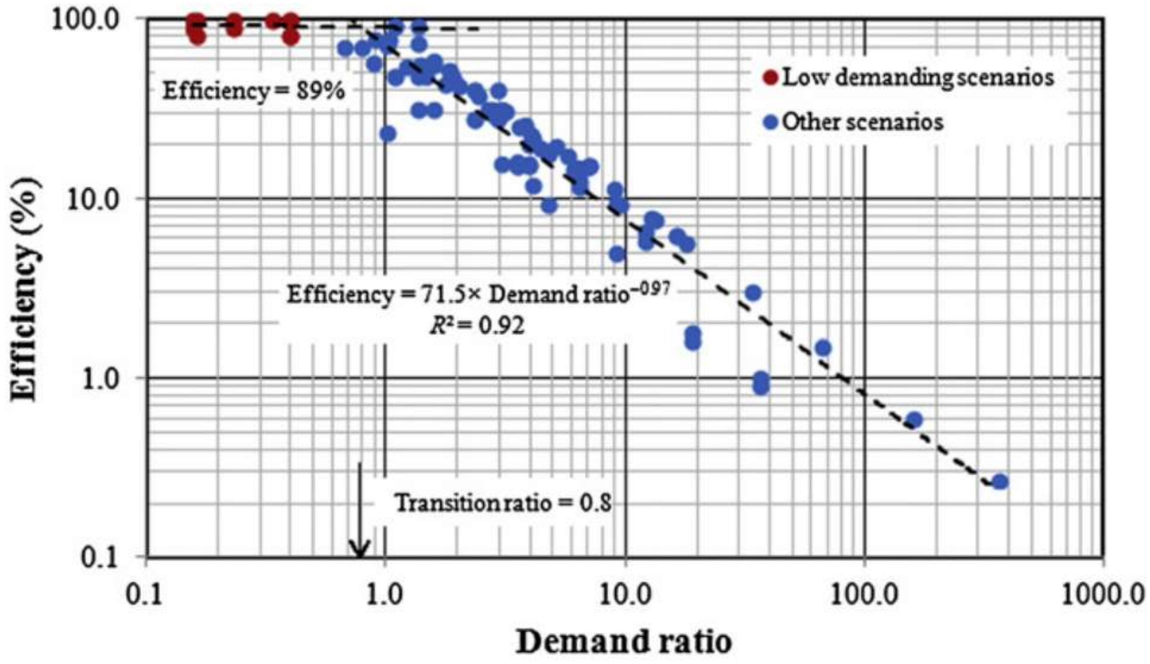
Efficiency is an increasingly good estimate of the system reliability as the **number of years of observation** increases.

Jordan	averaged monthly precipitation data	Domestic use	Abdulla et al., 2009
Brazilia(Brazil)	Dayly rainfall -30 years	washing vehicles	Sanches Fernandes et al., 2015
South Africa	data forecasted by 6 global circulation models	houshold water requirements	Mwenge Kahinda et al., 2010
Iran	Dayly rainfall -50 years	Reuse systems for residential buildings	Mehrabadi et al, 2013
Italy	Dayly rainfall -30 to 100 years	Reuse systems for residential buildings	Palla et al., 2011
Australia	Monthly average rainfall -80 years	Reuse systems for residential buildings	Zhang et al., 2009

Design is usually performed with reference to tank **efficiency**

In **high demanding scenarios** there will be a marked dependence of the RWH system efficiency on the water demand

(**Fernandes et al., 2015**).



non-dimensional analysis

demand ratio: ratio between annual demand and annually collected rainwater

storage fraction: ratio between storage capacity of the system and annually collected rainwater

Efficiency: Ratio between collected volume from catchment and expected volume at delivery node

Science of the Total Environment 529 (2015) 91–100

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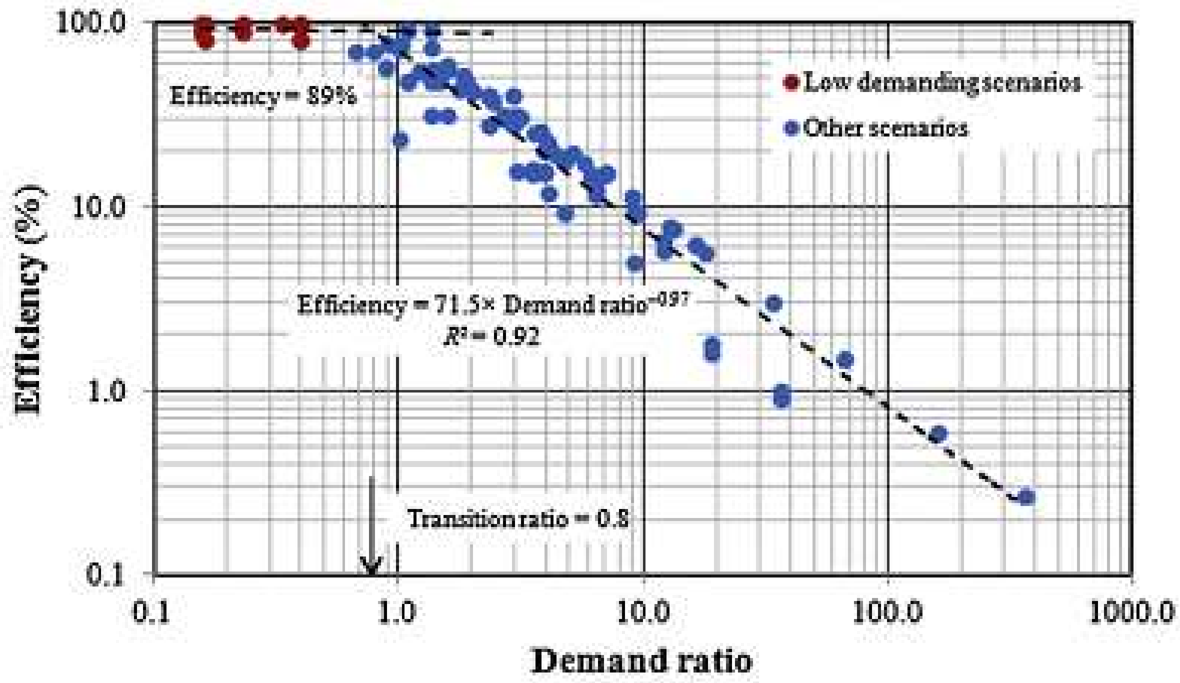
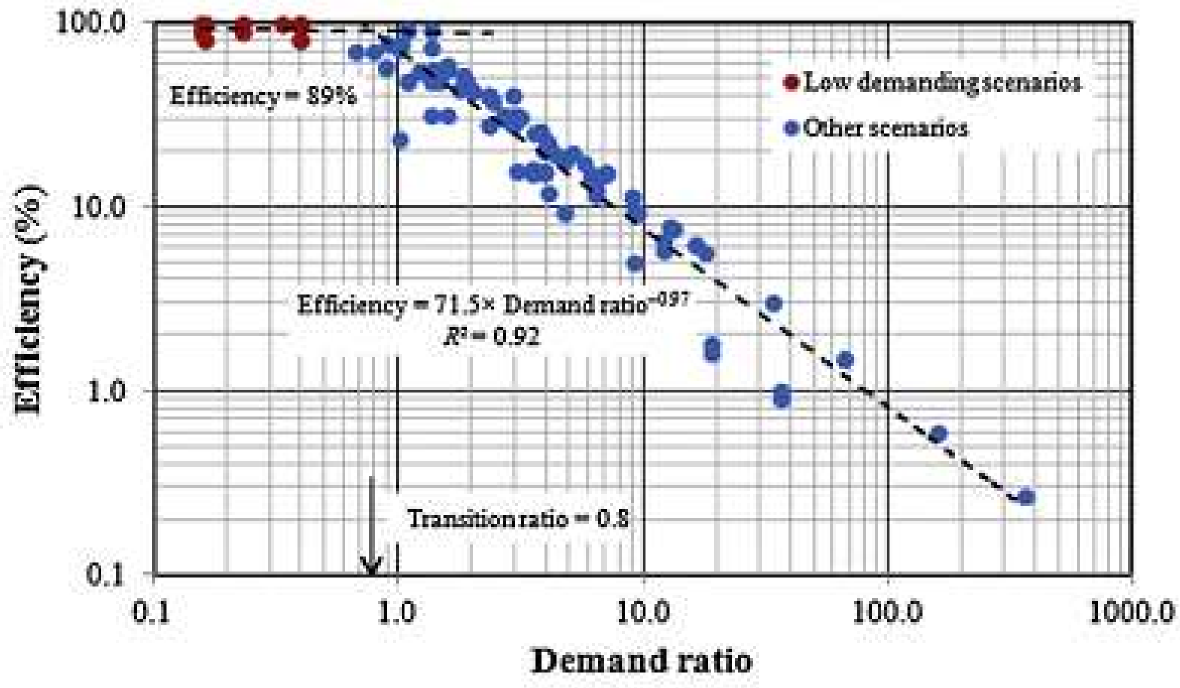
Rainwater harvesting systems for low demanding applications

Luís F. Sanches Fernandes [a,b], Daniela P.S. Terêncio [a], Fernando A.L. Pacheco [c,d,*]



L.F. Sanches Fernandes et al. / Science of the Total Environment 529 (2015) 91–100



RIESAME DEI DATI DI LETTERATURA SULLA BASE DELL'ANALISI DI PROBABILITA'

water

MDPI

Article

Risk Analysis Approach to Rainwater Harvesting Systems

Nadia Ursino

$$E = 1 - \left[\frac{b}{a+b}\, e^{-(a+b)} + \frac{a}{a+b}\right] = \frac{b}{a+b}\left[1 - e^{-(a+b)}\right]$$

$$E' = 1 - e^{-b}$$

$$DR = \frac{Q_0}{V_c \lambda} = \frac{(a+b)}{b\, a}\left[1 - e^{-(a+b)}\right]^{-1}$$

$$DR' = \frac{Q_0}{V_c' \lambda} = \frac{a}{b}\left(1 - e^{-a}\right)^{-1}$$

water

MDPI

Article

Risk Analysis Approach to Rainwater Harvesting Systems

Nadia Ursino

Melbourne

Journal of Environmental Management 91 (2009) 222–226

Contents lists available at ScienceDirect

Journal of Environmental Management

journal homepage: www.elsevier.com/locate/jenvman

Potential for rainwater use in high-rise buildings in Australian cities

…, Liang Chen[b], Stephanie Ashbolt[d]

