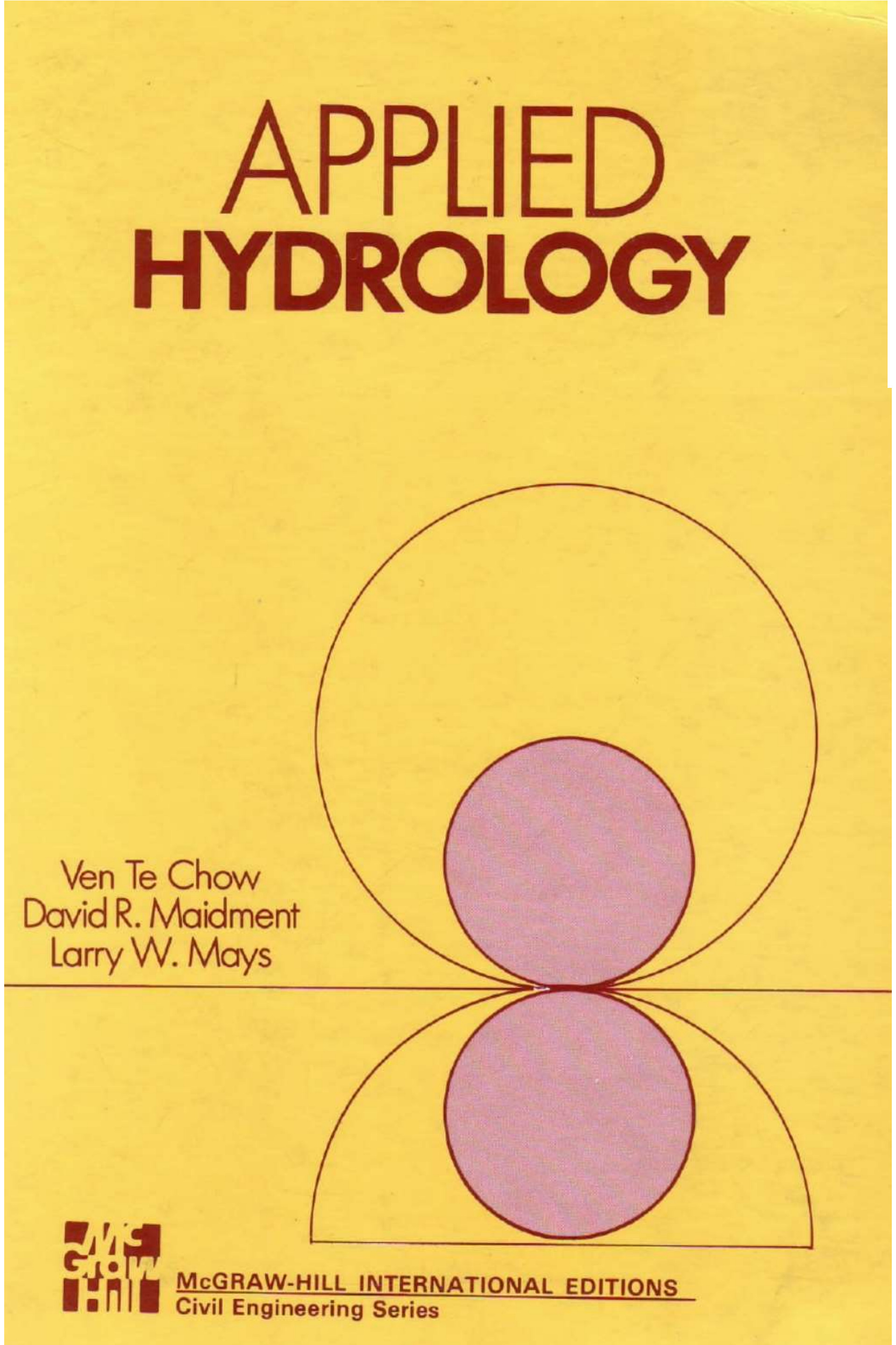


CHAPTER 11

HYDROLOGIC STATISTICS

La probabilità di un evento A, P(A), è una stima della possibilità che si verifichi, quando viene effettuata un osservazione/misura.

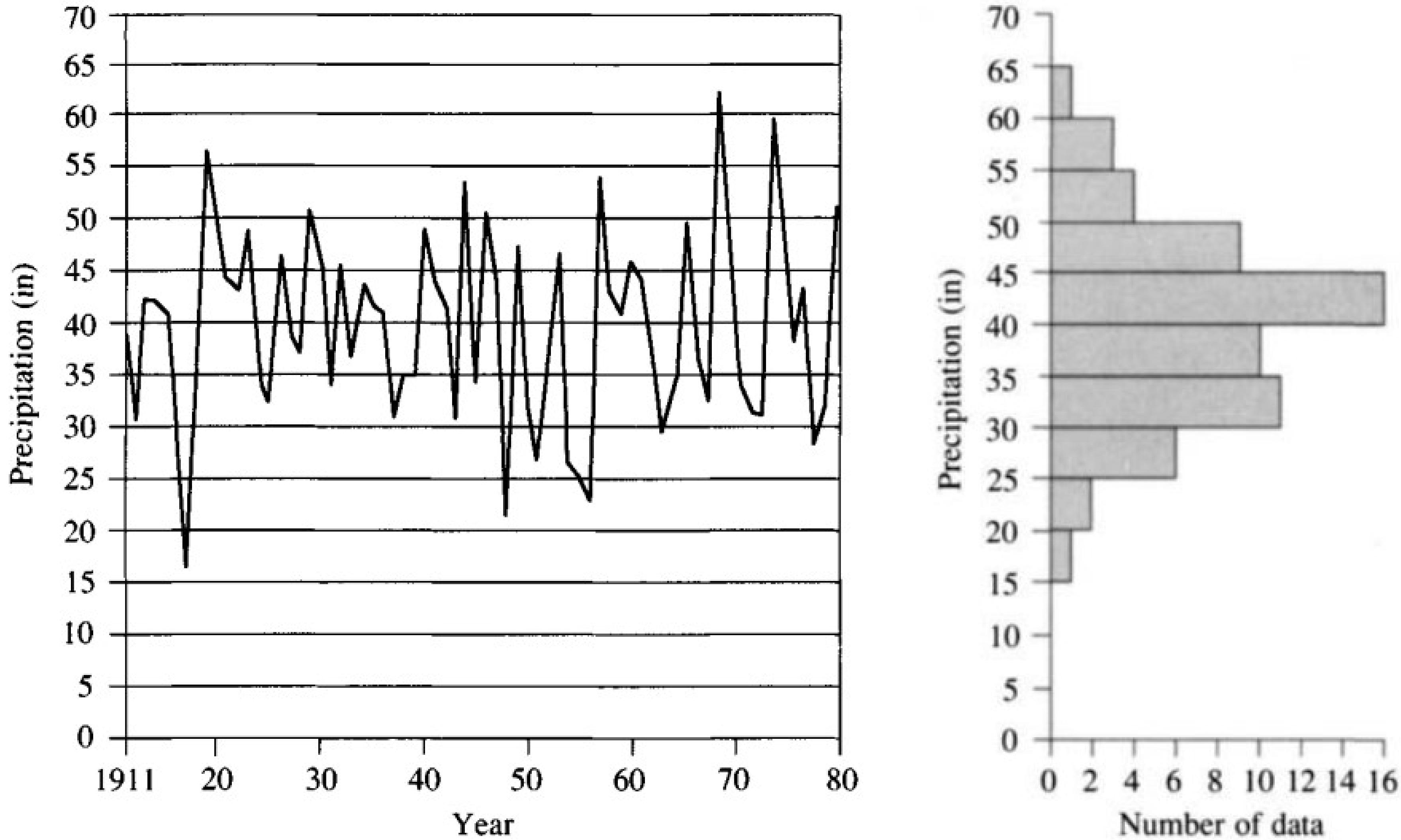
La probabilità di un evento può essere stimata. Se un campione di *n* osservazioni comprende n_A valori che appartengono ad A, la *frequenza probabile* di *A* è $\frac{n_A}{n}$.

All'aumentare della numerosità n del campione, la frequenza probabile di A approssima la probabilità di A

The *probability* of an event, $P(A)$, is the chance that it will occur when an observation of the random variable is made. Probabilities of events can be estimated. If a sample of n observations has n_A values in the range of event A, then the *relative frequency* of A is n_A/n. As the sample size is increased, the relative frequency becomes a progressively better estimate of the probability of the event, that is,

$$P(A) = \lim_{n \to \infty} \frac{n_A}{n} \tag{11.1.1}$$

11.2 FREQUENCY AND PROBABILITY FUNCTIONS



(*a*) Annual precipitation.

(*b*) Frequency histogram.

If the number of observations n_i in interval i, covering the range $[x_i - \Delta x, x_i]$, is divided by the total number of observations n, the result is called the *relative frequency function* $f_s(x)$:

$$f_s(x_i) = \frac{n_i}{n} \tag{11.2.1}$$

which, as in Eq. (11.1.1), is an estimate of $P(x_i - \Delta x \leq X \leq x_i)$, the probability that the random variable X will lie in the interval $[x_i - \Delta x, x_i]$. The subscript s indicates that the function is calculated from sample data.

The sum of the values of the relative frequencies up to a given point is the *cumulative frequency function* $F_s(x)$:

$$F_s(x_i) = \sum_{j=1}^{i} f_s(x_j) \tag{11.2.2}$$

This is an estimate of $P(X \leq x_i)$, the *cumulative probability* of x_i.

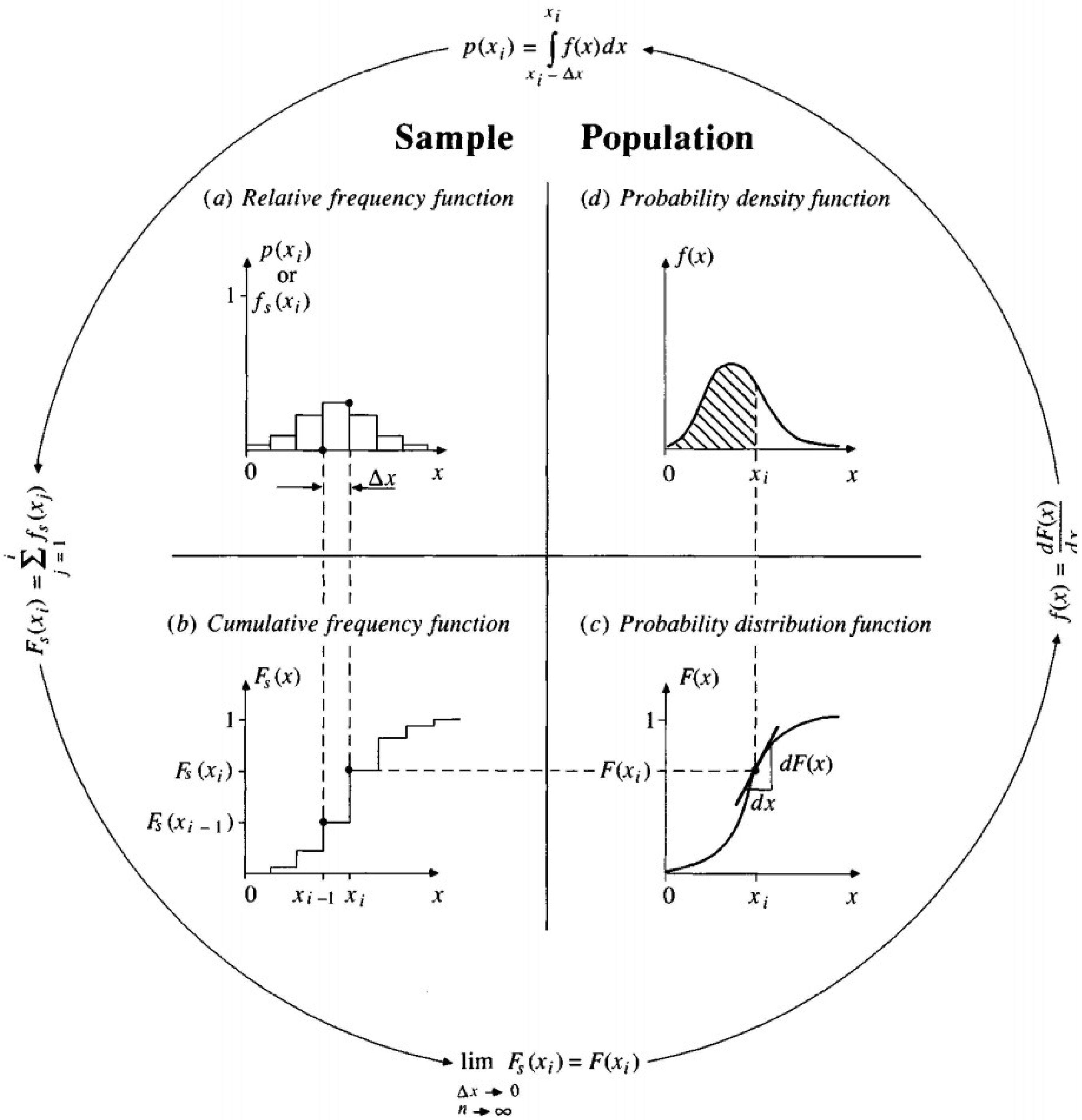
The relative frequency and cumulative frequency functions are defined for a sample; corresponding functions for the population are approached as limits as $n \rightarrow \infty$ and $\Delta x \rightarrow 0$. In the limit, the relative frequency function divided by the interval length Δx becomes the *probability density function* $f(x)$:

$$f(x) = \lim_{\substack{n \to \infty \\ \Delta x \to 0}} \frac{f_s(x)}{\Delta x} \tag{11.2.3}$$

The cumulative frequency function becomes the *probability distribution function* $F(x)$,

$$F(x) = \lim_{\substack{n \to \infty \\ \Delta x \to 0}} F_s(x) \tag{11.2.4}$$

whose derivative is the probability density function



$p(x_i) = \int_{x_i - \Delta x}^{x_i} f(x)dx$
Sample
Population
(a) Relative frequency function
(d) Probability density function
$p(x_i)$ or $f_s(x_i)$
1
0
Δx
x
$f(x)$
x_i
(b) Cumulative frequency function
(c) Probability distribution function
$F_s(x)$
$F_s(x_i)$
$F_s(x_{i-1})$
x_{i-1}
$F(x)$
$F(x_i)$
$dF(x)$
dx
$F_s(x_i) = \sum_{j=1}^{i} f_s(x_j)$
$f(x) = \frac{dF(x)}{dx}$
$\lim_{\substack{\Delta x \to 0 \\ n \to \infty}} F_s(x_i) = F(x_i)$

11.3 STATISTICAL PARAMETERS

TABLE 11.3.1
Population parameters and sample statistics

Population parameter	Sample statistic
1. *Midpoint*	
Arithmetic mean	
$\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$	$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
Median	
x such that $F(x) = 0.5$	50th-percentile value of data
Geometric mean	
antilog $[E(\log x)]$	$\left(\prod_{i=1}^{n} x_i\right)^{1/n}$

2. *Variability*

Variance

$$\sigma^2 = E[(x - \mu)^2] \qquad\qquad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Standard deviation

$$\sigma = \{E[(x - \mu)^2]\}^{1/2} \qquad\qquad s = \left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2\right]^{1/2}$$

Coefficient of variation

$$CV = \frac{\sigma}{\mu} \qquad\qquad CV = \frac{s}{\bar{x}}$$

11.4 FITTING A PROBABILITY DISTRIBUTION

A probability distribution is a function representing the probability of occurrence of a random variable. By fitting a distribution to a set of hydrologic data, a great deal of the probabilistic information in the sample can be compactly summarized in the function and its associated parameters. Fitting distributions can be accomplished by the *method of moments* or the *method of maximum likelihood*.

