

An Introduction to Persistent Homology

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Abstract

The fundamental idea behind topological data analysis (TDA) is that the global shape of data carries important information. TDA can thus be viewed as an approach to the analysis of datasets using techniques from algebraic topology and other mathematical fields to study their shape. It provides a framework for investigating the global and qualitative features of data, even in the presence of noise. A central problem, however, is that of scale: to study the shape of a point cloud one must choose a scale parameter, and different choices may reveal different topological features.

Persistent homology was introduced in this setting to address precisely this problem. It extends simplicial homology from a single complex to a filtration of complexes. Applying homology to a filtration yields a family of vector spaces connected by linear maps; the ranks of the induced maps between homology groups are the persistent Betti numbers, which encode which topological features persist across scales. This information can be represented by barcodes and persistence diagrams. Although it originated in TDA, the construction can also be formalised as a mathematical concept in its own right, within the framework of persistence modules.

The aim of this seminar is to introduce and motivate persistent homology and its topological foundations. Starting from a brief review of simplicial homology, we will define persistent homology, persistent Betti numbers and barcodes. We will then introduce constructions such as Čech, Vietoris–Rips and alpha complexes, together with their associated filtrations, illustrating how persistent homology is applied in TDA. We will also state a sampling theorem, which provides a theoretical justification for recovering topological information from data. Finally, we will briefly introduce persistence modules as a more general algebraic framework.

References

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