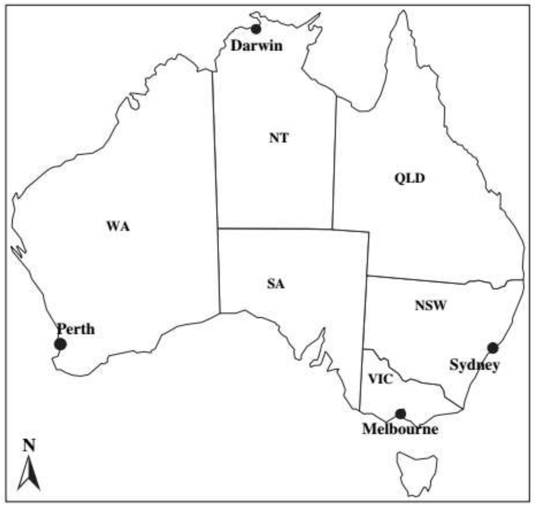
$$\zeta = 96 \quad m^{-1}$$

$$\lambda = 0{,}012 \quad hr^{-1}$$

$$a = \frac{\zeta V_s}{\varphi S} = 8$$

$$b = \frac{\lambda V_s}{Q_0} = 4$$

$$\left.\begin{array}{l} S = 600\ m^2 \\ \varphi = 0{,}5 \\ V_s = 25\ m^3 \\ Q_0 = 0{,}076\ m^3 hr^{-1} \end{array}\right\} \text{Zang et al. [2009]}, \quad E = 0{,}51 \quad DR = 0{,}81$$



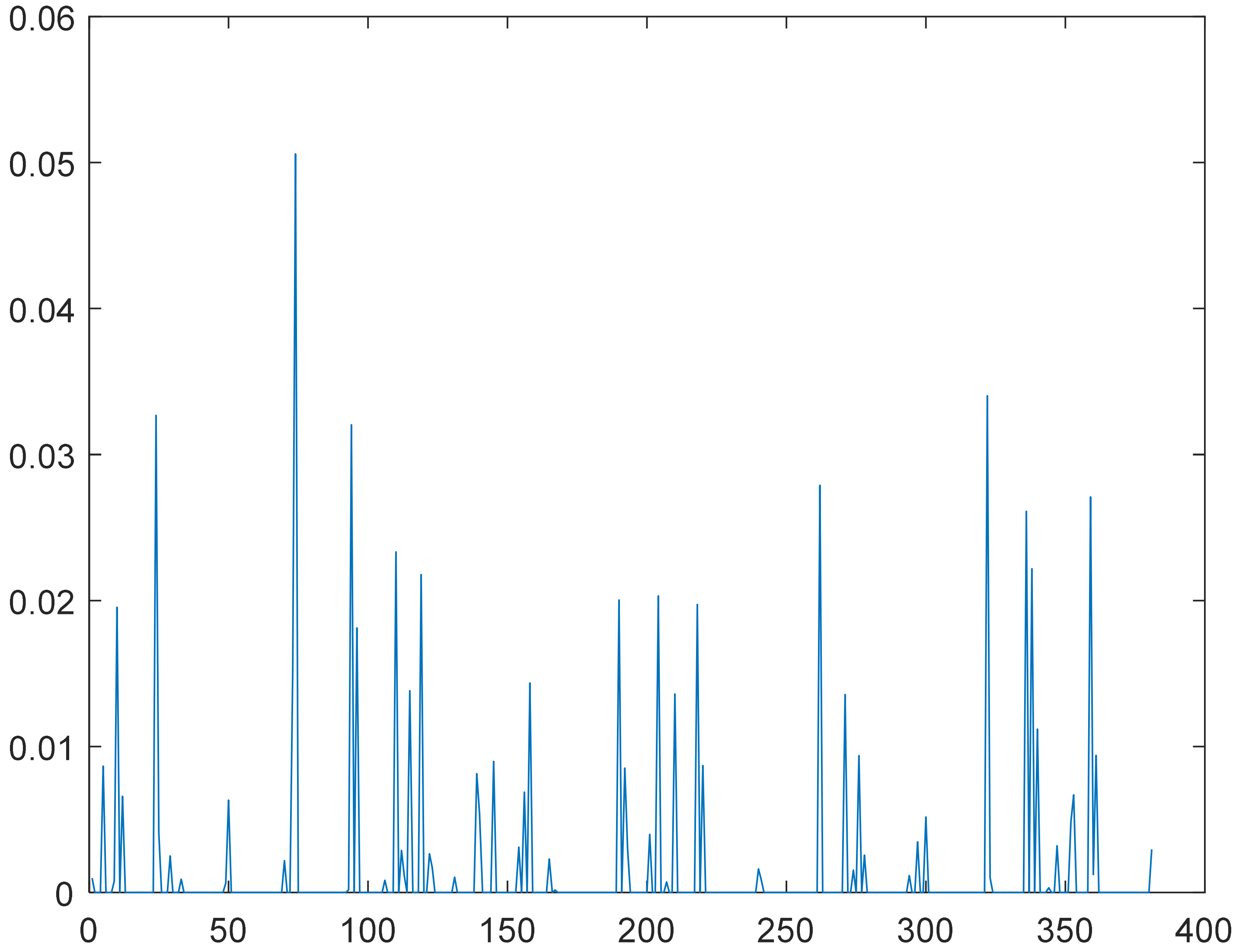
$$\Rightarrow \begin{cases} R_i = 0{,}67 \Rightarrow E = 0{,}33 \quad DR = 0{,}16 \\ R'_i = 0{,}02 \Rightarrow E' = 0{,}98 \quad DR = 2{,}01 \end{cases}$$

```
clear all
%raifall
lambda=0.012*24; %g^-1
zeta=96; %m^-1
%tempo
Nanni=1;
Ngiorni=365;
%allocate memory
Prec=[];
Dryspell=[];
VV=[];
Overflow=[];
Empty=[];

Prec=zeros(1,Nanni*Ngiorni);
VV=zeros(1,Nanni*Ngiorni);
Overflow=zeros(1,Nanni*Ngiorni);
Empty=zeros(1,Nanni*Ngiorni);
```

```
% variables
phi=0.5;
S=600.; %m2
Q0=0.076*24; %m3/g
% storage capacity
Vs=25; %m3
```

```matlab
hpr=-log(rand)/zeta; %m
i=1;
VV(i)=0.;
Prec(i)=hpr;
ii=0;
while i<(Nanni*Ngiorni)
inter=-log(rand)/lambda; %hr
ii=ii+1;
Dryspell(ii)=inter;
ninter=int32(inter);
Prec(i+1:ninter)=0;
hpr=-log(rand)/zeta; %m
i=i+ninter+1;
Prec(i+ninter+1)=hpr;
end
figure(1)
plot(Prec)
```


0.06
0.05
0.04
0.03
0.02
0.01
0
0
50
100
150
200
250
300
350
400

```
Nanni=100;
Ngiorni=365;
for t=1:Nanni*Ngiorni-1
    VV(t+1)=VV(t)-Q0*1+phi*S*Prec(t);
    if VV(t+1) >=Vs
        Overflow(t+1)=1;
        VV(t+1)=Vs;
    end
    if VV(t+1) <=0
        Empty(t+1)=1;
        VV(t+1)=0;
    end
end

Roverflow=mean(Overflow)
RWscarcity=mean(Empty)
Efficiency=1-mean(Empty)
```

Roverflow = 2.7397e-04

RWscarcity = 0.7368

Efficiency = 0.2632

Nanni=1000;

Efficiency =0.276

$$a = \frac{V_s \zeta}{\phi S}$$

$$b = \frac{\lambda V_s}{Q_0}$$

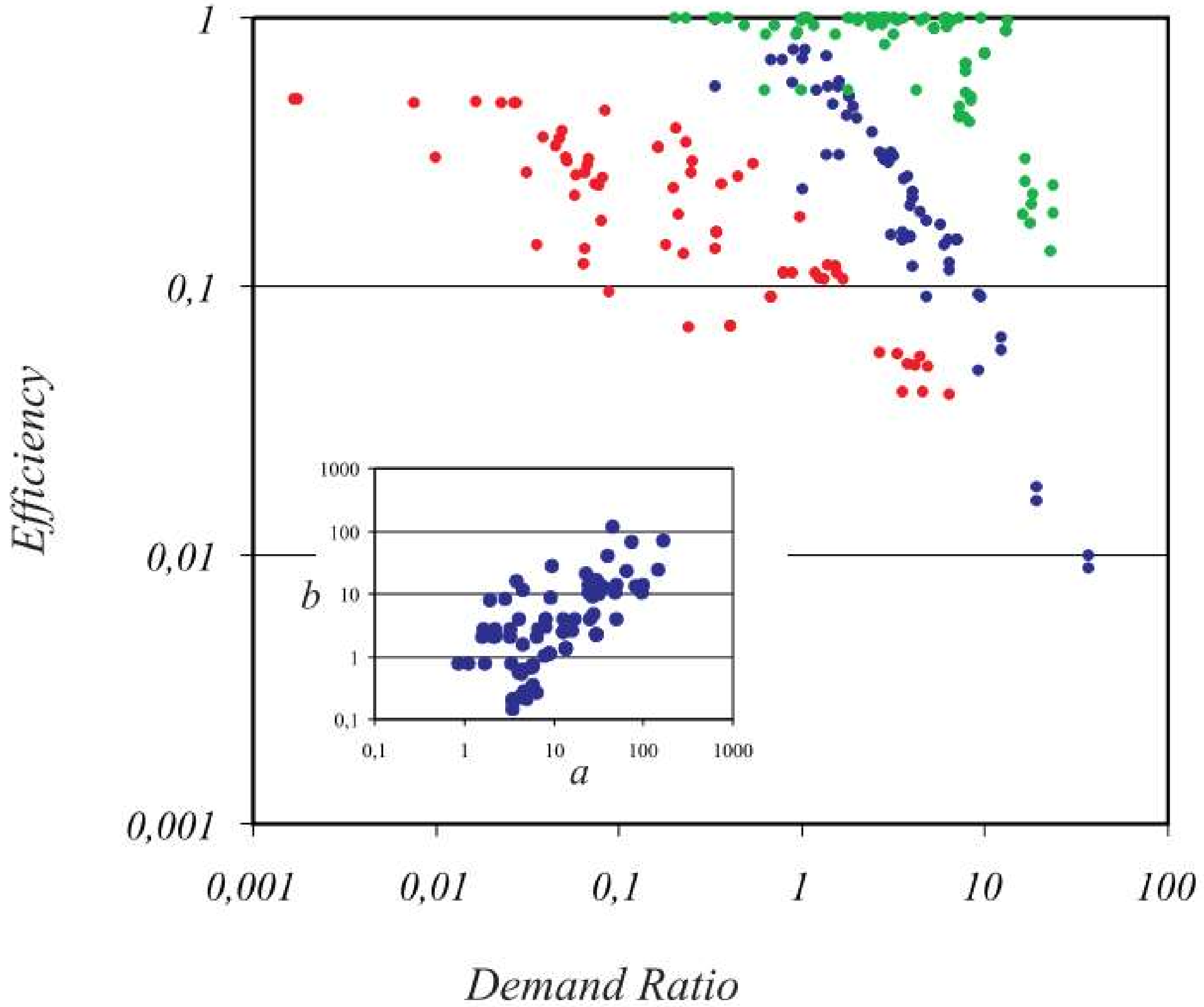
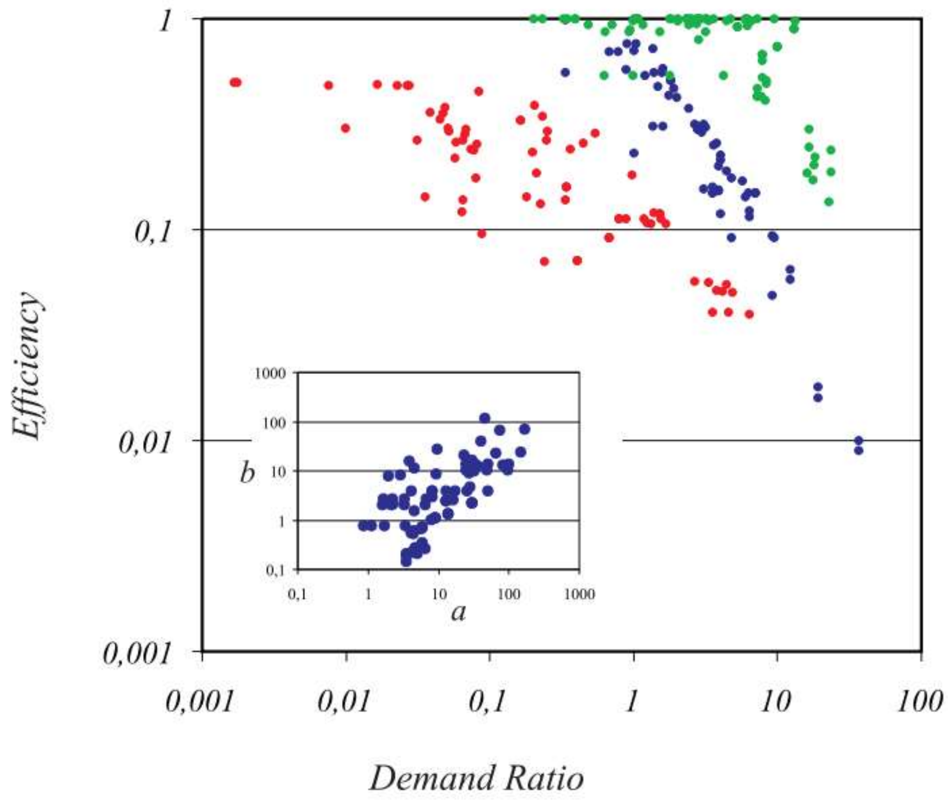