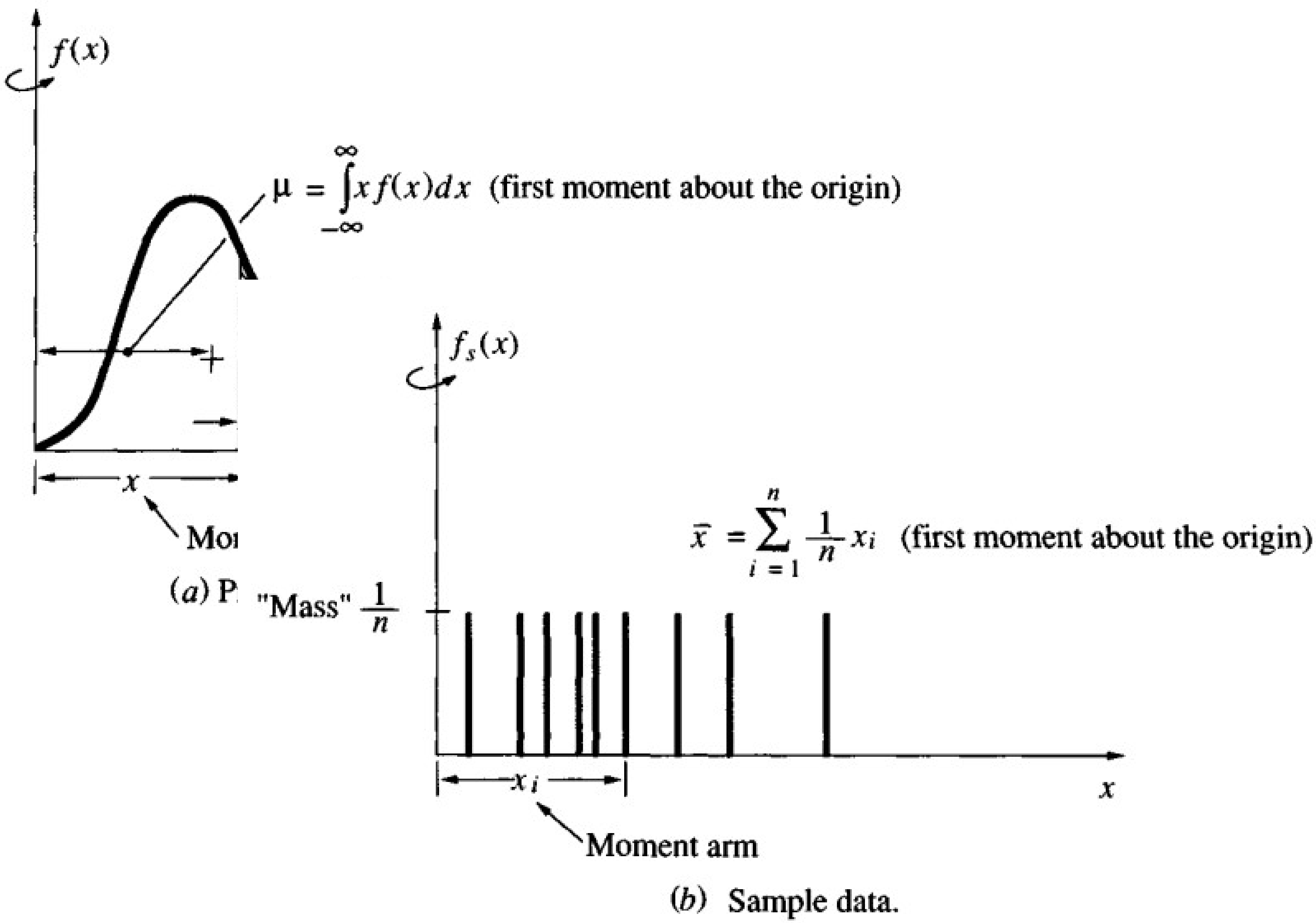


FIGURE 11.4.1
The method of moments selects values for the parameters of the probability density function so that its moments are equal to those of the sample data.

Method of Maximum Likelihood

The method of maximum likelihood was developed by R. A. Fisher (1922). He reasoned that the best value of a parameter of a probability distribution should be that value which maximizes the likelihood or joint probability of occurrence of the observed sample. Suppose that the sample space is divided into intervals of length dx and that a sample of independent and identically distributed observations x_1, $x_2, \ldots, x_n$ is taken. The value of the probability density for $X = x_i$ is $f(x_i)$, and the probability that the random variable will occur in the interval including x_i is $f(x_i)\, dx$. Since the observations are independent, their joint probability of occurrence is given from Eq. (11.1.5) as the product $f(x_1)\, dx\, f(x_2)\, dx \ldots f(x_n)\, dx = [\prod_{i=1}^{n} f(x_i)]\, dx^n$, and since the interval size dx is fixed, maximizing the joint probability of the observed sample is equivalent to maximizing the *likelihood function*

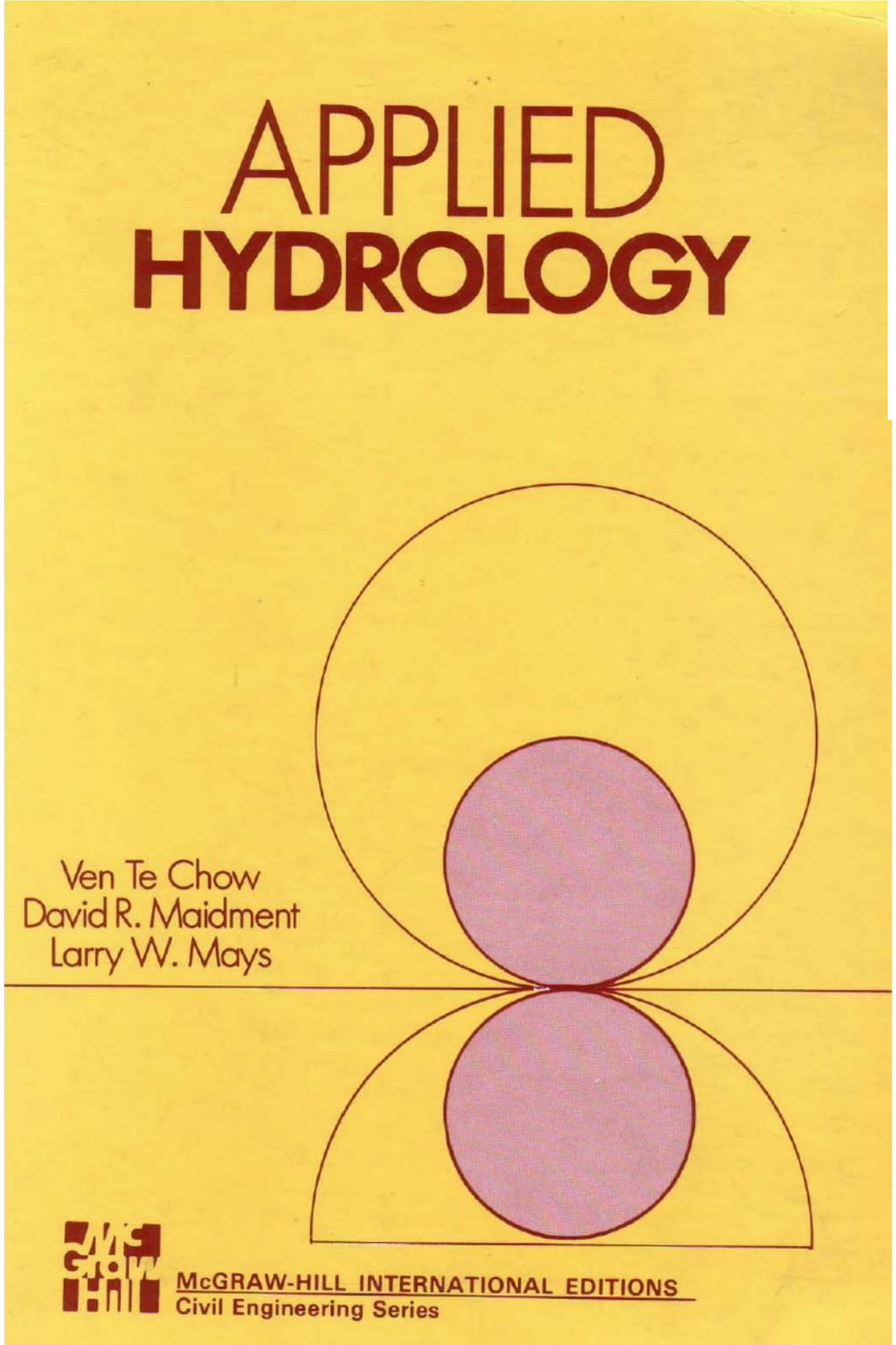
$$L = \prod_{i=1}^{n} f(x_i) \tag{11.4.2}$$

Testing the Goodness of Fit

The goodness of fit of a probability distribution can be tested by comparing the theoretical and sample values of the relative frequency or the cumulative frequency function. In the case of the relative frequency function, the χ^2 *test* is used. The sample value of the relative frequency of interval i is, from Eq. (11.2.1), $f_s(x_i) = n_i/n$; the theoretical value from (11.2.7) is $p(x_i) = F(x_i) - F(x_{i-1})$. The χ^2 *test statistic* χ_c^2 is given by

$$\chi_c^2 = \sum_{i=1}^{m} \frac{n[f_s(x_i) - p(x_i)]^2}{p(x_i)} \tag{11.4.4}$$

The χ^2 distribution function is tabulated in many statistics texts (e.g., Haan, 1977). In the χ^2 test, $\nu = m - p - 1$, where m is the number of intervals as before, and p is the number of parameters used in fitting the proposed distribution.



CHAPTER 12

FREQUENCY ANALYSIS

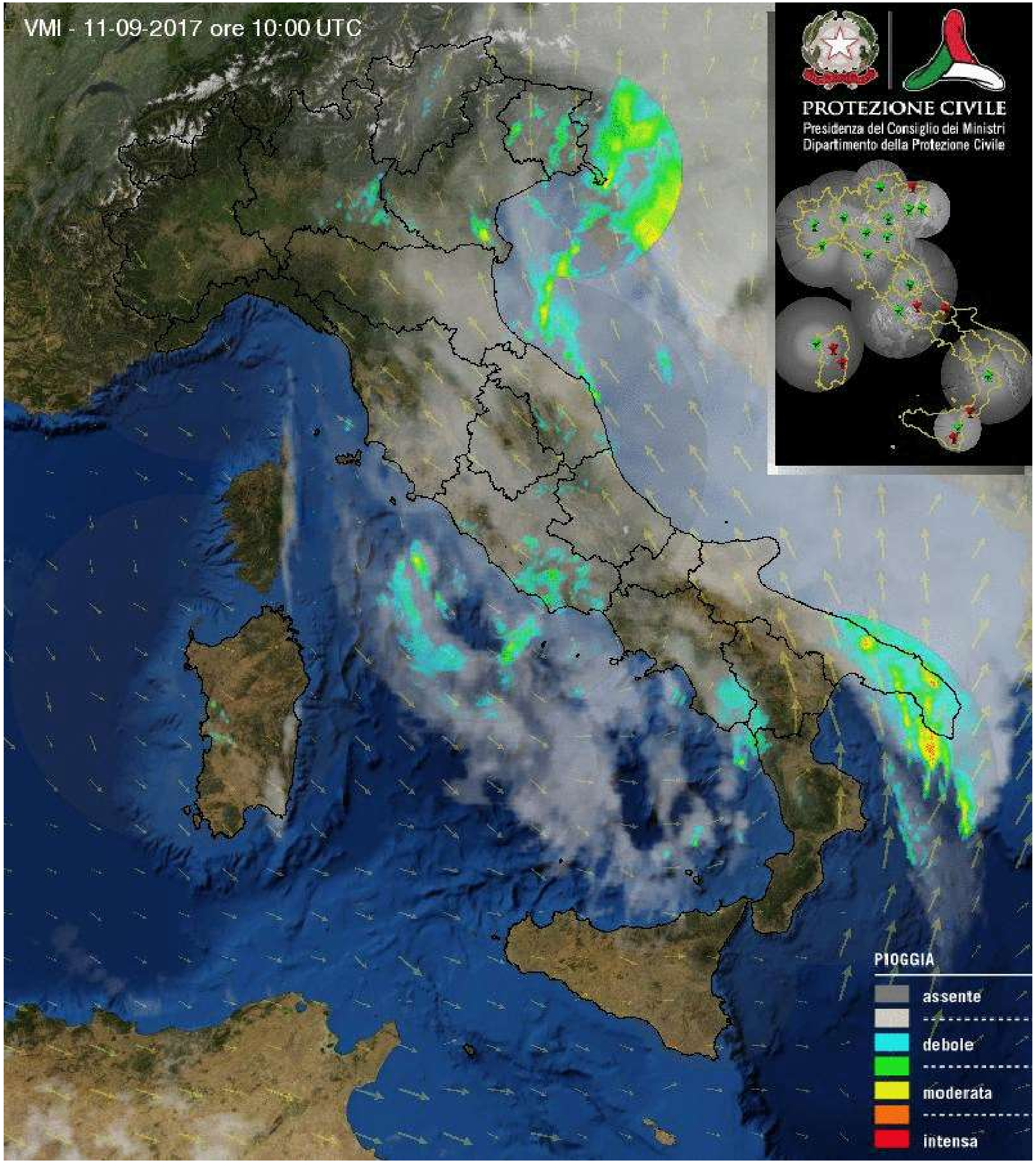
Le opere idrauliche, spesso devono far fronte ad **eventi estremi**, come portate di piena molto rilevanti o prolungati eventi siccitosi. L' **entità** di un evento estremo è **inversamente proporzionale alla sua probabilità di superamento**, gli eventi estremi sono rari.

Lo scopo dell'**analisi probabilistica** di dati idrologici è **stabilire la relazione esistente tra gli eventi estremi e la loro probabilità (di superamento).**

Si assume che i dati osservati siano indipendenti e estratti dalla stessa popolazione (identicamente distribuiti).

I massimi annuali della variabile idrologica che si analizza (e.g., la portata massima annuale in una sezione di un corso d’acqua) si ritengono indipendenti.