Figure 3. Efficiency vs- Demand Ratio. Blue: data taken from literature [14]. Red: Efficiency E and Demand Ratio DR are estimated under the most conservative assumptions. Green: Efficiency E' and Demand Ratio DR' are estimated under the least conservative assumptions. Inset: corresponding a and b, estimated from literature data.

Probability Analysis (PA) is computationally inexpensive and is useful instrument for quick cost benefit analysis and decision making

Upper and lower limits of Efficiency, estimated by PA, demonstrate a threshold behavior and clarify that the threshold behavior may be achieved if tank storage capacity is designed to match the average collected volume and the average reuse volume.



$$a = \frac{V_s \zeta}{\phi S} \qquad b = \frac{\lambda V_s}{Q_0}$$

$$\frac{a}{b} = \frac{Q_0}{\lambda} \frac{\zeta}{\phi S} = \frac{\text{expected demand during dry spell}}{\text{expected collected volume}}$$

The low demand limit cannot be achieved by increasing the tank size in systems characterized by a disproportion between harvesting and demand volume.

SE LA VASCA DI LAMINAZIONE FUNZIONA IN PARALLELO AD UNA STAZIONE DI SOLLEVAMENTO, E' NECESSARIO VALUTARE IL RISCHIO DI FALLANZA DI SISTEMI COMPOSTI

$$P_1 = \frac{\mu}{\mu + \lambda} = a = \text{affidabilità di un elemento riparabile}$$

$$P_2 = \frac{\lambda}{\mu + \lambda} = f = \text{probabilità di fallanza dell'elemento}$$

Si voglia valutare l’affidabilità A di un sistema composto da due elementi riparabili uguali, la cui affidabilità sia a.

ELEMENTI IN SERIE

Se i due elementi sono entrambi indispensabili per il corretto funzionamento del sistema.

La probabilità che il sistema funzioni è pari alla probabilità che tutti i suoi elementi si trovino in funzione

$$A = a \cdot a$$

La probabilità che il sistema si trovi in fallanza è pari alla probabilità che uno o tutti e due i suoi elementi si trovino in fallanza

$$1 - A = 2af + f^{2}$$

ELEMENTI IN PARALLELO

Se i due elementi non sono entrambi indispensabili per il corretto funzionamento del sistema (un elemento è ridondante).

La probabilità che il sistema funzioni è pari alla probabilità che uno o due elementi si trovino in funzione

$$A = a^2 + 2af$$

La probabilità che il sistema si trovi in fallanza è pari alla probabilità tutti e due i suoi elementi si trovino in fallanza

$$1 - A = f^2$$

$$\binom{n}{i} = \frac{n!}{(n-k)! \cdot k!}$$

Generalizzando...

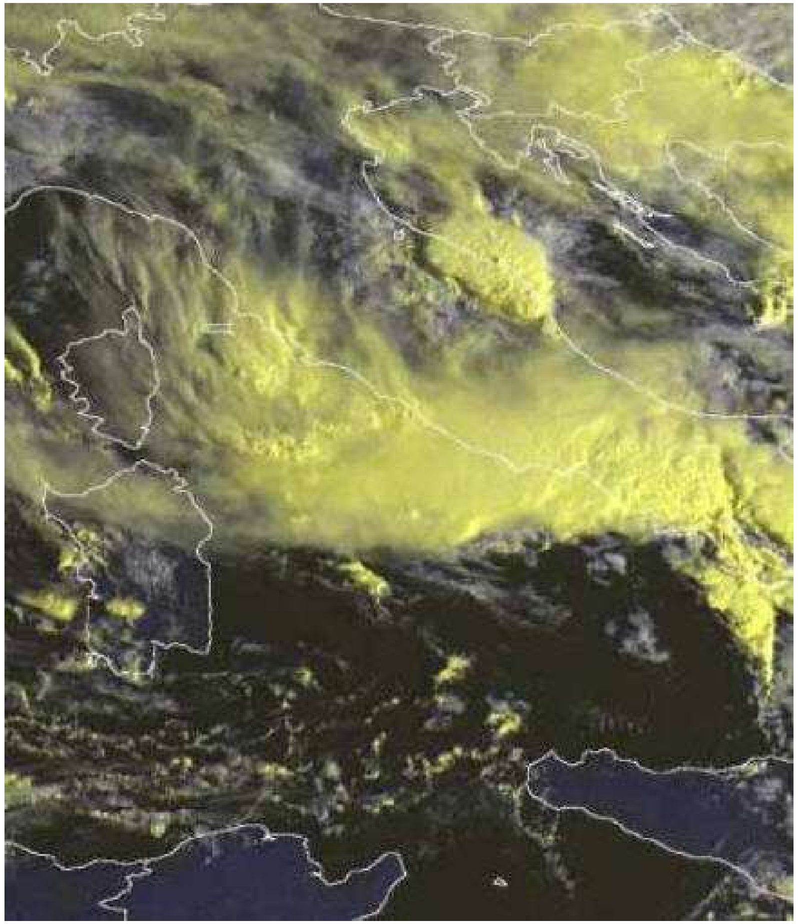
L'affidabilità di un sistema composto da n elementi di cui m necessari e n-m ridondanti è

$$A = \sum_{i=n-m}^{n} \binom{n}{i} \cdot a^i f^{(n-i)} \Rightarrow 1 - A = \sum_{i=1}^{n-m-1} \binom{n}{i} \cdot a^i f^{(n-i)}$$

Se m=0 (elementi in serie)

$$A = a^n \Rightarrow 1 - A = 1 - a^n$$

$\binom{n}{i} \cdot a^i f^{(n-i)}$ = probabilità che i elementi funzionino e (n - i) no



IL RISCHIO È DOVUTO A:

• ALEATORIETÀ IDROLOGICA

ED INCERTEZZA LEGATA A:

• MODELLAZIONE MATEMATICA

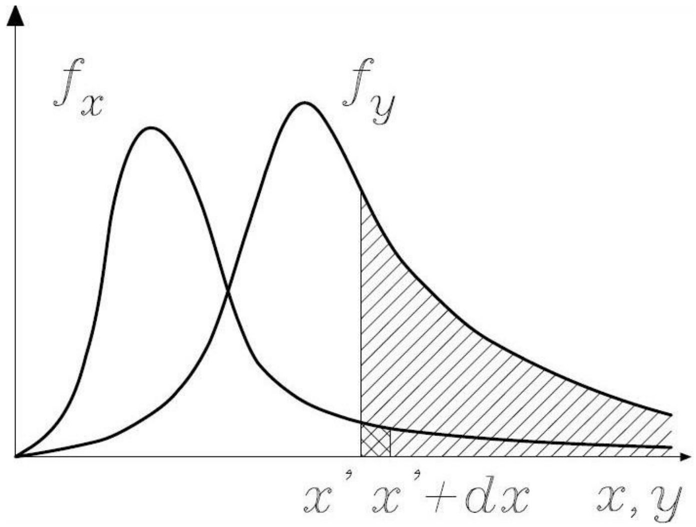
• REALIZZAZIONE DELLE OPERE CIVILI

• FUNZIONAMENTO DELLE COMPONENTI MECCANICHE

MODELLI A CARICO E RESISTENZA ALEATORI

Carico → x

Resistenza → y



$$f_y(y)\,dy = P[y]$$

$$\int_y^{\infty} f_x(x)\,dx = P[x > y]$$

$$R = \int_0^{\infty} f_y(y) \underbrace{\int_y^{\infty} f_x(x)\,dx}_{P[x>y]}\,dy$$

Un esempio di applicazione: il caso delle stazioni di sollevamento per fognatura.

Consideriamo la stazione di sollevamento per fognatura come un sistema complesso, composto da n elementi (pompe) in parallelo, capaci di allontanare ciascuna una portata Q.

- Quando tutte le pompe funzionano, la portata complessiva che la stazione è in grado di allontanare è nQ
- Se ci sono k pompe in fallanza la stazione è in grado di allontanare la portata (n-k)Q

a = affidabilità di una pompa
= probabilità che sia in funzione quando piove

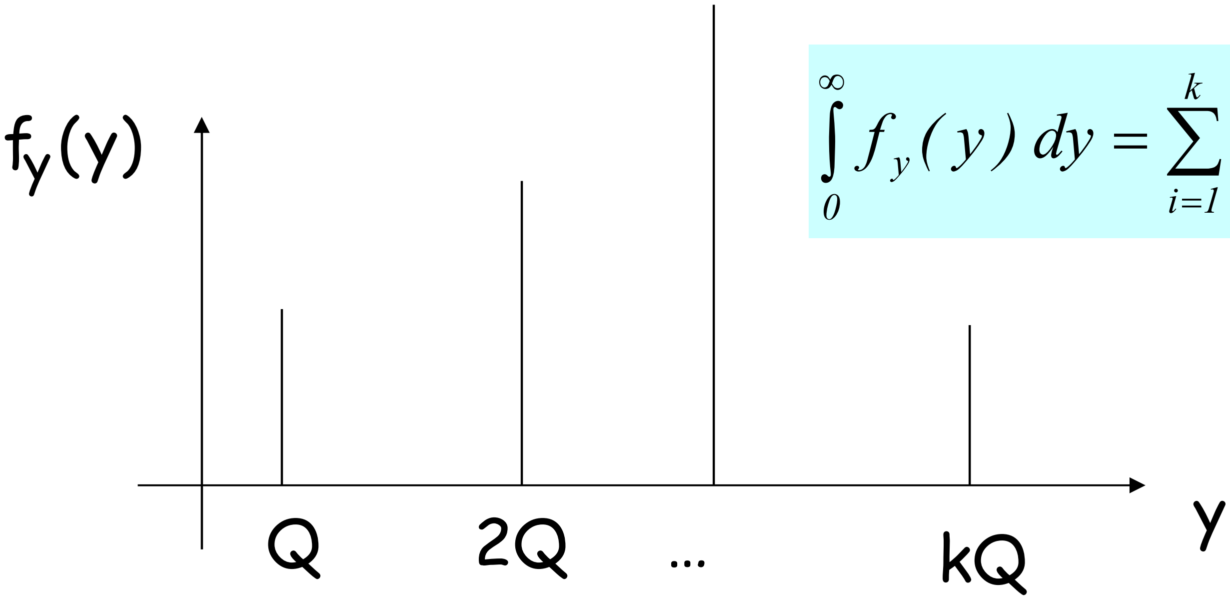
$$a = P[\, y = Q\,] = f_y(\, y\,)dy\Big|_{y=Q}$$

$$\binom{n}{n-k} a^{n-k}(1-a)^k = P[\, y = (n-k)Q\,] = f_y(\, y\,)dy\Big|_{y=(n-k)Q}$$

Distribuzione binomiale

$$f_y(y) = \binom{n}{n-k} a^{n-k}(1-a)^k \delta\left[(n-k)Q\right]$$

$$\delta(\kappa) = \begin{matrix} 0 & y \neq \kappa \\ 1 & y = \kappa \end{matrix}$$



$$\int_0^{\infty} f_y(y)\,dy = \sum_{i=1}^{k} \binom{n}{k} a^{n-k}(1-a)^k = 1$$

Distribuzione di probabilità del carico di progetto

$f_h(h) =$ densità di probabilità del'altezza di pioggia

$\int_0^h f_h(\tilde{h})\, d\tilde{h} = F_h(h) =$ probabilità di non superamento di h

$$Q = \frac{\varphi S h}{T_c} \Rightarrow h = \frac{T_c}{\varphi S} Q \Rightarrow dh = \frac{T_c}{\varphi S} dQ$$

$$\frac{T_c}{\varphi S} \int_0^{\frac{T_c}{\varphi S} Q} f_h\left(\frac{T_c \tilde{Q}}{\varphi S}\right) d\tilde{Q} = \int_0^Q f_Q(\tilde{Q})\, d\tilde{Q} = F_Q(Q)$$

$h =$ altezza di precipitazione aleatoria

S = superficie del bacino afferente alla stazione

$T_c =$ tempo di corrivazione

$\varphi =$ coefficiente di deflusso

Affidabilità della stazione di sollevamento

$$A = \int_0^{\infty} \binom{i}{n} a^i (1-a)^{n-i} \delta(iQ) \int_0^{iQ} f_Q(\tilde{Q})\, d\tilde{Q}\, dy =$$

$$= \sum_{i=0}^{n} \binom{i}{n} a^i (1-a)^{n-i} \int_0^{iQ} f_Q(\tilde{Q})\, d\tilde{Q}$$

$$\ldots$$

$$= \int_0^{\infty} f_Q(\tilde{Q}) \int_{\tilde{Q}}^{\infty} \delta(iQ) \binom{i}{n} a^i (1-a)^{n-i}\, dy\, d\tilde{Q}$$

$$\sum_{i=k}^{n} \binom{i}{n} a^i (1-a)^{n-i} \int_{(k-1)Q}^{kQ} f_Q(\tilde{Q})\, d\tilde{Q}$$

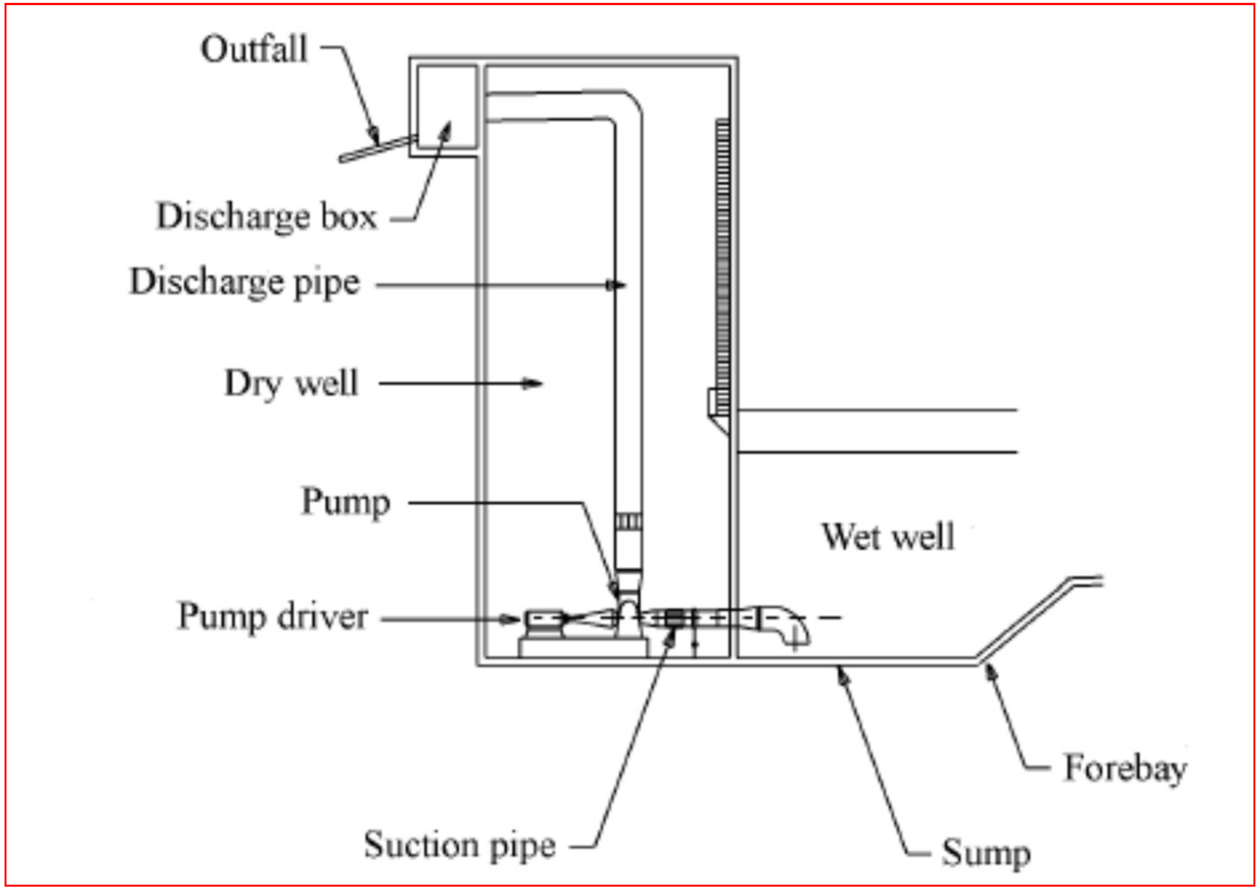
Dynamic reliability model

$$A = \int_0^{\infty} f_y(y) \int_0^{y} f_x(x)\, dx\, dy$$

$$A = \begin{cases} \displaystyle\sum_{i=0}^{N} \left[\binom{N}{i} a^i (1-a)^{N-i} \int_0^{i \cdot Q_p} p_Q(Q)\, dQ \right] = \\ \displaystyle\int_0^{NQ_P} p_Q(Q) \sum_{i=INT\left[\frac{Q}{Q_P}\right]}^{N} \binom{N}{i} a^i (1-a)^{N-i}\, dQ \end{cases}$$

COMBINED EFFECTS OF MECHANICAL AVAILABILITY AND STORAGE CAPACITY IN PUMPING STATION RISK ASSESSMENT

PUMP STATION DESIGN



MINIMUM VOLUME OF WET WELL

Allowable Minimum Cycling Time (t)

Design cycling times must exceed manufacturer's specified cycling times

The allowable minimum cycling time is dependent on the specific characteristics of the pump.

Manufacturers provide specifications for the maximum number of starts per hour which translates to a minimum cycling time for individual pumps.

$$t = \frac{4V}{Q_p}$$

Q_p = pumping rate of individual pump

V = cycle volume of pump

7.2.9 Pump Cycling Time

7.2.9.1 First Pump

The following provides proof that, for a given pump with a capacity, Q_p, the cycling time will be a minimum when the inflow, Q_i, is equal to half of the pump capacity.