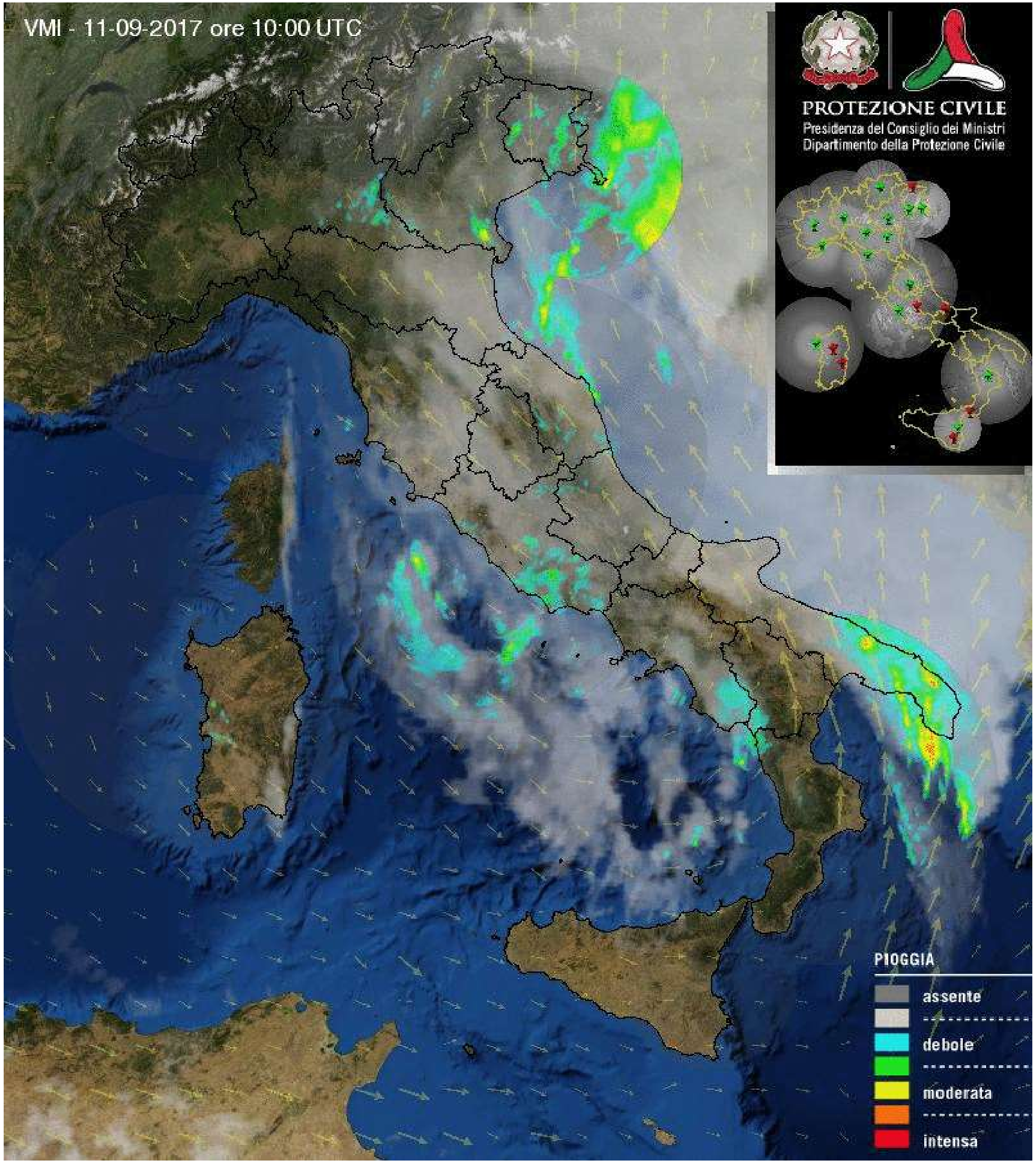
I risultati dell’analisi probabilistica dei massimi trovano numerose applicazioni: nella progettazione di opere idrauliche, nell’analisi di rischio idraulico e nell’analisi idroeconomica.



Le opere di difesa idraulica vengono progettate con riferimento ad un carico di progetto che ricorre con frequenza probabile assegnata.

Solo il carico è considerato aleatorio quando viene fatto riferimento al Tempo di Ritorno del carico di progetto.

Il Tempo di ritorno è il numero di anni che MEDIAMENTE intercorrono tra un superamento ed il successivo del carico di progetto.



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Reliability Theories

1) Quasi probabilistic methods → safety factors
2) Probabilistic methods → probability distribution of strength and/or load
3) Complex probabilistic methods → numerical simulation of random events

Reliability Theories

1) Quasi probabilistic methods → safety factors

2) Probabilistic methods → probability distribution of strength and/or load

3) Complex probabilistic methods → numerical simulation of random events

12.1 RETURN PERIOD

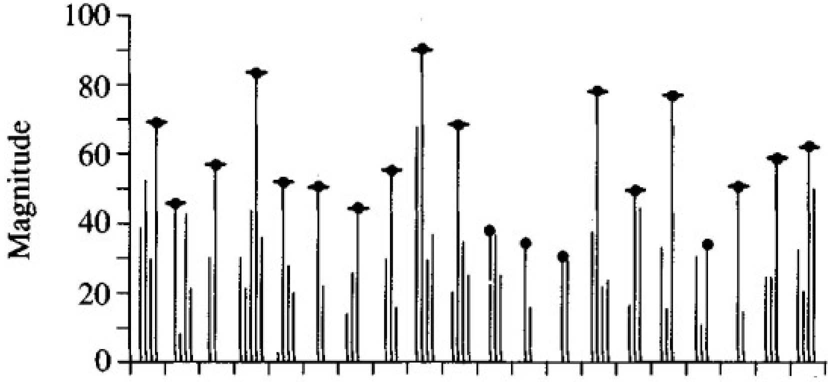
Suppose that an extreme event is defined to have occurred if a random variable X is greater than or equal to some level x_T. The *recurrence interval* τ is the time between occurrences of $X \geq x_T$.

The *return period* T of the event $X \geq x_T$ is the expected value of τ, $E(\tau)$, its average value measured over a very large number of occurrences.

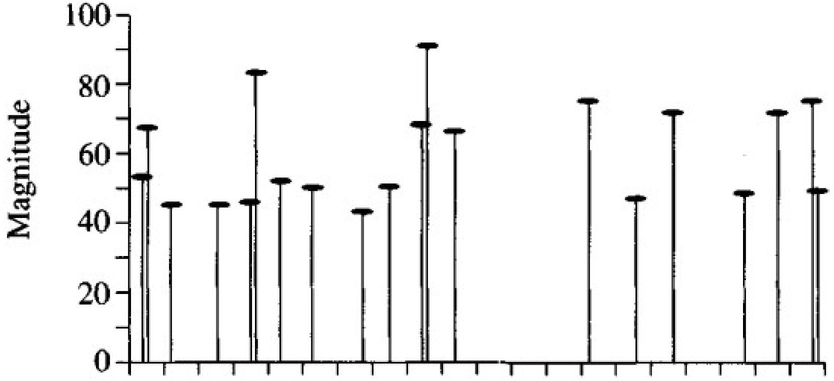
Data series

Hydrologic Data Series

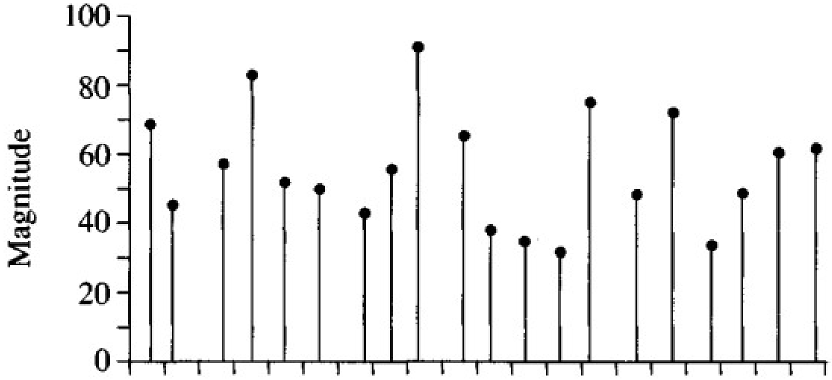
A *complete duration series* consists of all the data available as shown in Fig. 12.1.2(*a*). A *partial duration series* is a series of data which are selected so that their magnitude is greater than a predefined *base value*. If the base value is selected so that the number of values in the series is equal to the number of years of the record, the series is called an *annual exceedence series;* an example is shown in Fig. 12.1.2(*b*). An *extreme value series* includes the largest or smallest values occurring in each of the equally-long time intervals of the record. The time interval length is usually taken as one year, and a series so selected is called an *annual series*. Using largest annual values, it is an *annual maximum series* as shown in Fig. 12.1.2(*c*). Selecting the smallest annual values produces an *annual minimum series*.



(a) Original data; $N = 20$ years. Time



(b) Annual exceedences. Time



(c) Annual maxima. Time

PROBABILITY FUNCTIONS

Exponential

$$P(x) = 1 - e^{-\gamma x}$$

$$p(x) = \gamma\, e^{-\gamma x}$$

$$\mu_x = \frac{1}{\gamma}$$

$$\sigma_x = \frac{1}{\gamma}$$

Requires that the population standard deviation and mean have the same value describes the time between events in a Poisson Process, i.e. a process in which events occur continuously and independently at a constant average rate.
It has the key property of being memoryless
Daily rainfall may also follow an exponential distribution .

PROBABILITY FUNCTIONS

Gumbel

$$P(x) = e^{-e^{-\frac{x-\mu}{\beta}}}$$

$$p(x) = \frac{1}{\beta} e^{-\left(\frac{x-\mu}{\beta} + e^{-\frac{x-\mu}{\beta}}\right)}$$

$$\mu_x = \mu + \beta \cdot \gamma \quad \rightarrow \gamma = \text{Euler's constant} \approx 0.5772$$

$$\sigma_x = \frac{\pi}{\sqrt{6}} \beta$$

Gumbel distribution is a particular case of the generalized extreme value distribution and is used to model the distribution of the maximum (or the minimum) of a number of samples of various distributions.

According to the **theory of extreme values**, the largest or smallest value from a set of independent identically distributed random variables tends to an asymptotic distribution that only depends on the **tail** of the distribution of the basic variable.

the exponential distribution has the Gumbel distribution as its corresponding limiting extreme value distribution.

PROBABILITY FUNCTIONS

Binomial

$$P(x<k)=\sum_{i=0,k}\binom{n}{i}p^{i}(1-p)^{n-i}$$

$$p(x=k)=\binom{n}{k}p^{k}(1-p)^{n-k}$$

$$\mu_x = np$$

$$\sigma_x=\sqrt{np(1-p)}$$

the binomial distribution with parameters *k* and *p* is the discrete probability distribution of the number of successes (k) in a sequence of *n independent* yes/no experiments, if the probability of a success is p

PROBABILITY FUNCTIONS

Geometric

$$P(x < n) = 1 - (1 - p)^n$$

$$p(x = n) = (1 - p)^{n-1} p$$

$$\mu_x = \frac{1}{p}$$

$$\sigma_x = \frac{\sqrt{(1 - p)}}{p}$$

the geometric distribution with parameters *n* and *p* is the discrete probability distribution of the number of failure (n-1) in a sequence of *n* independent yes/no experiments needed to get one success (which has probability p)

The probability $p = P(X \geq x_T)$ of occurrence of the event $X \geq x_T$ in any observation may be related to the return period in the following way. For each observation, there are two possible outcomes: either "success" $X \geq x_T$ (probability p) or "failure" $X < x_T$ (probability $1-p$). Since the observations are independent, the probability of a recurrence interval of duration τ is the product of the probabilities of $\tau - 1$ failures followed by one success, that is, $(1-p)^{\tau-1}p$, and the expected value of τ is given by

$$
\begin{aligned}
E(\tau) &= \sum_{\tau=1}^{\infty} \tau(1-p)^{\tau-1}p \\
&= p + 2(1-p)p + 3(1-p)^2 p + 4(1-p)^3 p + \ldots \\
&= p[1 + 2(1-p) + 3(1-p)^2 + 4(1-p)^3 + \ldots]
\end{aligned}
\tag{12.1.1a}
$$

$$E(\tau) = \sum_{\tau=1}^{\infty} \tau(1-p)^{\tau-1}p$$

$$= p + 2(1-p)p + 3(1-p)^2p + 4(1-p)^3p + \ldots \tag{12.1.1a}$$

$$= p[1 + 2(1-p) + 3(1-p)^2 + 4(1-p)^3 + \ldots]$$

The expression within the brackets has the form of the power series expansion $(1+x)^n = 1 + nx + [n(n-1)/2]x^2 + [n(n-1)(n-2)/6]x^3 + \ldots$, with $x = -(1-p)$ and $n = -2$, so (12.1.1a) may be rewritten

$$E(\tau) = \frac{p}{[1-(1-p)]^2}$$

$$= \frac{1}{p} \tag{12.1.1b}$$

Hence $E(\tau) = T = 1/p$; that is, the probability of occurrence of an event in any observation is the inverse of its return period:

$$P(X \geq x_T) = \frac{1}{T} \tag{12.1.2}$$

$$\left(1 - P\left[x \le X\right]\right)$$

Probabilità che il massimo annuale di precipitazione sia x≥X, ed X sia la precipitazione di progetto

$$P\left[x \le X\right]$$

Probabilità che il massimo annuale di precipitazione sia x≤X

Gumbel

La probabilità che il massimo carico annuale (**x**) non ecceda il carico di progetto (**X**) per **m-1** anni e lo superi nell'anno **m-esimo** è espressa a mezzo della distribuzione geometrica.

m è il valore atteso del numero di anni entro cui avviene il primo superamento x>X

$$p_m(m) = \left(1 - P[x \le X]\right) \cdot P[x \le X]^{m-1}$$

Il tempo di ritorno(**Tr**) è il valore atteso di **m** (con probabilità $\mathbf{p_m}$)

$$\sum_{m=1}^{\infty} m \cdot z^m = \frac{z}{(1-z)^2}$$

$$Tr = \sum_{m=1}^{\infty} m \cdot p_m(m) =$$

$$= \sum_{i=1}^{\infty} m \cdot P[x \le X]^{m-1} \left(1 - P[x \le X]\right)$$

$$= \frac{1}{1 - P[x \le X]} = \frac{1}{P[x \ge X]}$$

What is the probability that a T-year return period event will occur at least once in N years? To calculate this, first consider the situation where no T-year event occurs in N years. This would require a sequence of N successive "failures," so that

$$P(X < x_T \text{ each year for } N \text{ years}) = (1 - p)^N$$

The complement of this situation is the case required, so by (11.1.3)

$$P(X \geq x_T \text{ at least once in } N \text{ years}) = 1 - (1 - p)^N \tag{12.1.3}$$

Since $p = 1/T$,

$$P(X \geq x_T \text{ at least once in } N \text{ years}) = 1 - \left(1 - \frac{1}{T}\right)^N \tag{12.1.4}$$