Assumptions:

- When pumping, the pump rate is constant and at pump capacity, Q_p
- The worst condition considered is a constant inflow rate, Q_i
- No pumping during filling cycle

The time to empty storage volume, V, is:

$$t = \frac{V}{Q_p - Q_i} \tag{7-1}$$

The time to fill the storage system (no pumping) is:

$$t = \frac{V}{Q_i} \tag{7-2}$$

If Q_i is expressed as a multiple of Q_p, $Q_i = xQ_p$, then the total cycle time is:

$$t = \frac{V}{Q_p - xQ_p} + \frac{V}{xQ_p} \tag{7-3}$$

For the minimum value of t,

$$\frac{dt}{dx} = -\frac{VQ_p}{\left(Q_p - xQ_p\right)^2} - \frac{V}{x^2 Q_p} = 0$$

Rearranging and dividing by V gives:

$$Q_p{}^2 x^2 + (Q_p - Q_p x)^2 = 0$$

For which,

$$x = 0.5$$

Thus,

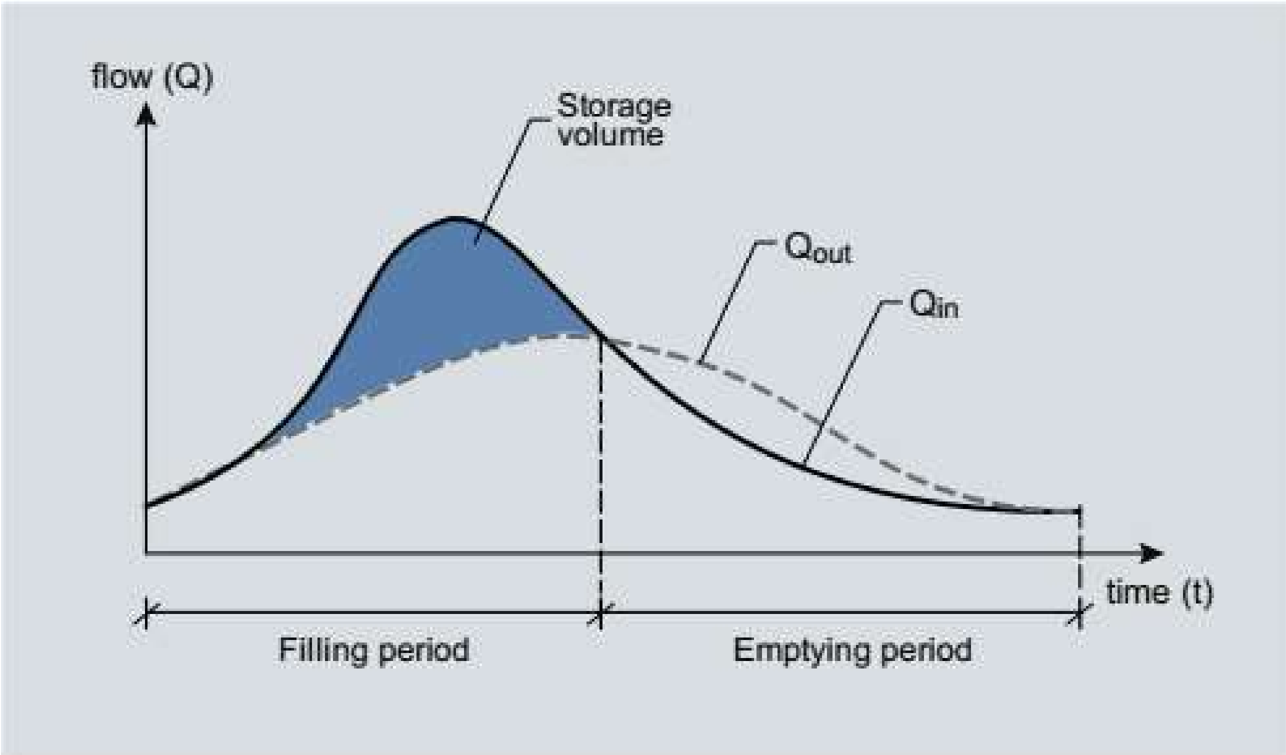
$$Q_i = 0.5Q_p$$

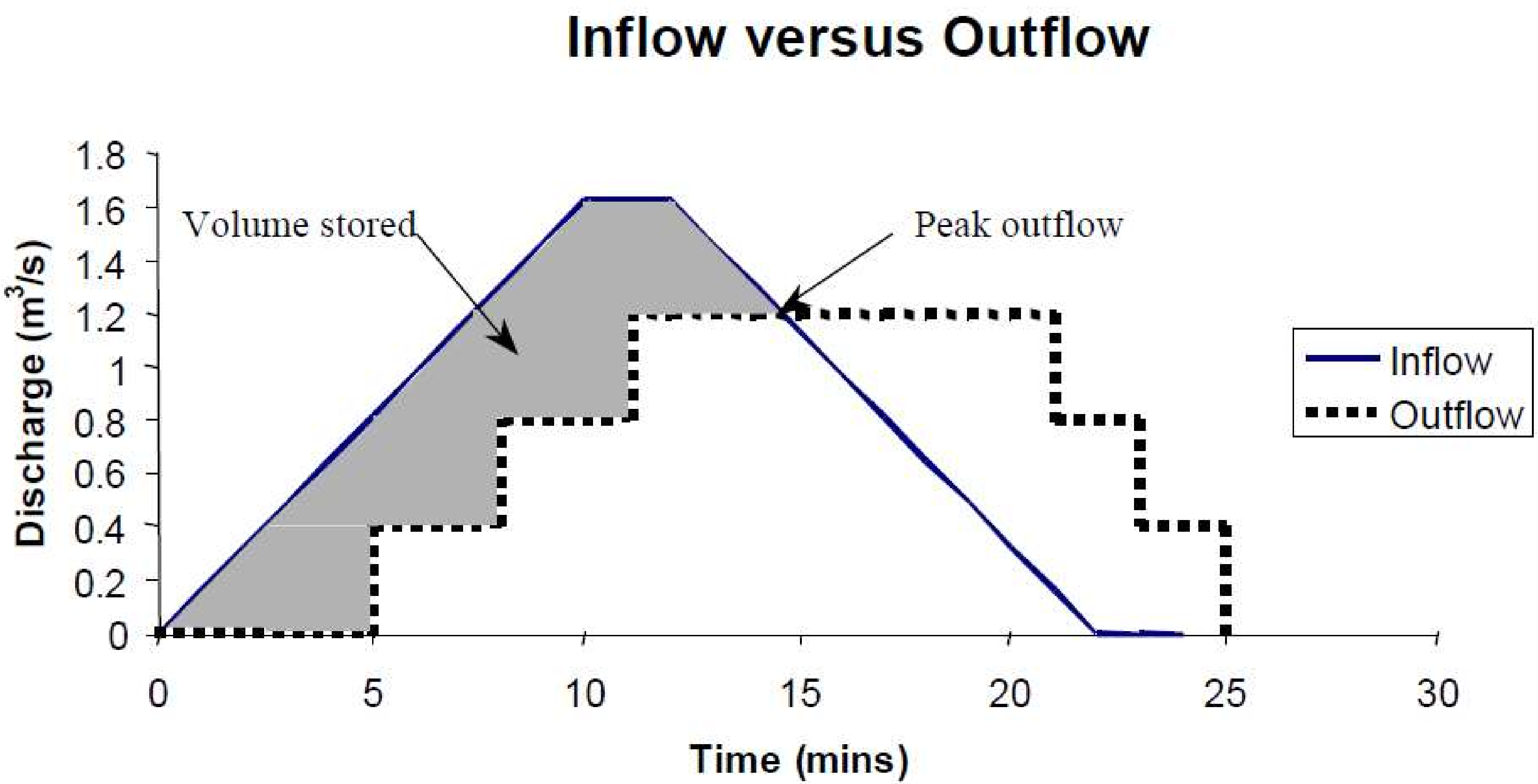
So, for the minimum cycle time, t, substituting for Q_i in Equation 7-1 gives:

$$\boxed{t = \frac{4V}{Q_p}} \tag{7-4}$$

The rate of outflow (Qout)
Is the difference between the rate of inflow (Qin)
And the rate of change of volume stored (dV/dt)

$$Q_{out} = Q_{in} - \frac{dV}{dt}$$





Inflow versus Outflow
Volume stored
Peak outflow
Inflow
Outflow
Discharge (m3/s)
Time (mins)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
5
10
15
20
25
30

The peack pumping capacity requirements
reduces with increasing storage

The total available storage comprehends
the collection system storage
The storage unit and the wet well storage capacity
Between the lowest pump operation elevation
And maximum allowable high water

An optimal condition balances the storage provided
with the size and number of pumps

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Risk analysis of combined rainwater detention and pumping systems

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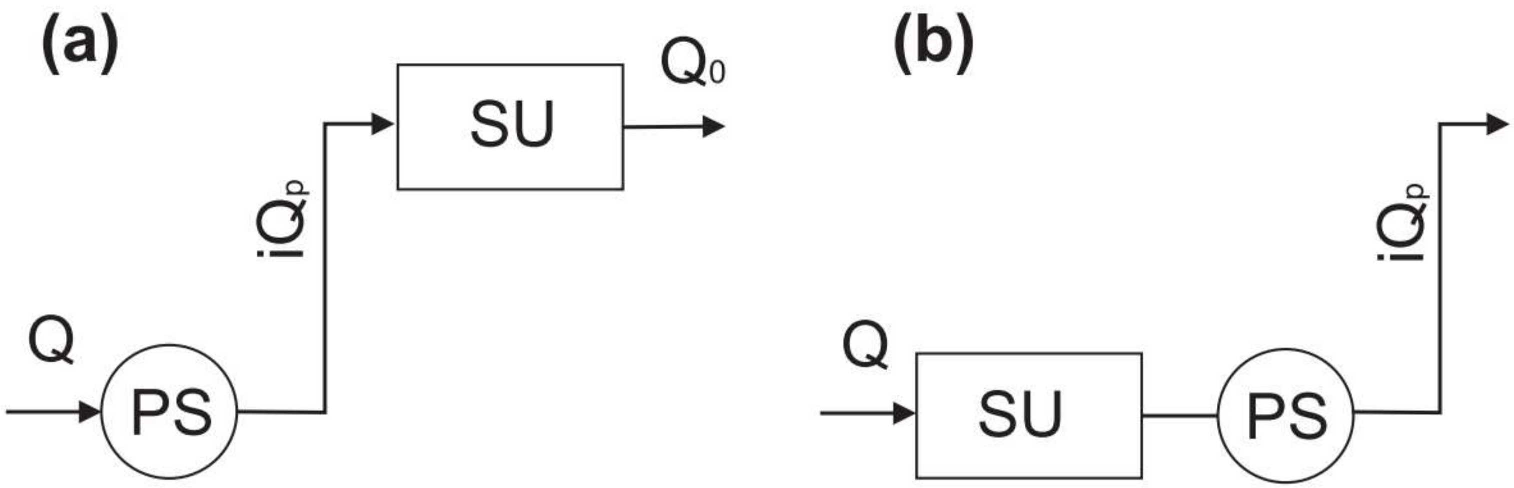
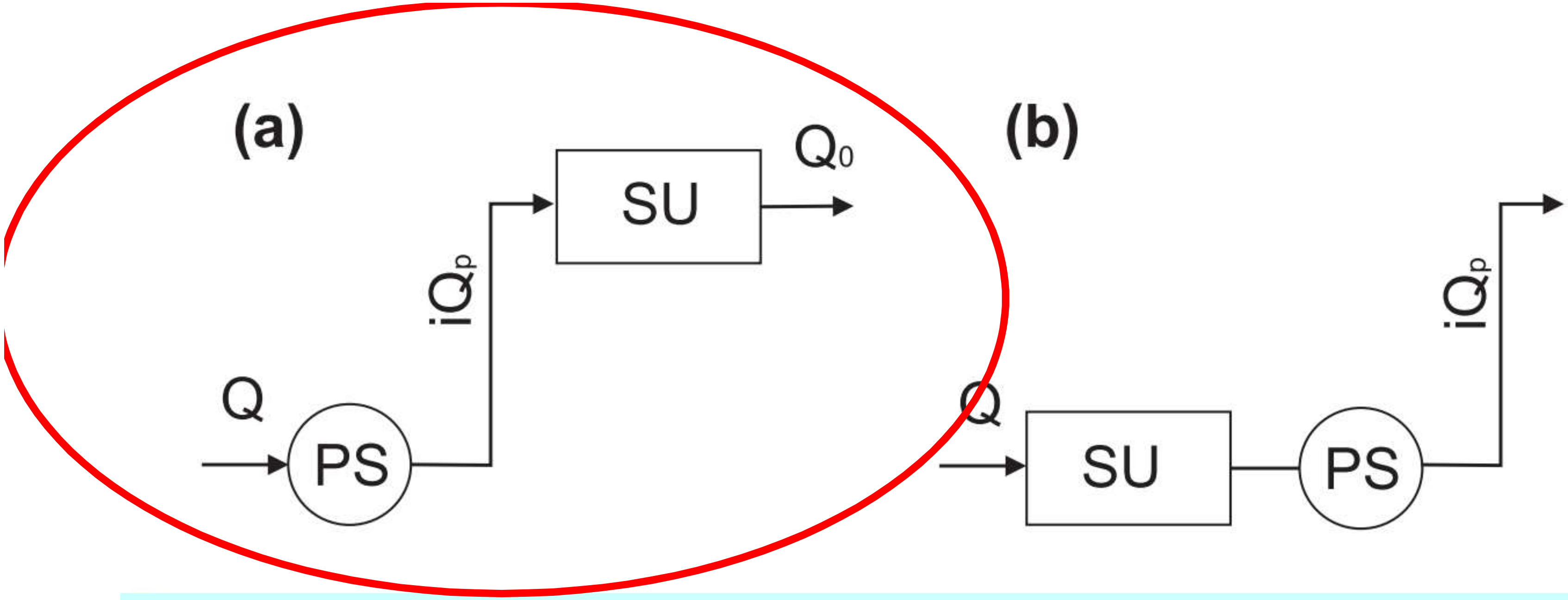


Figure 1. Conceptual model of low impact structural measures for storm sewer system (SSS). SU = storage unit, PS = pumping station.

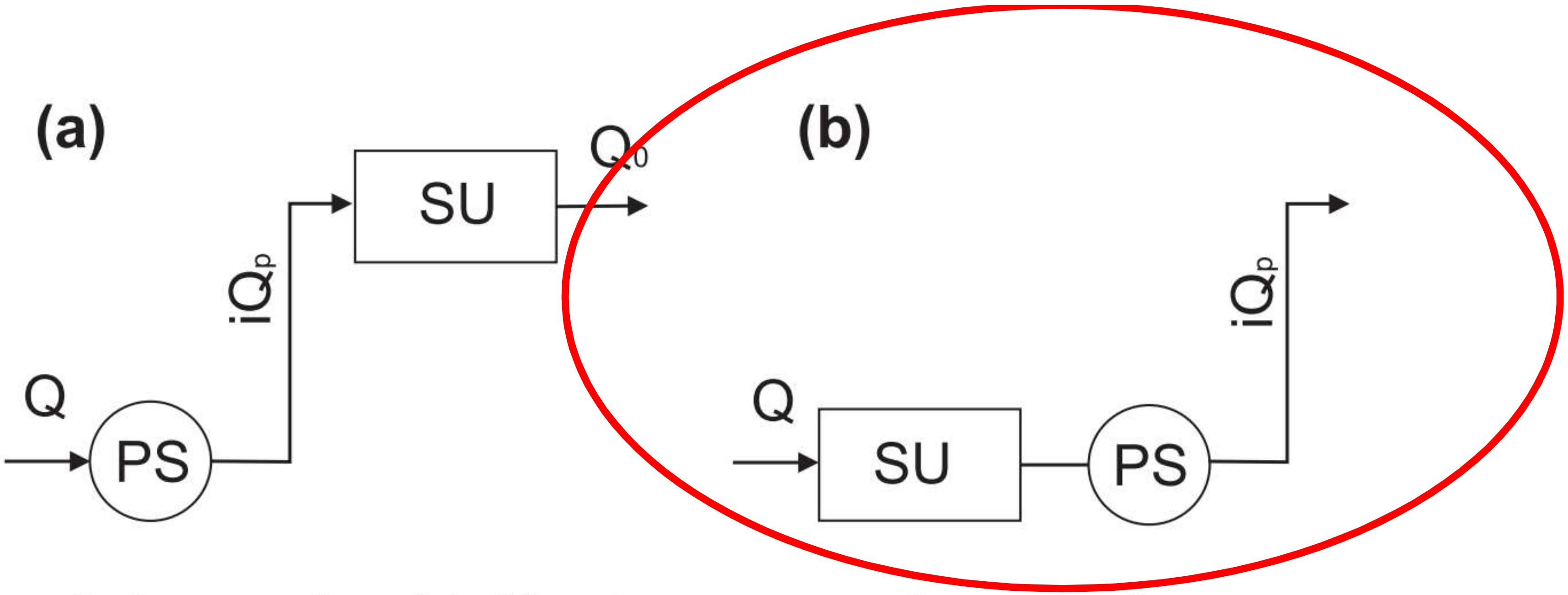
SU e PS sono elementi in serie la fallanza del sistema è legata alla fallanza di uno o entrambi gli elementi e viene calcolata come probabilità complementare all'affidabilità del sistema



$$R = 1 - \int_0^\infty f_Q \left\{ \sum_{i=1}^{INT\left[\frac{Q}{Q_P}+1\right]} P[iQ_P] \cdot A_{PS} \cdot A_{SU} \right\} dQ$$

$$A_{PS} = P[Q < iQ_P]$$

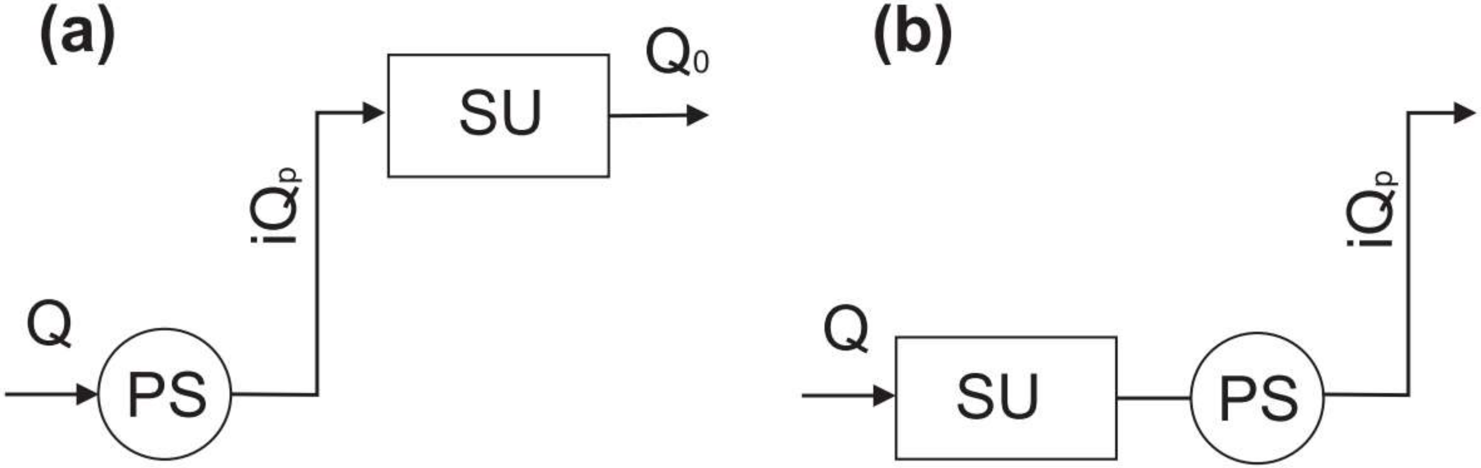
$$A_{SU} = P\left[t \geq \frac{V_s}{Q_0}\right] \cdot P\left[\tau < \frac{V_s}{iQ_P - Q_0}\right] + P\left[t < \frac{V_s}{Q_0}\right] \cdot P\left[\tau < \frac{t \cdot Q_0}{iQ_P - Q_0}\right]$$



$$R = 1 - \int_0^{\infty} f_Q \left\{ \sum_{i=1}^{n} P[iQ_P] \cdot A_{SU} \right\} dQ$$

$$A_{SU} = P\left[t \geq \frac{V_s}{iQ_P}\right] \cdot P\left[\tau < \frac{V_s}{Q - iQ_P}\right] + P\left[t < \frac{V_s}{iQ_P}\right] \cdot P\left[\tau < \frac{t \cdot iQ_P}{Q - iQ_P}\right]$$

PS non ha capacità di immagazzinamento, la sua fallanza determina il mancato o parziale vuotamento della SU



$$P[Q] = \begin{matrix} P\left[\dfrac{\phi S h}{\tau}\right] \\ \\ P\left[\phi S j\right] \end{matrix} \qquad h, \tau, j \quad \text{aleatorie}$$

$$f_h = \zeta\, e^{-\zeta h} \tag{1}$$

$$f_\tau = \lambda\, e^{-\lambda \tau} \tag{2}$$

$$f_t = \psi\, e^{-\psi t} \tag{3}$$

In equation (1) ζ is the inverse of the expected value of rainfall depth, in equation (2) λ is that of rainfall duration and in equation (3) ψ is that of the expected value of rainfall inter-arrival.