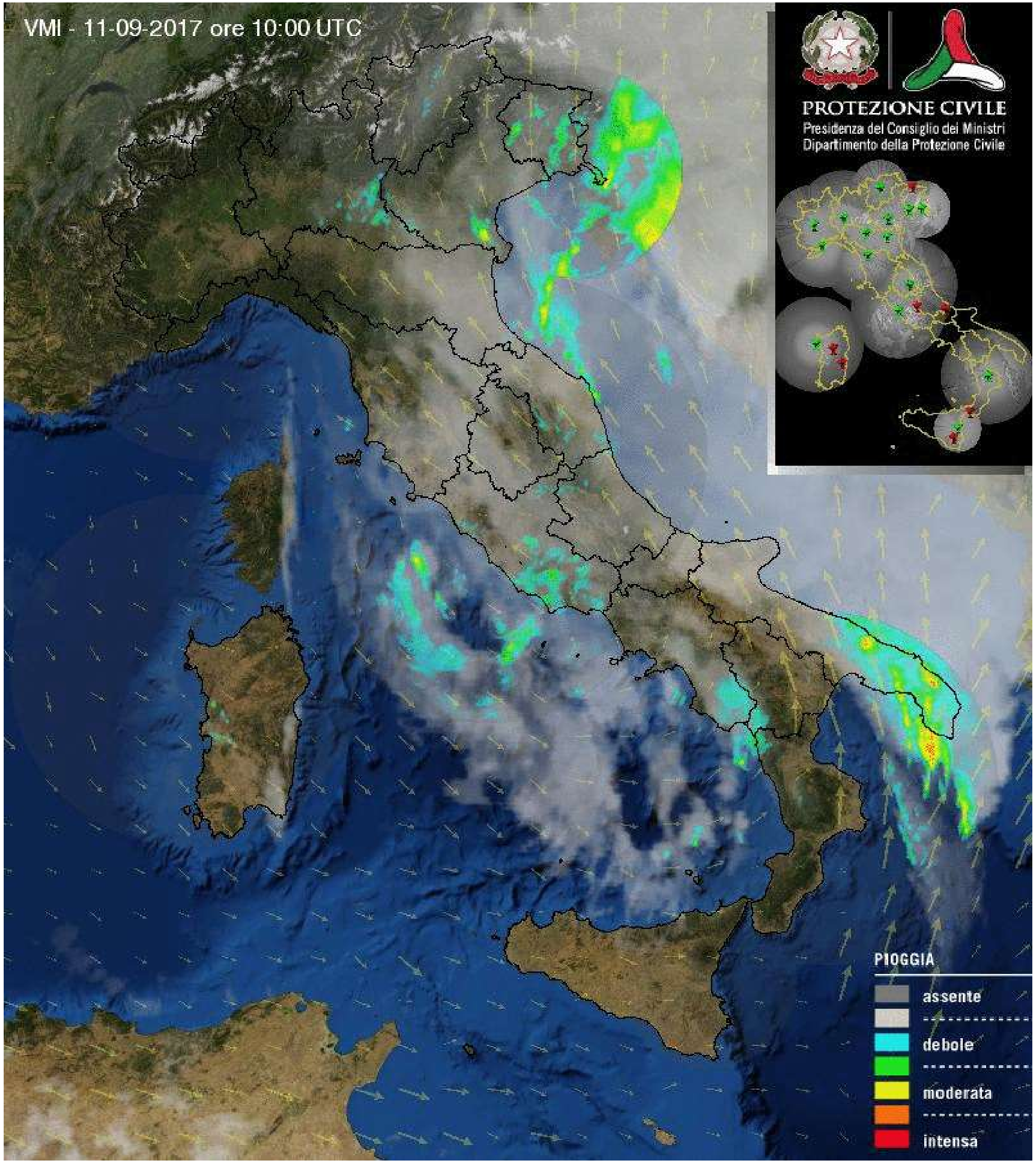
La probabilità che il massimo annuale (**x**) superi il carico di progetto solo una volta in **m** anni è

$$f_m(m) = \binom{m}{1}\left(1 - P[x \le X]\right) \cdot P[x \le X]^{m-1}$$
$$= m \cdot \left(1 - P[x \le X]\right) \cdot P[x \le X]^{m-1}$$

Il valore atteso di **m** con probabilità $\mathbf{f_m}$ è

$$\sum_{m=1}^{\infty} m \cdot f_m(m) =$$
$$= \sum_{i=1}^{\infty} m^2 \cdot P[x \le X]^{m-1}\left(1 - P[x \le X]\right)$$
$$= \frac{\left(1 + P[x \le X]\right)}{\left(1 - P[x \le X]\right)^2} = \frac{1}{P[x \ge X]} + \frac{2 \cdot P[x \le X]}{\left(1 - P[x \le X]\right)^2}$$

$$\sum_{m=1}^{\infty} m^2 \cdot z^m = \frac{z \cdot (1 + z)}{(1 - z)^3}$$



Le opere di difesa idraulica vengono progettate con riferimento ad un carico di progetto che ricorre con frequenza probabile assegnata.

Solo **il carico** è considerato aleatorio quando viene fatto riferimento al Tempo di Ritorno del carico di progetto.

Il Tempo di ritorno è il numero di anni che MEDIAMENTE intercorrono tra un superamento ed il successivo del carico di progetto.

EVI distribution

The Extreme Value Type I (EVI) probability distribution function is

$$F(x) = \exp\left[-\exp\left(-\frac{x-u}{\alpha}\right)\right] \qquad -\infty \le x \le \infty \tag{12.2.1}$$

The parameters are estimated, as given in Table 11.5.1, by

$$\alpha = \frac{\sqrt{6}\, s}{\pi} \tag{12.2.2}$$

$$u = \bar{x} - 0.5772\alpha \tag{12.2.3}$$

The parameter u is the mode of the distribution (point of maximum probability density). A *reduced variate* y can be defined as

$$y = \frac{x-u}{\alpha} \tag{12.2.4}$$

Substituting the reduced variate into (12.2.1) yields

$$F(x) = \exp\left[-\exp(-y)\right] \tag{12.2.5}$$

Solving for y:

$$y = -\ln\left[\ln\left(\frac{1}{F(x)}\right)\right] \tag{12.2.6}$$

$$
\begin{aligned}
\frac{1}{T} &= P(x \geq x_T) \\
&= 1 - P(x < x_T) \\
&= 1 - F(x_T)
\end{aligned}
$$

$$F(x_T) = \frac{T-1}{T}$$

and, substituting into (12.2.6),

$$y_T = -\ln\left[\ln\left(\frac{T}{T-1}\right)\right] \tag{12.2.7}$$

For the EVI distribution, x_T is related to y_T by Eq. (12.2.4), or

$$x_T = u + \alpha y_T \tag{12.2.8}$$

Extreme value distributions have been widely used in hydrology. They form the basis for the standardized method of flood frequency analysis in Great Britain (Natural Environment Research Council, 1975). Storm rainfalls are most commonly modeled by the Extreme Value Type I distribution (Chow, 1953; Tomlinson, 1980), and drought flows by the Weibull distribution, that is, the EVIII distribution applied to $-x$ (Gumbel, 1954, 1963).

Esempio: distribuzione di probabilità dei massimi annuali di precipitazione di durata assegnata

Example 12.2.1. Annual maximum values of 10-minute-duration rainfall at Chicago, Illinois, from 1913 to 1947 are presented in Table 12.2.1. Develop a model for storm rainfall frequency analysis using the Extreme Value Type I distribution and calculate the 5-, 10-, and 50-year return period maximum values of 10-minute rainfall at Chicago.

TABLE 12.2.1
Annual maximum 10-minute rainfall in inches at Chicago, Illinois, 1913–1947

Year	1910	1920	1930	1940
0		0.53	0.33	0.34
1		0.76	0.96	0.70
2		0.57	0.94	0.57
3	0.49	0.80	0.80	0.92
4	0.66	0.66	0.62	0.66
5	0.36	0.68	0.71	0.65
6	0.58	0.68	1.11	0.63
7	0.41	0.61	0.64	0.60
8	0.47	0.88	0.52	
9	0.74	0.49	0.64	

Mean = 0.649 in
Standard deviation = 0.177 in

Solution. The sample moments calculated from the data in Table 12.2.1 are $\bar{x} = 0.649$ in and $s = 0.177$ in. Substituting into Eqs. (12.2.2) and (12.2.3) yields

$$\alpha = \frac{\sqrt{6}\, s}{\pi}$$

$$= \frac{\sqrt{6} \times 0.177}{\pi}$$

$$= 0.138$$

$$u = \bar{x} - 0.5772\alpha$$

$$= 0.649 - 0.5772 \times 0.138$$

$$= 0.569$$

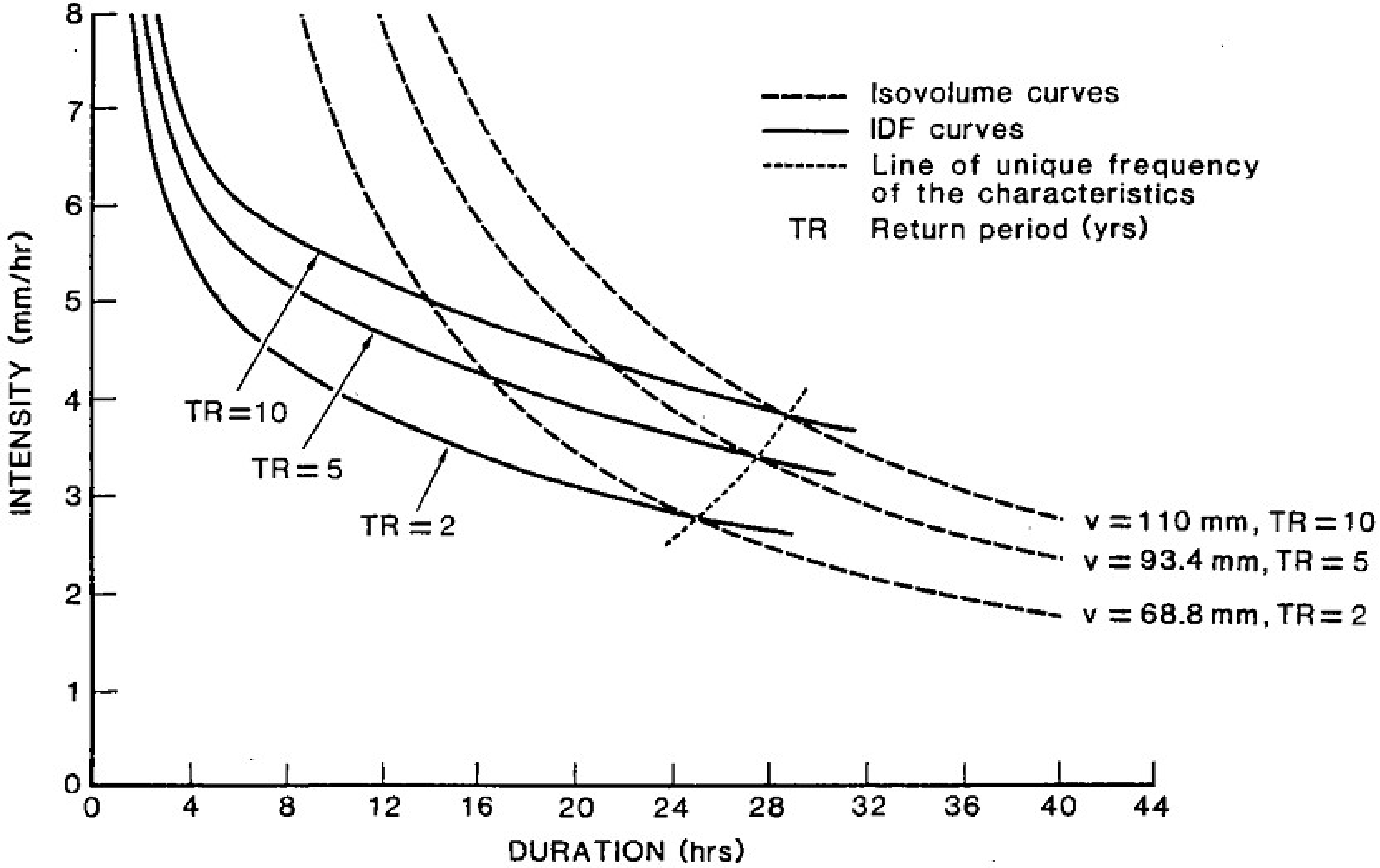
The probability model is

$$F(x) = \exp\left[-\exp\left(-\frac{x - 0.569}{0.138}\right)\right]$$

So the 10-minute, 5 year storm rainfall magnitude at Chicago is 0.78 in. By the same method, the 10- and 50-year values can be shown to be 0.88 in and 1.11 in, respectively. It may be noted from the data in Table 12.2.1 that the 50-year return period rainfall was equaled once in the 35 years of data (in 1936), and that the 10-year return period rainfall was equaled or exceeded four times during this period, so the frequency of occurrence of observed extreme rainfalls is approximately as predicted by the model.

Intensity-Duration-Frequency and Volume-Frequency Curves

(rainfall recorded at Vancouver City Hall)



1. Il tempo di ritorno è una variabile aleatoria?

2. Qual è la distribuzione di probabilità di un evento estremo?

3. Qual è la distribuzione di probabilità del tempo che intercorre tra due superamenti di un valore di soglia?

4. Definizione di tempo di ritorno.

5. Quali sono le ipotesi alla base della definizione della distribuzione di probabilità del tempo che intercorre tra due superamenti di un valore di soglia?

1. **Definizione di rischio idraulico per una struttura dimensionata con riferimento a un tempo di ritorno assegnato**

2. **Specifica le ipotesi alla base della definizione data**

3. **Qual è la differenza tra rischio idraulico e tempo di ritorno?**

Esercizi

Esercizio:

Per un sistema di drenaggio, la probabilità che la massima precipitazione annuale sia maggiore della precipitazione di progetto sia $p=0.02$.

Qual è la probabilità che la precipitazione di progetto sia superata almeno una volta nell'arco della vita utile dell'opera, se questa è 30 anni?

… si supponga che il sistema di drenaggio venga dimensionato con riferimento ad un carico di progetto che ha $p'=0.01$.

$$P[X>1]=1-p_X(0)=1-\binom{30}{0}0.02^0\cdot 0.98^{30}\approx 0.44$$

$$....1-(1-0.01)^{30}\approx 0.26$$

Esercizio:

Sia p'=0.01 la probabilità di non superamento del carico di progetto di un opera idraulica.

Il numero N-1 di anni per cui il carico di progetto di un opera idraulica non viene uguagliato o superato, assumendo che il primo superamento avvenga nell'anno N-esimo, è una variabile aleatoria discreta con distribuzione geometrica g(N).

Calcolare il valore atteso di N

Distribuzione di probabilità di N:

$g(N)=(1-p')^{N-1}p'$

Valore atteso di N = tempo di ritorno

$$m_N = \frac{1}{p'} = \frac{1}{0.01} = 100\ anni = Tr$$

Esercizio (continuazione):

Qual è la probabilità che sia N>10 anni?

Qual è la probabilità che sia N>30 anni?

$$p' = 0.01 = P \text{ di superamento nell'anno generico}$$

$$P[N > 10] = \underbrace{(1 - p')^{10}}_{\substack{P \text{ di non superamento 10 volte} \\ \text{(per 10 anni)}}} = (1 - 0.01)^{10} \approx 0.92$$

$$P[N > 30] = (1 - 0.01)^{30} = 0.74$$

Esercizio:

In un alalisi costibenefici, il progettista di un opera idraulica confronta il costo di realizzazione dell'opera con il costo del danno derivante da una fallanza

Sia **c** il costo del danno derivante dalla singola fallanza (indipendentemente dall'entità del carico quando questo supera il carico di progetto).
Il costo di realizzazione dell'opera sia **I=150c, più un incremento linearmente proporzionale al logaritmo di Tr, con costante di proporzionalità b=0.2**

Qual è il Tr che minimizza il costo totale considerato che la vita utile dell'opera è 30 anni?

$$E[\text{costo totale}] = I + Ib\ln(Tr) + \sum_{i=0}^{30} i \cdot c \cdot P[i \text{ fallanze in 30 anni}]$$

$$= I(1 + b\ln(Tr)) + E[ci] = I(1 + b\ln(Tr)) + cE[i]$$

$$= I + Ib\ln(Tr) + c\frac{30}{T_r}$$

$$\frac{dE[\text{costo totale}]}{dT_r} = Ib\frac{1}{Tr} - c\frac{30}{T_r^{\,2}} = 0$$

$$\Rightarrow T_r = \frac{30c}{Ib}$$

$$T_r = \frac{30c}{Ib} = \frac{30c}{150c \cdot 0.01} = 20 \text{ anni}$$

$$P(x<k) = \sum_{i=0,k} \binom{n}{i} p^i (1-p)^{n-i}$$

$$p(x=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu_x = np$$

$$\sigma_x = \sqrt{np(1-p)}$$

La definizione di Tempo di ritorno è basata sull'ipotesi di stazionarietà della distribuzione dei massimi (P) ed omogeneità dei dati (che dovrebbero derivare dalla stessa popolazione)

Stoch Environ Res Risk Assess
DOI 10.1007/s00477-014-0916-1

SHORT COMMUNICATION

Dismissing return periods!