Table 1. Risk model. Dimensionless groups.

Parameter	definition
k_{ζ}	$\frac{\zeta\ V_s}{\varphi\ S}$
k_{λ}	$\frac{\lambda\ V_s}{i\ Q_p}$
$k_{\psi,0}$	$\frac{\psi\ V_s}{Q_0}$
$k_{T,a}$	$\frac{IETD\ Q_0}{V_s}$
$k_{T,b}$	$\frac{IETD\ i\ Q_p}{V_s}$
k_{ψ}	$\frac{\psi\ V_s}{i\ Q_p}$
k_{λ,T_d}	$\frac{\lambda Q_0 T_d}{iQ_p}$
k_{ζ,T_d}	$\frac{\zeta\, iQ_p T_d}{\varphi S}$

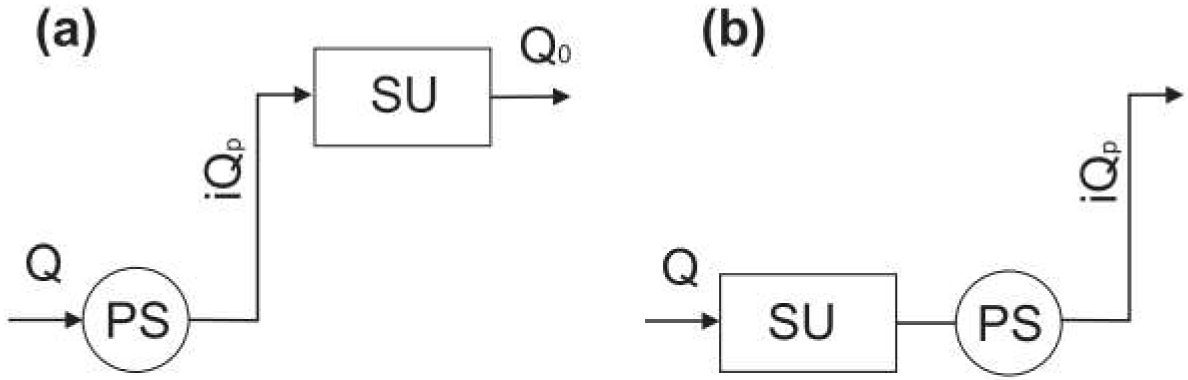
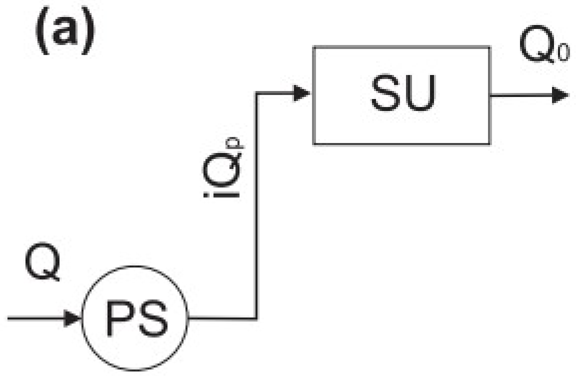


Figure 1. Conceptual model of low impact structural measures for storm sewer system (SSS). SU = storage unit, PS = pumping station.

Assuming that the sorage unit (SU) is full at the end of the first of two consecutive rain events, the risk of overflow for storage unit above ground downstream the pumping station (PS) is

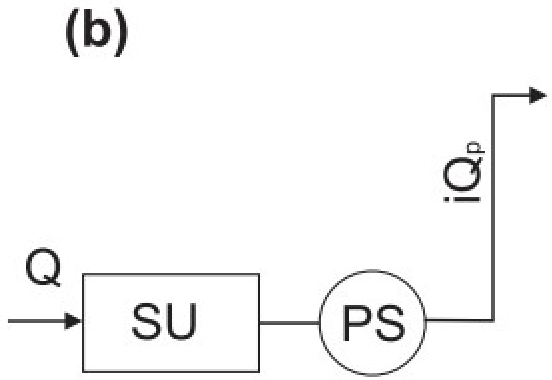


$$R_a = \frac{k_\lambda}{k_\zeta + k_\lambda} + \frac{k_{\psi,0} \cdot e^{-k_{T,a}\left(k_\lambda + k_{\psi,0}\right)} - k_\lambda \cdot e^{-\left(k_\lambda + k_{\psi,0}\right)}}{k_\lambda + k_{\psi,0}} + \frac{k_{\psi,0} \cdot e^{-k_{T,a}\left(k_\zeta + k_\lambda + k_{\psi,0}\right)} - k_\lambda \cdot e^{-\left(k_\zeta + k_\lambda + k_{\psi,0}\right)}}{k_\zeta + k_\lambda + k_{\psi,0}}$$

Assuming that the SU is empty at the beginning of any rain event, the risk of overflow for storage unit above ground downstream the PS is

$$R'_a = \frac{k_\lambda}{k_\zeta + k_\lambda} + e^{-k_\lambda} - \frac{k_\lambda}{k_\zeta + k_\lambda} \cdot e^{-\left(k_\zeta + k_\lambda\right)}$$

Assuming that the sorage unit (SU) is full at the end of the first of two consecutive rain events, the risk of overflow for storage unit below ground upstream the pumping station (PS) is



$$R_b = \frac{k_\psi}{k_\zeta + k_\psi} \cdot e^{-k_{T,b}(k_\zeta + k_\psi)} + \frac{k_\zeta}{k_\zeta + k_\psi} \cdot e^{-(k_\zeta + k_\psi)}$$

Assuming that the SU is empty at the beginning of any rain event, the risk of overflow for storage unit below ground upstream the PS is

$$R'_b = e^{-k_\zeta}$$

The PS is modeled as a complex system with n identical components (pumps), which are connected in parallel, each pump having discharge capacity Q_p and reliability a, given by equation (4). Backup capacity is provided by r additional pumps with capacity Q_p and reliability a, which are redundant in standby. The probability that $n + r - i$ of $n + r$ pumps are not working at time t and thus, that the discharge capacity of the PS is $i\ Q_p$ may be expressed by a binomial distribution (Ursino and Salandin, 2014), according to equation (6):

$$P_i = \binom{n+r}{i} a^i \cdot f^{n+r-i} \tag{6}$$

where $i = 0 \div n$.

Probability $P(\theta)$ that the SSS does not perform its task in stationary conditions is obtained by combining the risks of overflow, too short detention times within the SU, and PS failure, as:

$$P(\theta) = \sum_{i=0}^{n} \binom{n+r}{i} a^i \cdot f^{n+r-i} \cdot R(i) \tag{16}$$

where $R(i)$ is the combined risk of overflow and too short detention time.

Equation (16) can estimate the risk of failure of the SSS whenever rain falls, as a function of the constant parameters: $\theta = [k_\zeta, k_\lambda, k_\psi, k_{T,a}, k_{T,b}, k_{\lambda,T_d}, k_{\zeta,T_d} a, n, r]$.

(1) the annual risk of SSS failure (not shown here),

$$P_N(\theta) = 1 - \left[1 - P(\theta)\right]^N \tag{17}$$

where N is the average number of rainfall events in one year and $1 - P(\theta)^N$ is the annual reliability of the SSS;

(2) the risk of failure of the SSS over its design life M (not shown here):

$$P_M(\theta) = 1 - \left[1 - P_N(\theta)\right]^M \tag{18}$$

MODELLI DI MARKOV
(UN APPROCCIO ALL'UTILIZZO DI TASSI COSTANTI DI FALLANZA E RIPARAZIONE)

The pumping station is here modelled as a complex system with *N* components all equal and connected in parallel.

$$P(y = iQ_P) = \binom{i}{N} a^i f^{N-i} \qquad a = \frac{\mu}{\lambda + \mu}$$

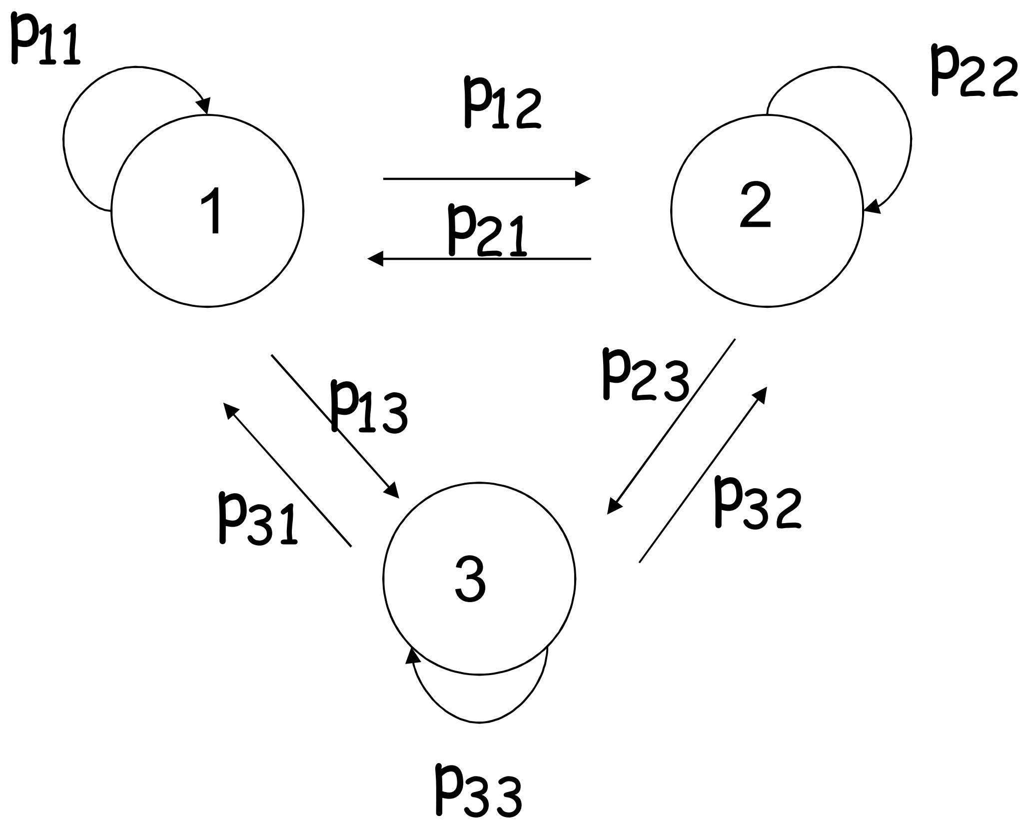
When $\lambda(t)$ and $\mu(t)$ are time independent, the reliability of each pumping unit may be evaluated as the steady state solution of a **Markov process** with constant rates according to the following expression [Henley & Kuamamoto, 1992].

Un sistema al quale può essere applicato un modello di Markov presenta le seguenti caratteristiche:

•può trovarsi in un numero finito (n) di stati

•passa da uno stato (i) ad uno stato (j) con probabilità data nell'unità di tempo (p_{ij})

DIAGRAMMA DI TRANSIZIONE (n=3)



MATRICE DI TRANSIZIONE

$$P = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{n1} \\ p_{12} & p_{22} & & p_{n2} \\ \vdots & & \ddots & \vdots \\ p_{1n} & p_{2n} & \cdots & p_{nn} \end{bmatrix}$$

p_{ij} = probabilità di passare dallo stato i allo stato j nell'unità di tempo

VETTORE DELLE PROBABILITA' DI STATO

$$\begin{bmatrix} P_1 \\ P_2 \\ \cdots \\ P_n \end{bmatrix}$$

$P_i = P_i(t)$ = probabilità di risiedere nello stato i al tempo t

Stato assorbente: stato con una probabilità nulla di uscirne, ovvero

$$p_{ij} = 0$$

Se v(t) è un vettore di probabilità di stato (iniziale) al tempo t, Pv è il vettore di probabilità di stato al tempo successivo

$$\begin{bmatrix} P_1(t+dt) \\ P_2(t+dt) \\ \vdots \\ P_n(t+dt) \end{bmatrix} = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{n1} \\ p_{12} & p_{22} & & p_{n2} \\ \vdots & & \ddots & \vdots \\ p_{1n} & p_{2n} & \dots & p_{nn} \end{bmatrix} \cdot \begin{bmatrix} P_1(t) \\ P_2(t) \\ \vdots \\ P_n(t) \end{bmatrix}$$

Ovvero,

$$P_i(t+dt) = \sum_{j=1}^{n} p_{ji} P_j(t)$$

$p_{ii} P_i(t) =$ probabilità di essere in i al tempo t e di rimanerci nell'intervallo dt successivo

$p_{ji} P_i(t) =$ probabilità di essere in j al tempo t e di passare in i nell'intervallo dt successivo

Considerando istanti di tempo dt successuvi, a partire da un vettore di probabilità di stato v, si ha che le probabilità di stato sono:

$$P \cdot Pv = P^2 v \quad \text{dopo 2 dt}$$

$$P^3 v \quad \text{dopo 3 dt}$$

$$\vdots$$

$$\mathrm{P}^{\mathrm{n}} v \quad \text{dopo n dt}$$

Il sistema raggiunge una condizione stazionaria quando le probabilità di stato rimangono invariate nel tempo, ovvero è soddisfatta la relazione:

Pv=v

ESEMPIO:

Consideriamo un elemento che può trovarsi in funzione o no.

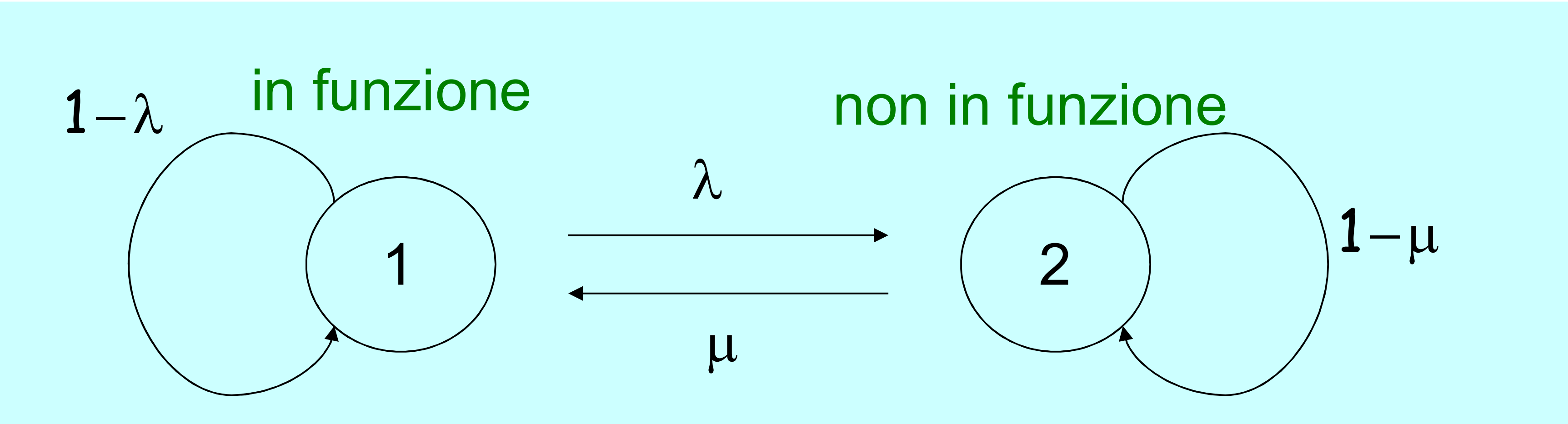
$$n = 2$$

Si rompa inoltre solo per cause aleatorie con tasso costante

$$\lambda = \frac{1}{MTTF}$$

e venga riparato mediamente in un tempo pari all'inverso del suo tasso di riparazione (anche costante)

$$\mu = \frac{1}{MTTR}$$



$$P_1(t+dt) = P_1(t)\cdot(1-\lambda dt) + P_2\cdot\mu dt$$

$$P_2(t+dt) = P_1(t)\cdot\lambda dt + P_2\cdot(1-\mu)dt$$

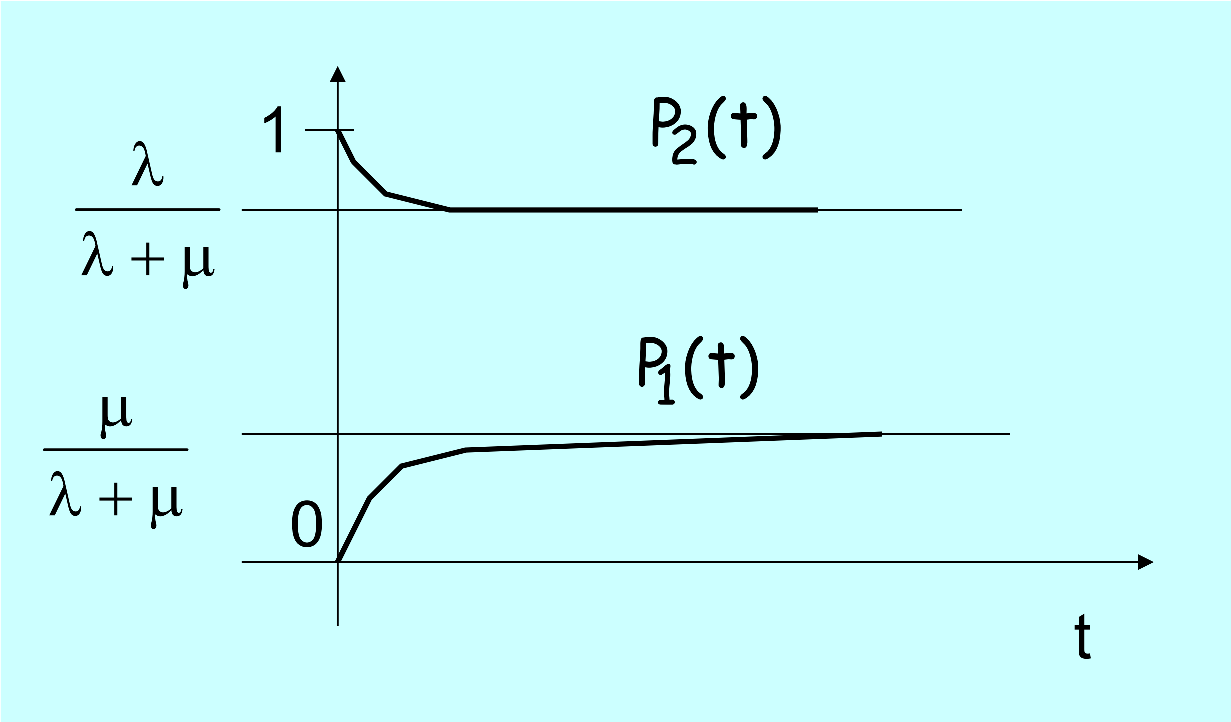
Da cui

$$\frac{dP_1}{dt} = -P_1\cdot\lambda + P_2\cdot\mu$$

$$\frac{dP_2}{dt} = P_1(t)\cdot\lambda - P_2\cdot\mu$$

$$\begin{cases} \dfrac{dP_1}{dt} = -P_1 \cdot \lambda + P_2 \cdot \mu \\ P_1 + P_2 = 1 \end{cases}$$

$$\Rightarrow \begin{aligned} P_2 &= \frac{\lambda}{\lambda + \mu}\left(1 - e^{-(\lambda+\mu)t}\right) \\ P_1 &= 1 - P_2 \end{aligned}$$



La condizione stazionaria può ricavarsi anche ponendo

$$\frac{dP_1}{dt} = 0$$

$$\frac{dP_2}{dt} = 0$$

$$-P_1 \cdot \lambda + P_2 \cdot \mu = 0$$

E quindi

$$P_1(t) \cdot \lambda - P_2 \cdot \mu = 0$$

Che poste a sistema con la

$$P_1 + P_2 = 1$$

Forniscono

$$P_1 = \frac{\mu}{\mu + \lambda}$$

$$P_2 = \frac{\lambda}{\mu + \lambda}$$

Il problema scritto in forma matriciale è il seguente

$$\begin{bmatrix} P_1(t+dt) \\ P_2(t+dt) \end{bmatrix} = \begin{bmatrix} 1-\lambda dt & \mu dt \\ \lambda dt & (1-\mu)dt \end{bmatrix} \cdot \begin{bmatrix} P_1(t) \\ P_2(t) \end{bmatrix}$$

Per dt =1 si ricava la matrice di transizione

$$\begin{bmatrix} 1-\lambda & \mu \\ \lambda & (1-\mu) \end{bmatrix}$$

E da questa la probabilità di stato stazionaria

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 1-\lambda & \mu \\ \lambda & (1-\mu) \end{bmatrix} \cdot \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

Sapendo che $P_1 + P_2 = 1 \Rightarrow$

$$P_1 = \frac{\mu}{\mu+\lambda}$$

$$P_2 = \frac{\lambda}{\mu+\lambda}$$

Per i componenti non riparabili $\mu = 0$

E quindi $$\frac{dP_1}{dt} = -\lambda P_1 \Rightarrow P_1 = e^{-\lambda t}$$

Esiste uno stato assorbente (stato 2), per cui $P_1 \xrightarrow{t \to 0} 0$