Francesco Serinaldi

Abstract The concept of return period in stationary univariate frequency analysis is prone to misconceptions and misuses that are well known but still widespread. In this study we highlight how nonstationary and multivariate extensions of such a concept are affected by additional misconceptions, thus easily resulting in further ill-posed procedures and misleading conclusions. We also show that the concepts of probability of exceedance and risk of failure over a given design life period provide more coherent, general and well devised tools for risk assessment and communication.

Il concetto di tempo di ritorno, necessita di una reinterpretazione nel momento in cui le osservazioni non possano ritenersi indipendenti e/o identicamente distribuite, richiedendo un analisi multivariata e non stazionaria

Return period $\mathcal{T}$ is probably one of the most used and misused concepts in hydrological and geophysical risk analysis. $\mathcal{T}$ is commonly written as:

$$\mathcal{T} = \frac{\mu}{p} = \frac{\mu}{\mathbb{P}[X > x]} = \frac{\mu}{1 - F(x)} \tag{1}$$

where X is a random variable describing the process under study (e.g. flow or rainfall peaks above a given threshold), $\mu > 0$ denotes the average inter-arrival time between two realizations of the process, $p = \mathbb{P}[X > x]$ is the probability to observe realizations exceeding a specific value x, and $F(x) = 1 - p = \mathbb{P}[X \leq x]$ indicates the distribution function of X. Even though the attitude of considering observations of physical processes as realizations of random variables is questionable (e.g., Klemeš 1986, 2000, 2002), pure statistical frequency analyses and computation of $\mathcal{T}$ values and corresponding return levels x are widespread in engineering, environmental sciences, and many other disciplines.

In fact, what really matters in systems' design and planning is not $\mathcal{T}$ and the corresponding x values obtained by inverting Eq. 1, but the risk of failure, which is the probability p_M to observe a critical event at least once in M years of design life. Under *iid* conditions, p_M is defined as (Chow et al. 1988, p. 383):

$$p_M := 1 - \prod_{j=1}^{M}(1 - p_j) = 1 - (1 - p)^M = 1 - (F(x_d))^M. \tag{3}$$

However, in the common practice, Eq. 3 is not used directly to define the design value x_d as it should, but only to verify the value of p_M corresponding to the $\mathcal{T}$-year value x, resulting in the well-known expression

$$p_M = 1 - (F(x))^M = 1 - \left(1 - \frac{1}{\mathcal{T}}\right)^M. \tag{4}$$

In other words, Eq. 3 allows us to compute a design value x_d with an appropriate probability p_M describing the actual risk of observing at least a failure in the entire design life period

$$x_d = F^{-1}((1 - p_M)^{1/M}), \tag{5}$$

3 Univariate nonstationary analyses: highlighting the limits of $\mathcal{T}$

When we move from stationary to nonstationary conditions (i.e. independent non-identically distributed *i/nid* data), the concept of return period becomes further ambiguous (Cooley 2013). However, it can still be defined in two ways for operational purposes. The first definition is the extension to nonstationary conditions of the concept of expected occurrence interval (expected waiting time until an exceedance occurs; Olsen et al 1998; Salas and Obeysekera 2014). In more detail, under nonstationarity, $p_j = \mathbb{P}[X_j > x] = 1 - F_j(x)$ is no longer constant and equal to p but changes for each trial (time step) j along the time series. Therefore, the return period in Eq. 1 becomes (Cooley 2013; Salas and Obeysekera 2014)

$$\mathcal{T} = 1 + \sum_{k=1}^{\infty} \prod_{j=1}^{k} (1 - p_j) = 1 + \sum_{k=1}^{\infty} \prod_{j=1}^{k} F_j(x). \tag{6}$$

Parey et al. (2007, 2010) extended to *i/nid* conditions an alternative definition of return period such that x is the value for which the expected number of exceedances in $\mathcal{T}$ years (trials) is equal to one. Therefore, x is the solution of the equation (Cooley 2013)

$$1 = \sum_{j=1}^{\mathcal{T}} p_j = \sum_{j=1}^{\mathcal{T}} (1 - F_j(x)). \tag{7}$$

Unlike the *iid* case, both Eqs. 6 and 7 need to be solved numerically to obtain the required x value corresponding to the assigned $\mathcal{T}$ (see e.g., Cooley 2013; Salas and Obeysekera 2014, for numerical details).

$$Tr = \sum_{m=1}^{\infty} mF^{m-1}(1-F) = \frac{1}{1-F} = \frac{1}{P} = \sum_{m=1}^{\infty} F^{m}$$

$$E[m, Tr] = \sum_{m=1}^{\infty} m \binom{Tr}{m} (1-P)^{Tr-m} P^{m} = Tr\ P$$

$$\boxed{\begin{aligned} Tr &= \sum_{m=1}^{\infty} \prod_{i=1}^{m} (1-P_i) = \sum_{m=1}^{\infty} \prod_{i=1}^{m} F_i \\ E[m, Tr] &= 1 = \sum_{1=1}^{Tr} P_i \end{aligned}}$$

ANALISI STATISTICA DEI MINIMI ANNUALI E SICCITA’

Il lago di Tiberiade è il lago d'acqua dolce più grande d'Israele. Insieme al fiume Giordano, che lo attraversa, e al mar Morto più a sud, è tra le principali risorse idriche del paese (oltre che della Palestina e della Giordania). Secondo gli esperti, questi bacini saranno sempre più colpiti dagli effetti del riscaldamento globale: al clima già arido d'Israele si aggiungeranno il calo delle precipitazioni e l'aumento delle temperature.

Nel 2017, dopo cinque anni di siccità, le acque del lago di Tiberiade hanno raggiunto il livello più basso dell'ultimo secolo. Il governo ha fatto costruire cinque impianti di desalinizzazione lungo la costa del mar Mediterraneo, che attualmente forniscono il 70 per cento dell'acqua potabile del paese. Nel maggio del 2018 è stata lanciata una campagna per chiedere ai cittadini di usare meno acqua. Il Giordano, che negli anni sessanta forniva il 95 per cento dell'acqua per l'agricoltura, oggi ne fornisce meno di un quarto. Il prosciugamento del fiume ha inoltre contribuito al restringimento del mar Morto, il lago salato che si trova nella depressione più bassa della Terra e rischia di scomparire.

Nonostante siano già minacciati dal cambiamento climatico, questi bacini continuano a essere sfruttati dal settore turistico per attirare persone. Secondo il ministero del turismo, nella prima metà del 2018 sono arrivati in Israele 2,1 milioni di visitatori, il 19 per cento in più del 2017.

Internazionale 1274 | 21 settembre 2018

I problemi legati al verificarsi di portate minime nei corpi idrici superficiali sono numerosi e riguardano:

- L'approvigionamento idrico
- La diluizione degli inquinanti provenienti dagli scarichi (immissioni nel corpo idrico)
- La sussistenza degli ecosistemi aquatici
- La navigabilità dei corsi d'acqua

Il venir meno di una o piu' funzionalità del corpo idrico a causa di un period siccitoso e quindi del verificarsi di portate minime, è associato ad una specifica situazione di rischio idraulico

POLICYFORUM

WATER MANAGEMENT

Water Security: Research Challenges and Opportunities

Karen Bakker

An estimated 80% of the world's population faces a high-level water security or water-related biodiversity risk

This increase in research and policy activity reflects growing concerns—particularly among practitioners, who have been at the vanguard of this agenda—over water-related human and ecosystem vulnerability, notably:

24 AUGUST 2012 VOL 337 SCIENCE www.sciencemag.org
Published by AAAS

1. threats to drinking water supply systems

2. threats to economic growth and human livelihoods

3. threats to water-related ecosystem services

4. increased hydrological variability

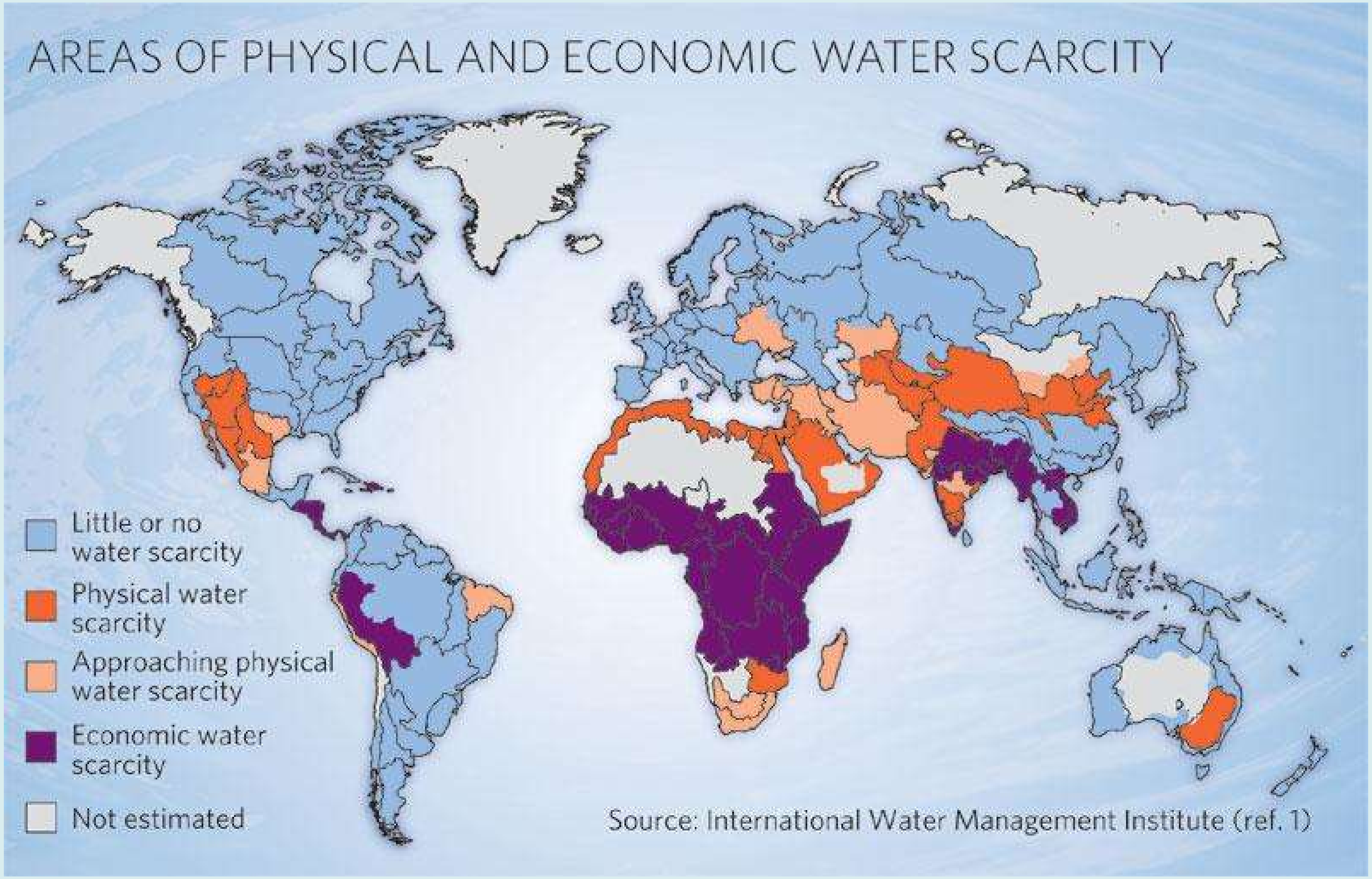
1. threats to drinking water supply systems

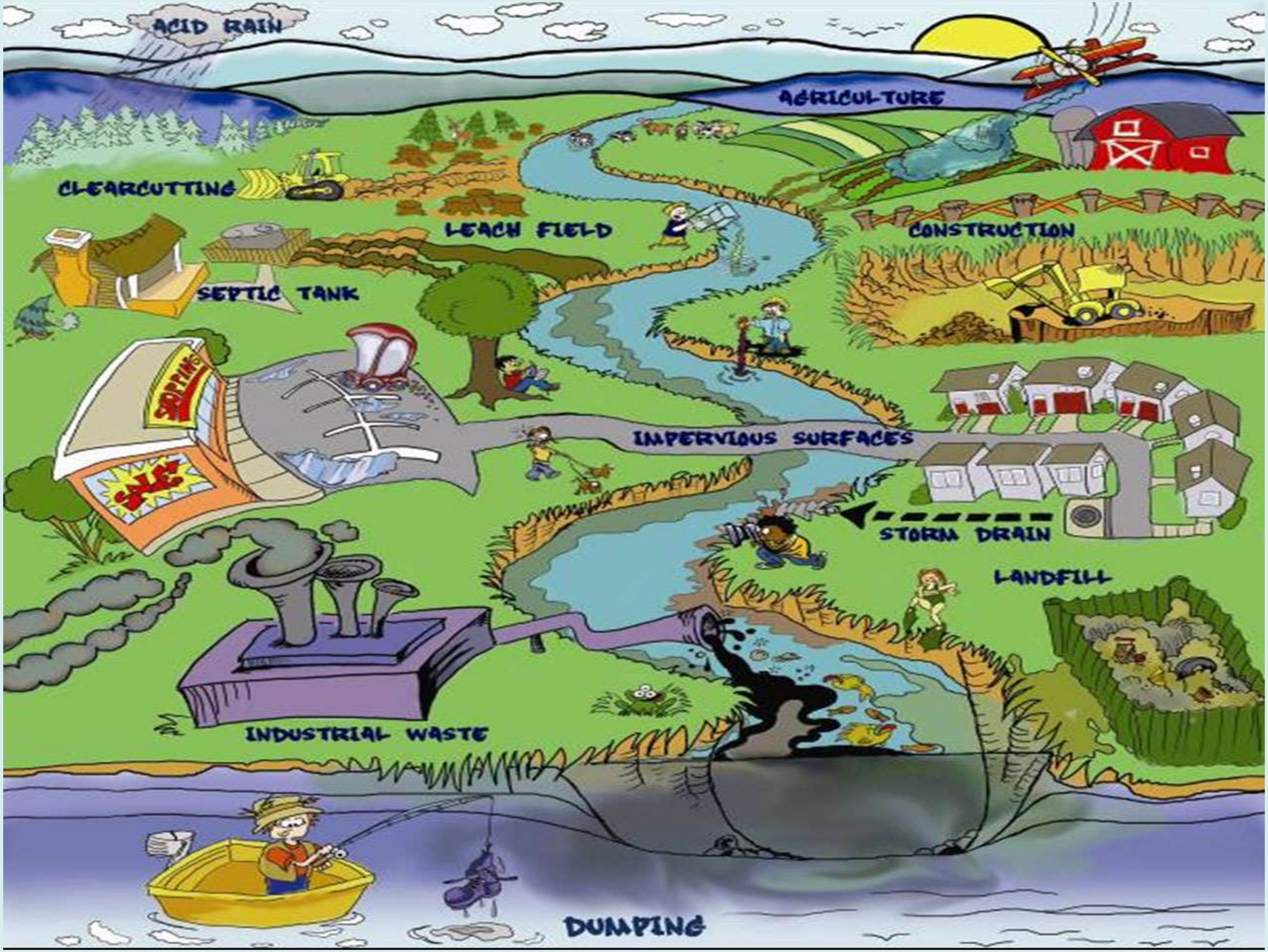
- lack of drinkable water access,
- human impact on aquatic ecosystems,
- contamination,

Implying:

- investment to meet the needs of the more than 1 billion people worldwide without access to safe drinking water;
- the need for enhanced monitoring and emergency preparedness,

GLOBAL SCALE (storage/reuse to contrast the water crisis)




ACID RAIN
AGRICULTURE
CLEARCUTTING
LEACH FIELD
CONSTRUCTION
SEPTIC TANK
SHOPPING
SALE
IMPERVIOUS SURFACES
STORM DRAIN
LANDFILL
INDUSTRIAL WASTE
DUMPING

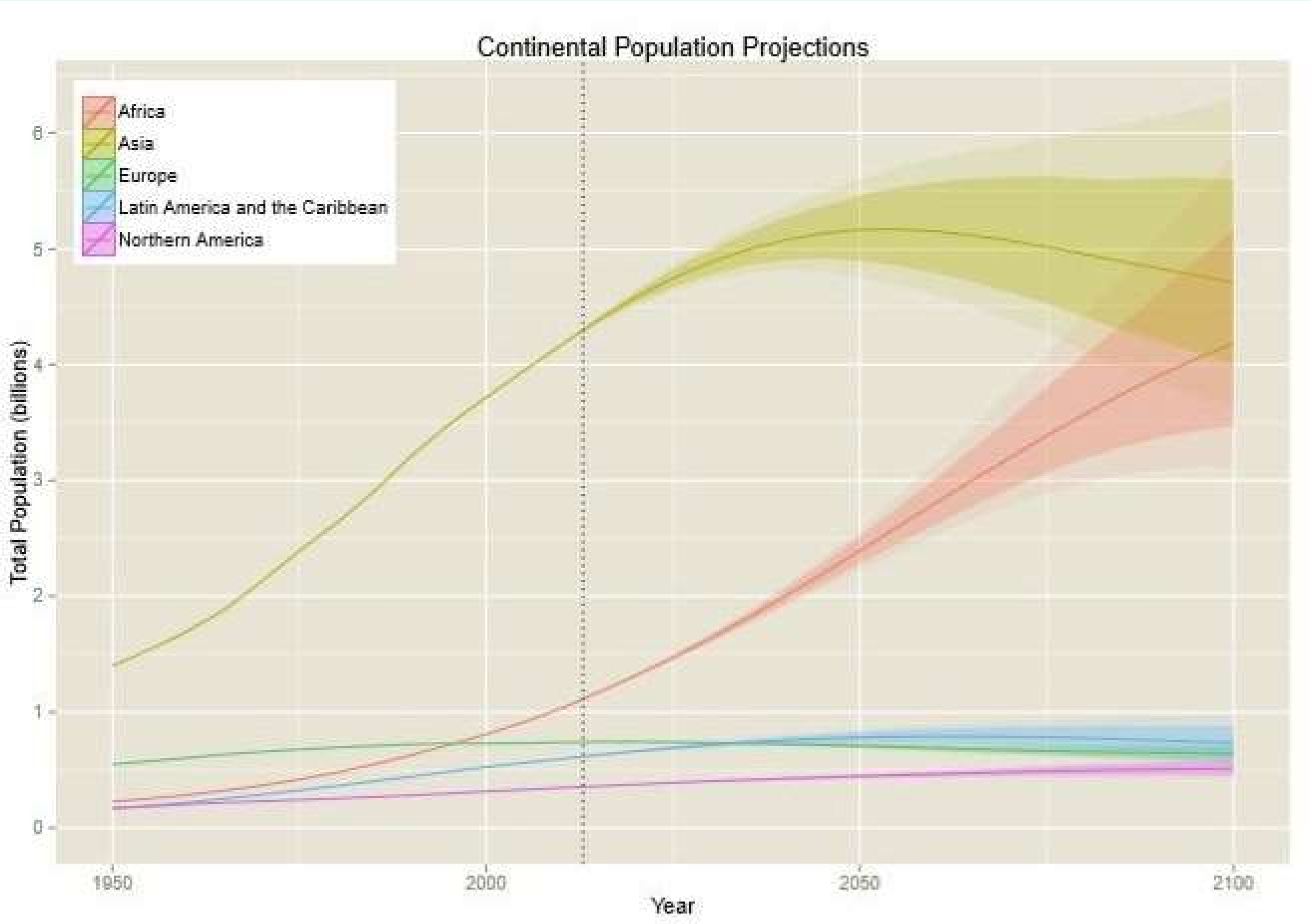
2. threats to economic growth and human livelihoods

•water stress, and water scarcity,

with respect to:
•food security and
•energy security

implying the need for:
•technological innovation and
•water conservation.

Projections of world population by the United Nations have spanned a wide range of scenarios.
These include a 'medium' projection
based on the current and expected global fertility rate, and higher and lower estimates that take into account the uncertainty in that fertility rate.



Asia's population will probably peak and then decline, while Africa's growth will continue unabated, the study predicts.

NATURE | NEWS 18 September 2014

Water uses

- Municipal (8%)
- Agricultural (70%)
- Industrial (22%)

nature OUTLOOK
AGRICULTURE AND DROUGHT
26 September 2013 / Vol 501 / Issue No. 7468

Central to long term food security is implementing a sustainable agriculture.

Farmers need to know how to grow more while using less water in a climate change scenario

3. threats to water-related ecosystem services

- point- and non-point source pollution
- increased water consumption,
- biodiversity loss,

Implying:

the need to co-manage water for human and ecosystem needs, particularly given potential "tipping points" in socioecological systems

3. threats to water-related ecosystem services

- point- and non-point source pollution
- increased water consumption,
- biodiversity loss,

Implying:

the need to co-manage water for human and ecosystem needs, particularly given potential "tipping points" in socioecological systems

4. increased hydrological variability

in the context of climate change (notably increased amplitude and frequency of droughts and floods),

implying the need to:

develop innovative strategies for dealing with uncertainty (reuse) including governance and social learning as key strategies for more effective water management

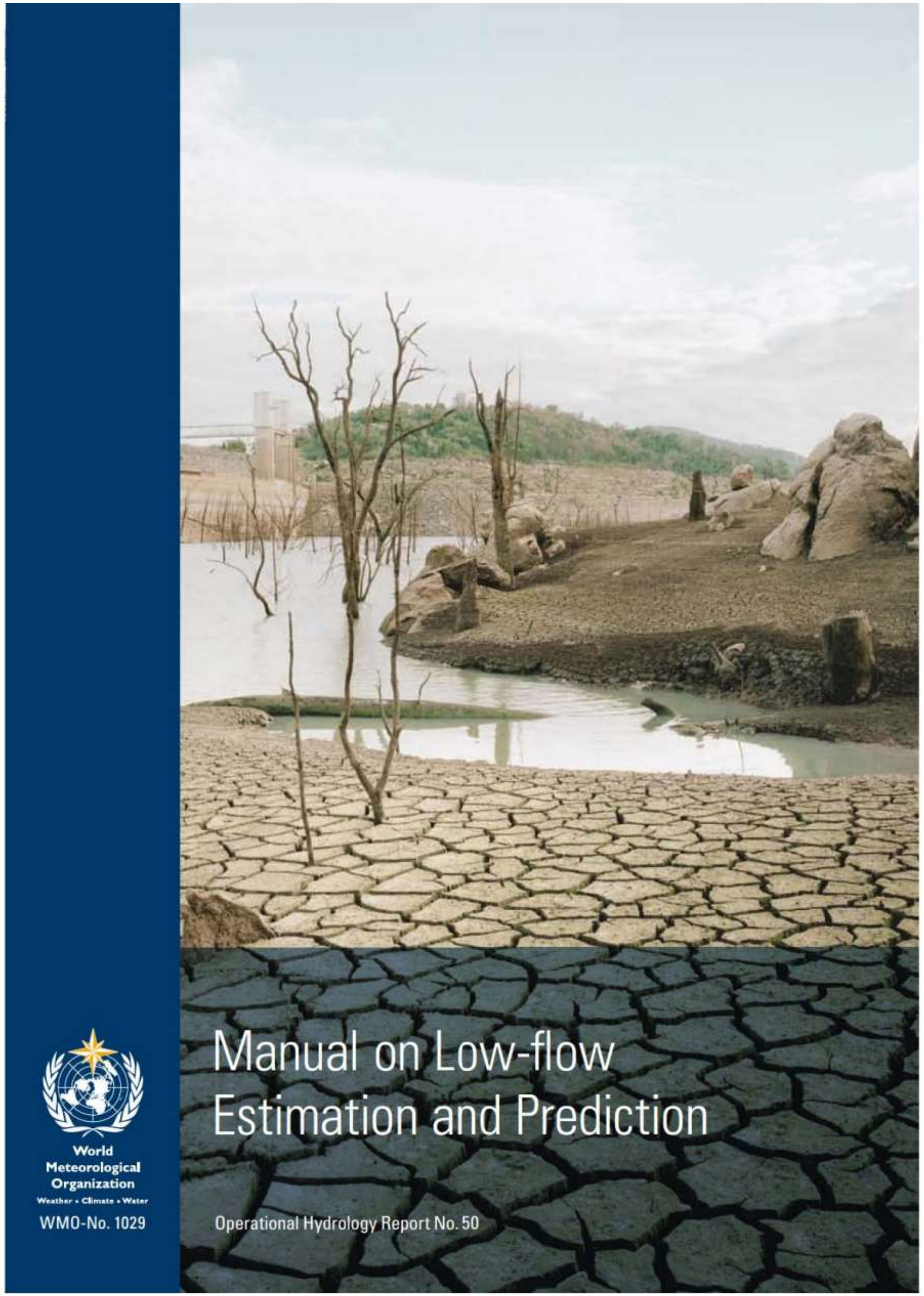
Il rischio di sperimentare uno o piu’ minimi annuali inferiori al valore di riferimento con tempo di ritorno T, nell’arco di N anni è

$$R=1-(1-1/T)^N$$

La definizione di tempo di ritorno è basata sull’ipotesi che le osservazioni usate per la caratterizzazione della distribuzione di probabilità dei minimi siano indipendenti ed estratte da una popolazione omogenea e stazionaria (siano generate dagli stessi processi)

Le cause che maggiormente influenzano il regime delle portate minime dei corsi d’acqua sono:

- Prelievo di portate da corsi d’acqua
- Estrazione dal sottosuolo
- Scarichi nei corpi d’acqua superficiali
- Realizzazione di serbatoi
- Cambio d’uso del territorio



The Probability Distribution of Block Minima

There are situations where the extreme value are the prime interest, as in the case of **minima values in low flow analysis**.

Estimates of the **probability of occurrence of low-flow events** can be derived from historical records using frequency analysis.

The focus of interest in low-flow analysis is the non-exceedance probability p, which is defined for any time interval Δt. The non-exceedance (p) and exceedance probability (1–p) is frequently expressed in terms of the return period, T, which for minimum values is defined as:

$$T = \frac{1}{p} \tag{7.14}$$

where T is the mean time interval between occurrence of an event $X \leq xp$ and the T-year event is given by the corresponding value for xp. Annual non-exceedance probabilities are defined for Δt equal to one year.

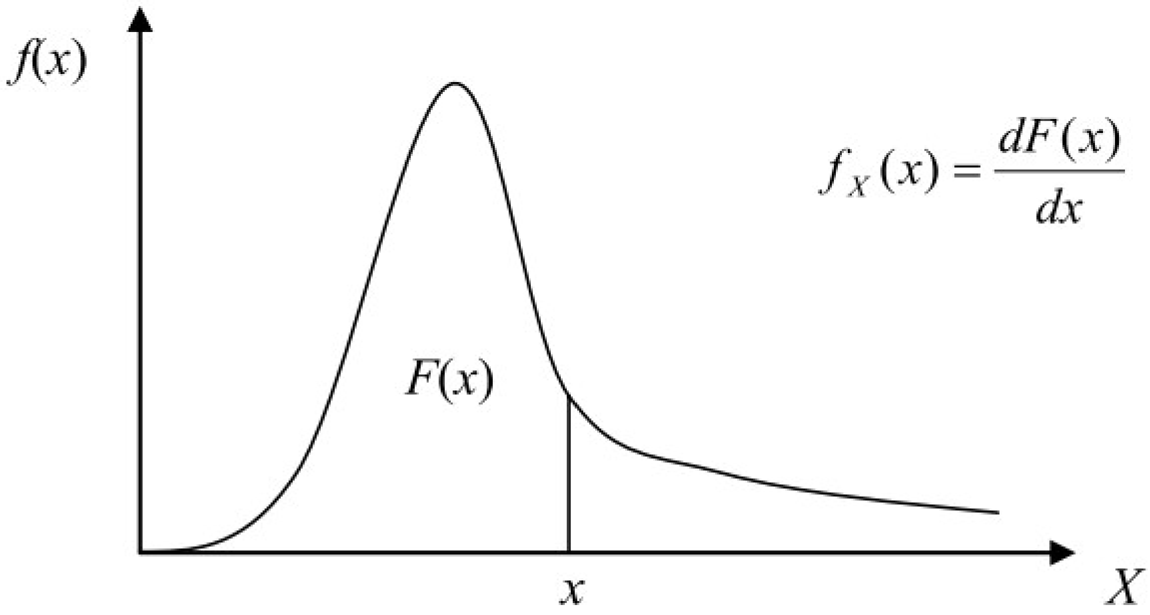


Figure 7.2 *Probability density function, f(x), and non-exceedance probability, F(x)*

The GEV distribution is a general mathematical form that encompasses the three types of limiting distributions (Coles, 2001). Following the Fisher-Tippett theorem or limit laws for maxima (Fisher and Tippett, 1928), the GEV distribution can be defined by:

$$F_X(x) = \exp\left\{-\left[1 - \frac{\kappa(x-\xi)}{\alpha}\right]^{1/\kappa}\right\} \tag{7.5}$$

Per κ=0 la (7.5) perde di significato e viene rimpiazzata dal limite per κ→0 (Gumbel)

The model has three parameters: a location parameter, ξ, a scale parameter, α, and a shape parameter, κ, which controls the tail of the distribution (Figure 7.7). When $\kappa = 0$ it reduces to the Gumbel distribution (EV I); for $\kappa < 0$ it equals the Fréchet-type distribution (EV II); and for $\kappa > 0$ it equals the Weibull-type distribution with a finite upper bound (EV III). The three types of extreme distributions can be compared on a Gumbel probability plot, which plots the observations (x) against the Gumbel reduced variate, y, where $y = -\ln(-\ln F(x))$. A reduced variate, y, substitutes the variable x in the expression for F(x) and is linearly related to x (here, $y = (x-\xi)/\alpha$).

The Weibull probability distribution for minimum values is given as:

$$F(x) = 1 - \exp\left[-\left(\frac{x-\xi}{\alpha}\right)^{\kappa}\right] \qquad (7.6)$$

which for $\kappa = 1$ equals the exponential distribution. The value of ξ (lower bound) should be greater than, or equal to, zero. A two-parameter Weibull distribution (Weibull, 1961) is the EV III distribution for minima bound below by zero ($\xi = 0$).

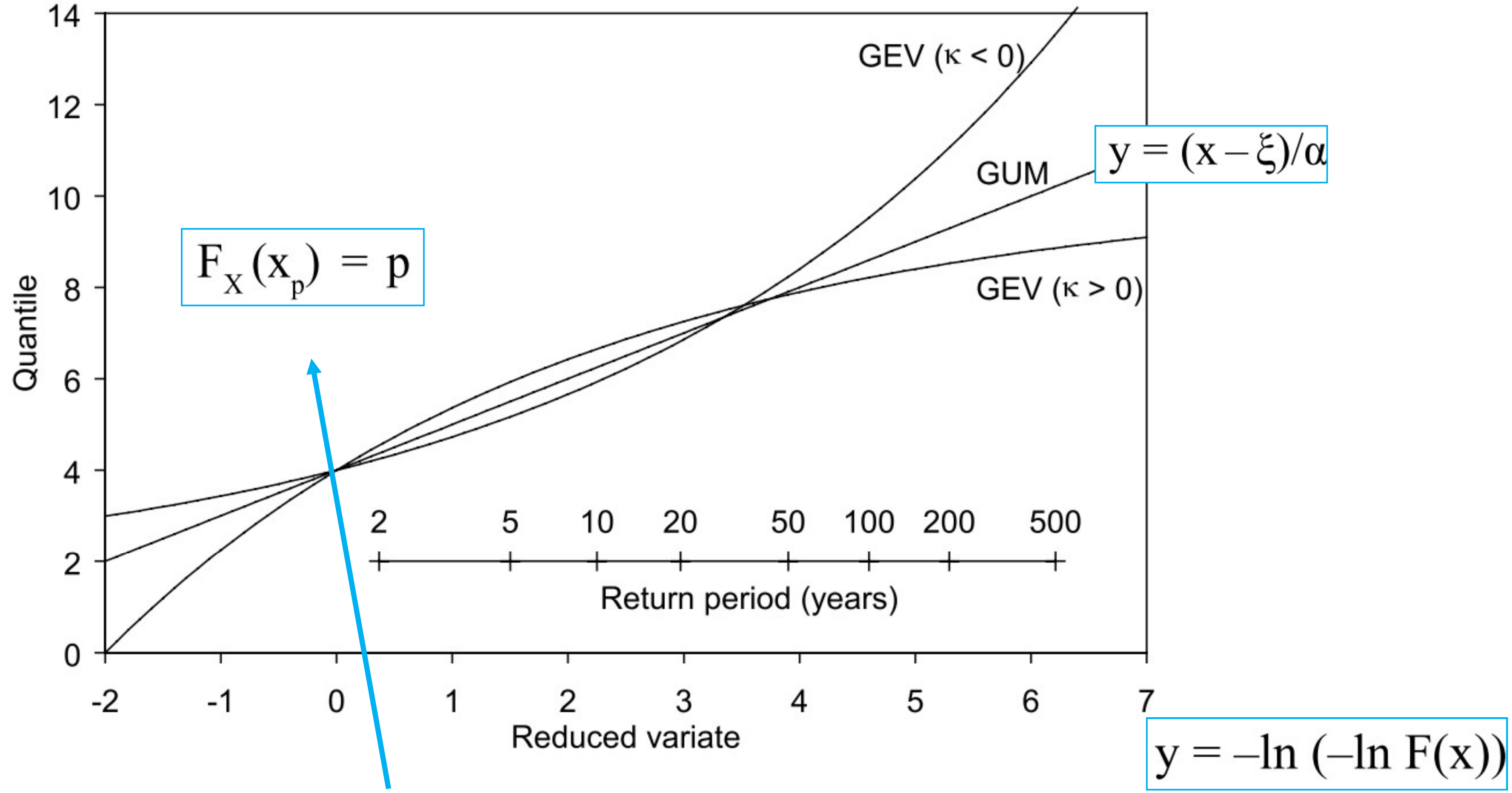


Figure 7.7 *The quantile, x_T, for the three types of GEV distributions plotted against the Gumbel reduced variate, y (from Tallaksen and others, 2004) (the return period is defined here as $1/(1 - F(x))$ as for the traditional case of annual maxima)*

The results confirm the suitability of the Weibull distribution for low-flow frequency analysis.

7.6 Parameter estimation methods

Once the distribution function has been selected, the next step is to estimate the parameters of the distribution from the sample data. In this Manual, the method of moments, including P-moments and L-moments, is presented.

The moment estimators of the parameters are obtained by replacing the theoretical moments for the specified distribution with the sample moments (for example, mean value, variance and skewness).

$$F(x) = 1 - \exp\left[-\left(\frac{x-\xi}{\alpha}\right)^{\kappa}\right]$$

$$\mu = \xi + \alpha\Gamma\left(1+\frac{1}{\kappa}\right)$$

$$\sigma^2 = \alpha^2\left[\Gamma\left(1+\frac{2}{\kappa}\right) - \left[\Gamma\left(1+\frac{1}{\kappa}\right)\right]^2\right]$$

where Γ is the gamma function; ξ is the location parameter; α is the scale parameter; and κ is the shape parameter. If ξ is known, the moment estimate of κ can be obtained by combining equations 7.15 and 7.16, which can be solved using Newton-Raphson iteration.

$$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt$$

7.6.1 P-moments

The first product moment about X = 0 is the mean, μ, or the expected value of X, E{X}. The second moment about the mean is called the variance, σ^2. The standard deviation, σ, is a measure of the spread around the central value and equals the square root of the variance.

$$\hat{\mu} = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{7.7}$$

$$\hat{\sigma}^2 = s^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2 \tag{7.8}$$

The first two moment estimators (equations 7.7 and 7.8) are unbiased and independent of the distribution and sample size

7.6.2 L-moments

The Hosking (1990) method of L-moments has found widespread application in the statistical analyses of hydrological data. L-moments are weighted linear (the "L" is therefore introduced) combinations of the expected order statistics and are analogous to the conventional moments used to summarize the statistical properties of a probability function or an observed dataset. Let X be a real-valued random variable with the cumulative distribution function F(x), and let $X(1{:}n) \le X(2{:}n) \le \ldots \le X(n{:}n)$ be the order statistics of a random sample of size n drawn from the distribution of X. The first four L-moments are then defined as:

the ***k*th order statistic** of a statistical sample is equal to its *k*th-smallest value

$$
\begin{aligned}
\lambda_1 &= E\left\{X_{(1:1)}\right\} \\
\lambda_2 &= \tfrac{1}{2} E\left\{X_{(2:2)} - X_{(1:2)}\right\} \\
\lambda_3 &= \tfrac{1}{3} E\left\{X_{(3:3)} - 2X_{(2:3)} + X_{(1:3)}\right\} \qquad (7.11) \\
\lambda_4 &= \tfrac{1}{4} E\left\{X_{(4:4)} - 3X_{(3:4)} + 3X_{(2:4)} - X_{(1:4)}\right\}
\end{aligned}
$$

E{..}=Select randomly j observation, order $X_{(i,j)}$ and average

$$
\begin{aligned}
\lambda_1 &= E\left\{X_{(1:1)}\right\} \\
\lambda_2 &= \tfrac{1}{2} E\left\{X_{(2:2)} - X_{(1:2)}\right\} \\
\lambda_3 &= \tfrac{1}{3} E\left\{X_{(3:3)} - 2X_{(2:3)} + X_{(1:3)}\right\} \\
\lambda_4 &= \tfrac{1}{4} E\left\{X_{(4:4)} - 3X_{(3:4)} + 3X_{(2:4)} - X_{(1:4)}\right\}
\end{aligned}
\tag{7.11}
$$

The first moment equals the mean ($E\{X\}$), and the second moment is a measure of variation based on the expected difference between two randomly selected observations.

> The main advantage of L-moments is that they suffer less from the effect of sample variability compared with product moment estimators because the calculation does not involve squaring or cubing the observations.

L-moments: Analysis and Estimation of Distributions using Linear Combinations of Order Statistics

By J. R. M. HOSKING†

IBM Research Division, Yorktown Heights, USA

[Received January 1989]

SUMMARY

L-moments are expectations of certain linear combinations of order statistics. They can be defined for any random variable whose mean exists and form the basis of a general theory which covers the summarization and description of theoretical probability distributions, the summarization and description of observed data samples, estimation of parameters and quantiles of probability distributions, and hypothesis tests for probability distributions. The theory involves such established procedures as the use of order statistics and Gini's mean difference statistic, and gives rise to some promising innovations such as the measures of skewness and kurtosis described in Section 2, and new methods of parameter estimation for several distributions. The theory of L-moments parallels the theory of (conventional) moments, as this list of applications might suggest. The main advantage of L-moments over conventional moments is that L-moments, being linear functions of the data, suffer less from the effects of sampling variability: L-moments are more robust than conventional moments to outliers in the data and enable more secure inferences to be made from small samples about an underlying probability distribution. L-moments sometimes yield more efficient parameter estimates than the maximum likelihood estimates.

2. L-MOMENTS OF PROBABILITY DISTRIBUTIONS

2.1. *Definitions and Basic Properties*

Let X be a real-valued random variable with cumulative distribution function $F(x)$ and quantile function $x(F)$, and let $X_{1:n} \leqslant X_{2:n} \leqslant \ldots \leqslant X_{n:n}$ be the order statistics of a random sample of size n drawn from the distribution of X. Define the *L-moments* of X to be the quantities

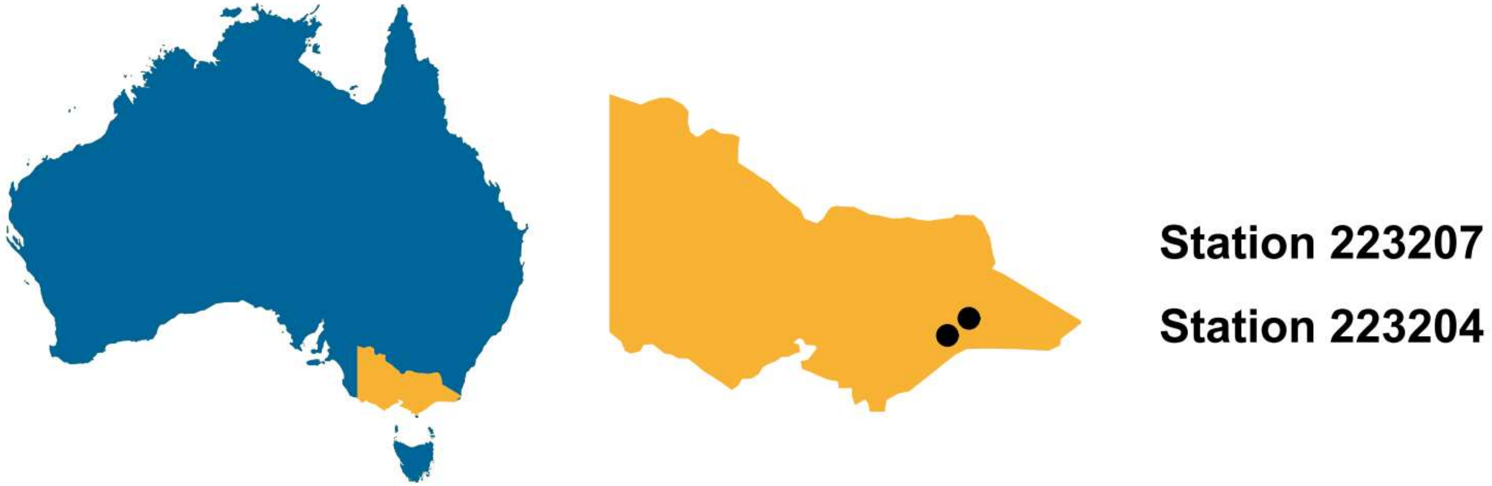
$$\lambda_r \equiv r^{-1} \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} EX_{r-k:r}, \qquad r = 1, 2, \ldots. \tag{2.1}$$

The L in 'L-moments' emphasizes that λ_r is a *linear* function of the expected order statistics. Furthermore, as will be seen in Section 3, the natural estimator of λ_r based on an observed sample of data is a linear combination of the ordered data values, i.e. an L-statistic. The expectation of an order statistic may be written as

$$EX_{j:r} = \frac{r!}{(j-1)!(r-j)!} \int x\{F(x)\}^{j-1}\{1 - F(x)\}^{r-j}\, \mathrm{d}F(x)$$

$$
\begin{aligned}
\lambda_1 &= EX = \int_0^1 x(F)\,\mathrm{d}F, \\
\lambda_2 &= \tfrac{1}{2}E(X_{2:2} - X_{1:2}) = \int_0^1 x(F)\,(2F - 1)\,\mathrm{d}F, \\
\lambda_3 &= \tfrac{1}{3}E(X_{3:3} - 2X_{2:3} + X_{1:3}) = \int_0^1 x(F)\,(6F^2 - 6F + 1)\,\mathrm{d}F, \\
\lambda_4 &= \tfrac{1}{4}E(X_{4:4} - 3X_{3:4} + 3X_{2:4} - X_{1:4}) = \int_0^1 x(F)\,(20F^3 - 30F^2 + 12F - 1)\,\mathrm{d}F.
\end{aligned}
\tag{2.4}
$$

The use of L-moments to describe probability distributions is justified by the following theorem.



Station 223204: 8.0, 13.0, 2.0, 2.7, 9.1, 0.0, 0.7, 21.6, 35.1, 12.7, 1.3, 9.1, 26.0, 22.7, 21.3, 7.9, 23.0, 6.0, 3.1, 4.1, 0.0, 4.9, 2.3, 12.4, 4.4, 2.1, 3.1, 7.6, 9.0, 9.0, 20.4, 13.6, 3.3, 10.3

Station 223207: 27.3, 38.0, 72.9, 53.6, 45.7, 37.0, 21.7, 36.6, 43.1, 12.6, 20.7, 66.7, 76.6, 53.1, 15.7, 66.6, 78.0, 53.1, 62.6, 40.7, 51.6, 23.1, 23.4, 23.0, 6.0

The moments of the Weibull distribution can be calculated from:

$$\mu = \xi + \alpha\Gamma\left(1+\frac{1}{\kappa}\right) \tag{7.15}$$

$$\sigma^2 = \alpha^2\left[\Gamma\left(1+\frac{2}{\kappa}\right)-\left[\Gamma\left(1+\frac{1}{\kappa}\right)\right]^2\right] \tag{7.16}$$

$$\gamma_3 = \frac{\Gamma\left(1+\frac{3}{\kappa}\right)-3\Gamma\left(1+\frac{1}{\kappa}\right)\Gamma\left(1+\frac{2}{\kappa}\right)+2\left[\Gamma\left(1+\frac{1}{\kappa}\right)\right]^3}{\left[\Gamma\left(1+\frac{2}{\kappa}\right)-\left[\Gamma\left(1+\frac{1}{\kappa}\right)\right]^2\right]^{3/2}} \tag{7.17}$$

where Γ is the gamma function; ξ is the location parameter; α is the scale parameter; and κ is the shape parameter. If ξ is known, the moment estimate of κ can be obtained by combining equations 7.15 and 7.16, which can be solved using Newton-Raphson iteration. The moment estimate of α is then given by:

$$\hat{\alpha} = \frac{\hat{\mu} - \xi}{\Gamma\left(1 + \frac{1}{\hat{\kappa}}\right)} \tag{7.18}$$

The L-moments of the Weibull distribution can be estimated from:

$$\lambda_1 = \xi + \alpha\Gamma\left(1+\frac{1}{\kappa}\right) \tag{7.19}$$

$$\lambda_2 = \alpha\left(1-2^{-1/\kappa}\right)\Gamma\left(1+\frac{1}{\kappa}\right) \tag{7.20}$$

$$\tau_3 = 3 - \frac{2\left(1-3^{-1/\kappa}\right)}{1-2^{-1/\kappa}} \tag{7.21}$$

If ξ is known, L-moment estimates of κ and α are given by:

$$\hat{\kappa} = -\frac{\ln 2}{\ln\left(1-\frac{\hat{\lambda}_2}{\hat{\lambda}_1}\right)} \quad , \quad \hat{\alpha} = \frac{\hat{\lambda}_1 - \xi}{\Gamma\left(1+\frac{1}{\hat{\kappa}}\right)} \tag{7.22}$$

If ξ is unknown, expressions for the parameters ξ, κ and α can be derived from the first three moments

Step I: Derivation of empirical quantiles

(a) Rank the AM(7) values in ascending order and give the smallest value rank 1 (i =1);

(b) Derive the non-exceedance probability, F(x) = p, for each value using the Weibull plotting position formula (equation 7.3 for a = 0);

(c) Plot the AM(7) values against F(x) in a probability plot (section 7.4).

$$p_i = i – a/(n+1–2a) \qquad (7.3)$$

$$a = 0 \rightarrow p_i = \frac{i}{n+1}$$

Step II: Fitting the 2-parameter Weibull distribution by P-moments

(a) Calculate an estimate of the first two P-moments for the AM(7) series (Table 7.1);

(b) Combine equations 7.15 and 7.16 to obtain an estimate of κ, using a Newton-Rapshon iteration scheme. Alternatively, create a table (or plot) to depict the relationship between the skewness of the Weibull distribution (equation 7.17) and $1/\kappa$, and use the table (or plot) to interpolate $1/\kappa$ corresponding to the sample skewness (equation 7.9) of the low-flow series;

(c) Estimate α by substituting the interpolated $1/\kappa$ into equation 7.18 for $\xi = 0$;

(d) Estimate $F(x)$ as given by equation 7.6 (here, ξ is zero).

Step III: Fitting the 2-parameter Weibull distribution by L-moments

(a) Calculate an estimate of the first two L-moments for the AM(7) series, both with and without the zero values included (Table 7.1);

(b) Obtain an estimate of κ and then α from equation 7.22 for both F(x) and G(x), where G(x) is fitted to the non-zero values;

(c) Estimate F(x) (based on all values including the zero observations), G(x) and F(x)* as given by equation 7.4 (the percentage of zero values is six for Station 223204, that is, $p_0 = 0.06$).

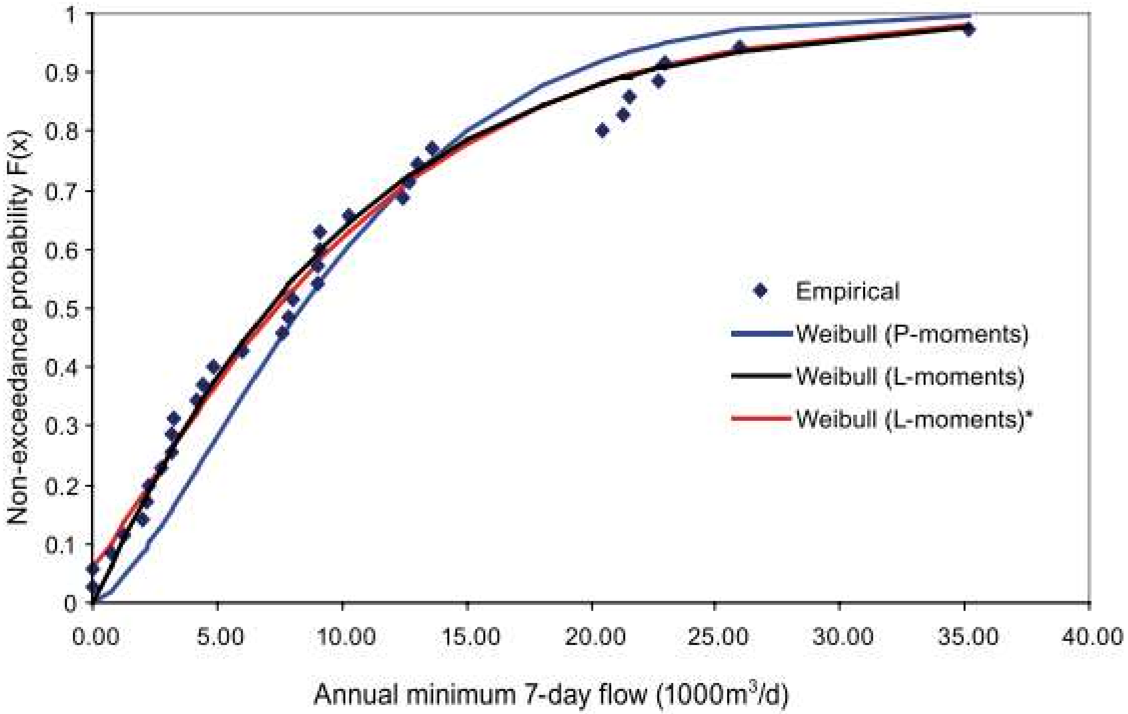


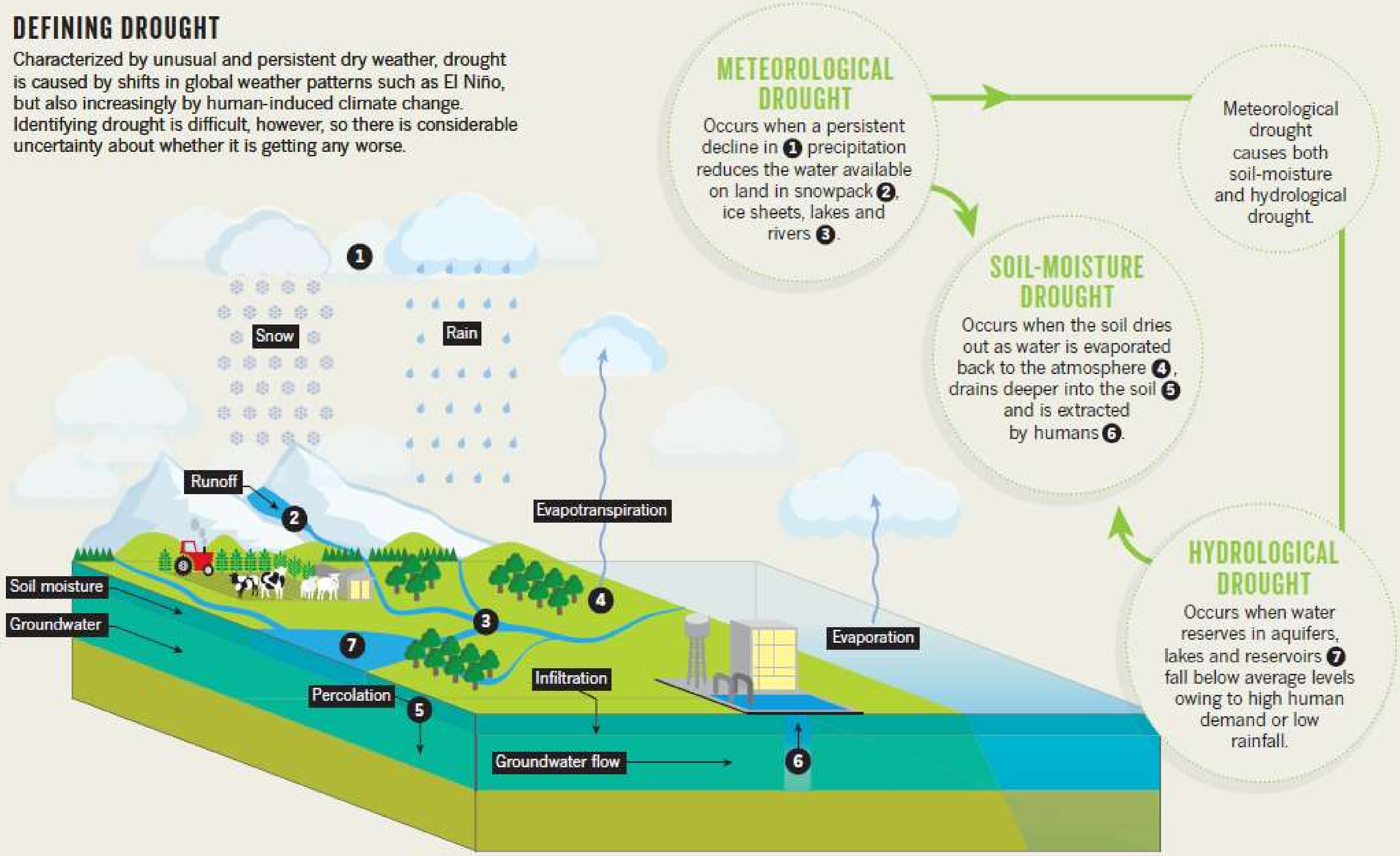
Figure 7.8 *Probability plot showing the empirical quantiles (observations) against the Weibull distribution for the AM(7) flow series of the Nicholson River (Station 223204) (* the estimate is obtained using the expression for F(x)*)*

Station	Method of P-moments			Method of L-moments		
	$1/\kappa$	κ	α	$1/\kappa$	κ	α
223204; F(x)	0.695	1.44	10.76	0.953	1.05	9.953
223204; G(x)				0.863	1.16	10.926
223207; F(x)	0.3265	3.06	46.96	0.500	2.00	47.365

drought

DEFINING DROUGHT

Characterized by unusual and persistent dry weather, drought is caused by shifts in global weather patterns such as El Niño, but also increasingly by human-induced climate change. Identifying drought is difficult, however, so there is considerable uncertainty about whether it is getting any worse.



(i) **Meteorological drought:** It is a situation when there is a significant decrease in rainfall from the normal over an area.

(ii) **Hydrological drought:** Meteorological drought, if prolonged, results in hydrological drought with marked depletion of surface water and consequent drying up of inland water bodies such as lakes, reservoirs, streams and rivers and fall in level of water table.

(iii) **Agricultural drought:** It occurs when soil moisture and rainfall are inadequate to support crop growth to maturity and cause extreme crop stress leading to the loss of yield.

Socioeconomic drought is also defined. Socioeconomic drought occurs when physical water shortages start to affect the health, well being and quality of life of the people or when the drought starts to affect the supply and demand of an economic product.

Water security is the availability of an acceptable **quantity and quality of water** for health, livelihoods, ecosystems and production, coupled with an acceptable **level of water-related risks to people, environments and economies**.

Grey & Sadoff, Water Policy, 2007

HYDRAULIC RISK ANALYSIS = complexity + interdisciplinarity + variability + multiscale perspective + non stationariety…

STIMA DELLE PORTATE MINIME IN ASSENZA DI SERIE STORICHE

Metodi empirici per la stima di indici basati sulla definizione di equazioni matematiche semplici (non descrivono funzioni fisiche del bacino nè la caratterizzazione statistica delle portate)

Metodi statistici (basati su analisi di correlazione)

Metodi basati sulla modellazione della trasformazione afflussi deflussi (l'aleatorietà viene attribuita alla precipitazione e la previsione dipende dalla risposta idrologica del bacino ad un impulse di precipitazione)

Metodi empirici

La serie delle misurazioni disponibile in una sezione di chiusura di un bacino viene adattata ad una sezione di chiusura di un altro bacino caratterizzato da risposta idrologica simile

Streamflow estimation for ungauged catchments by transposing gauged streamflow data from an analogue catchment is a widely used technique requiring the re-scaling of the flow regime to the ungauged target catchment. These techniques all take the following form:

$$QX_T = \mathrm{fn}\left(\frac{A_T}{A_A}\right)QX_A \tag{9.1}$$

where:

QX_T = the flow in the target ungauged catchment T;
QX_A = the corresponding flow in the analogue catchment A;
A_T = the catchment area for the ungauged catchment T;
A_A = the catchment area for the analogue catchment A;
fn = a scaling constant or function.

Metodi statistici

Mettono in relazione portate e caratteristiche fisiche e climatiche del bacino con un procedimento di regionalizzazione (che spesso è alla basa della descrizione della distribuzione spaziale e statistica delle precipitazioni)

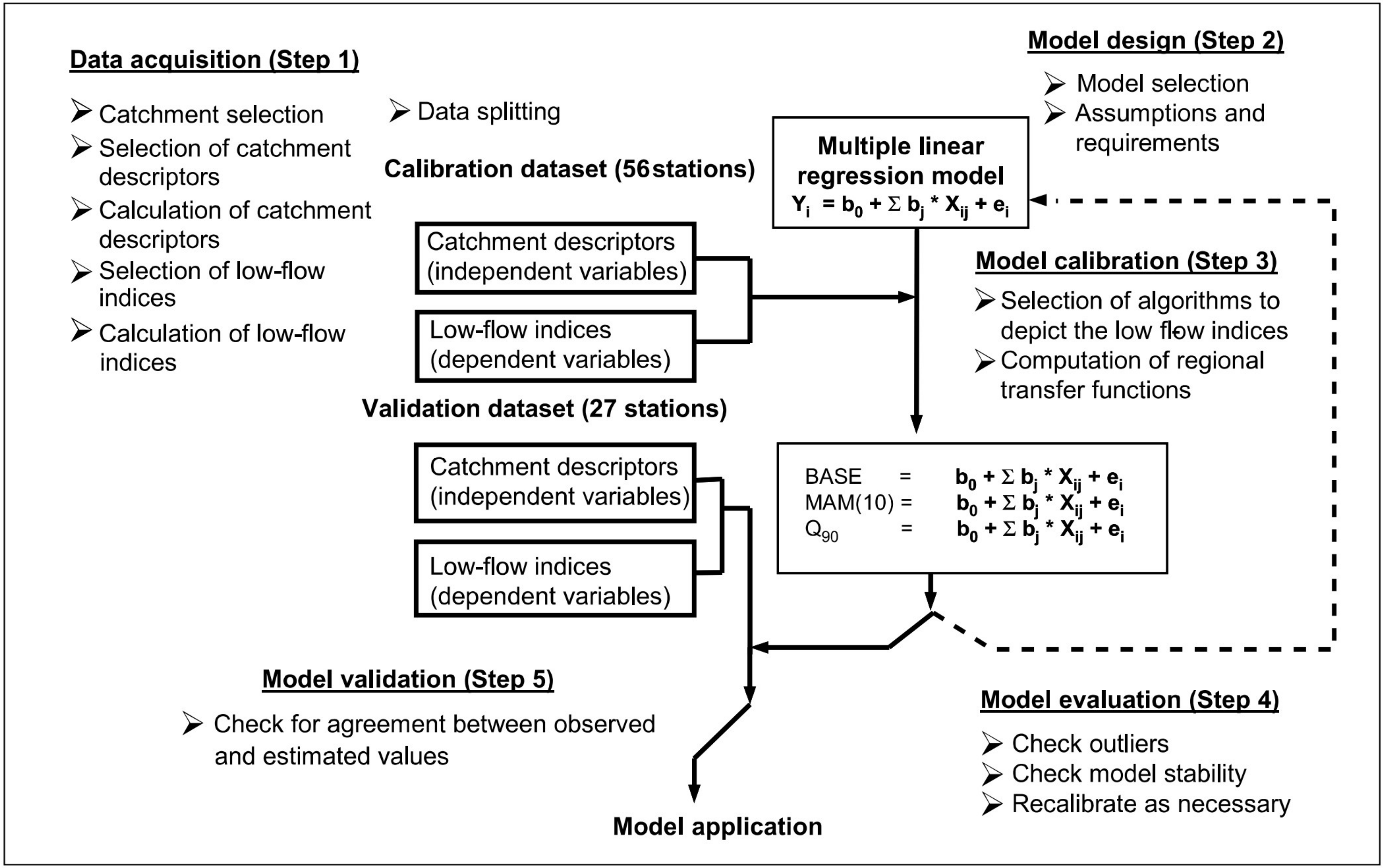


Figure 9.1 *Regionalization procedure using multivariate regression (modified according to Demuth, 2004)*

Metodi basati sulla modellazione della trasformazione afflussi deflussi

I modelli di trasformazione afflussi deflussi sono rappresentazioni dell’idrologia del bacino, usati per elaborare serie di dati di precipitazione per ottenere serie di variabili idrologiche (portate: scorrimento superficiale, infiltrazione e deflussi superficiali e sotterranei unitamente ai volumi invasati sopra suolo, sotto suolo e nei corpi idrici)

I modelli concettuali ed I modelli fisicamente basati, possono tenere conto dei cambiamenti climatici, delle politiche di gestione delle risorse idriche e dei cambi d’uso del territorio.