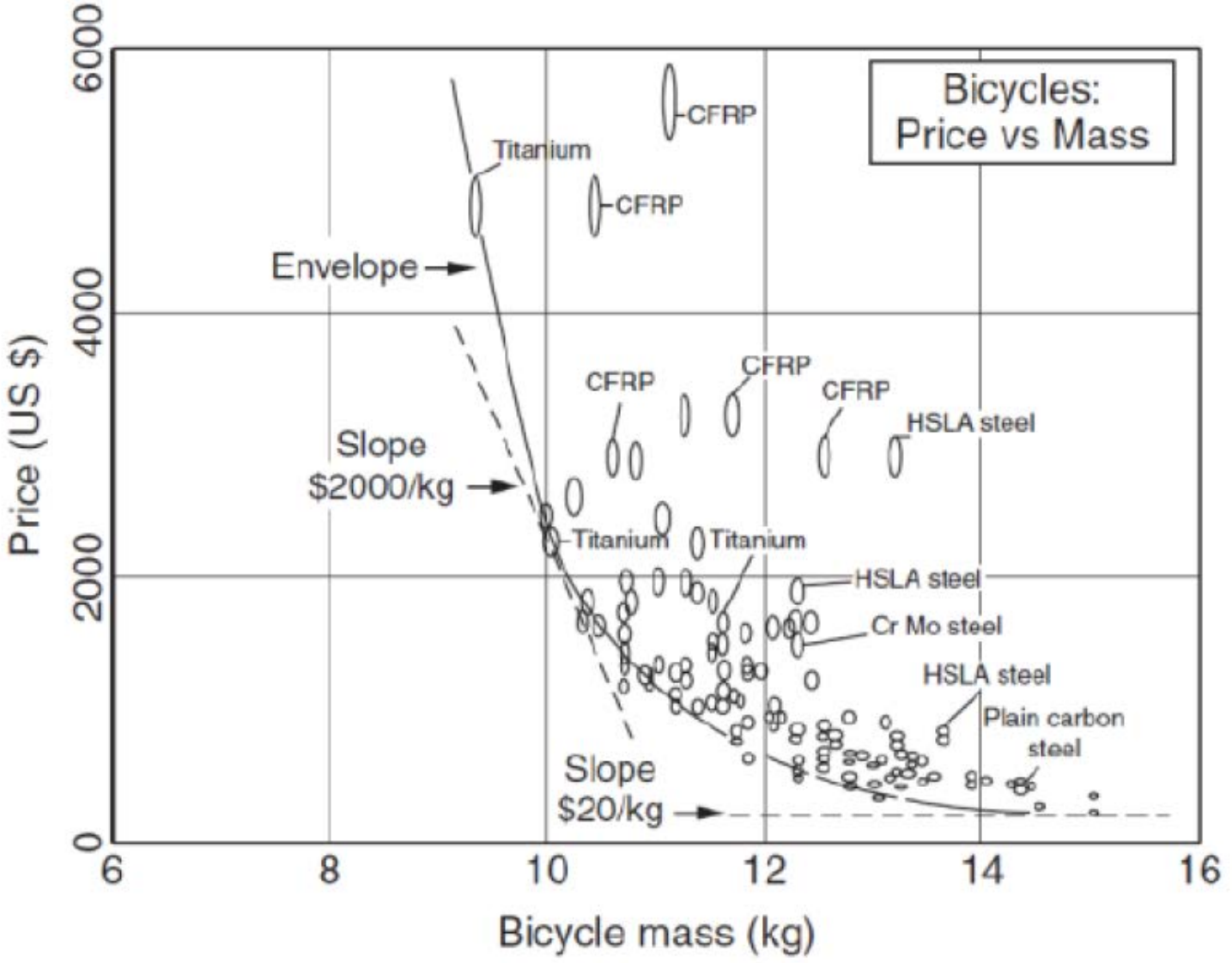
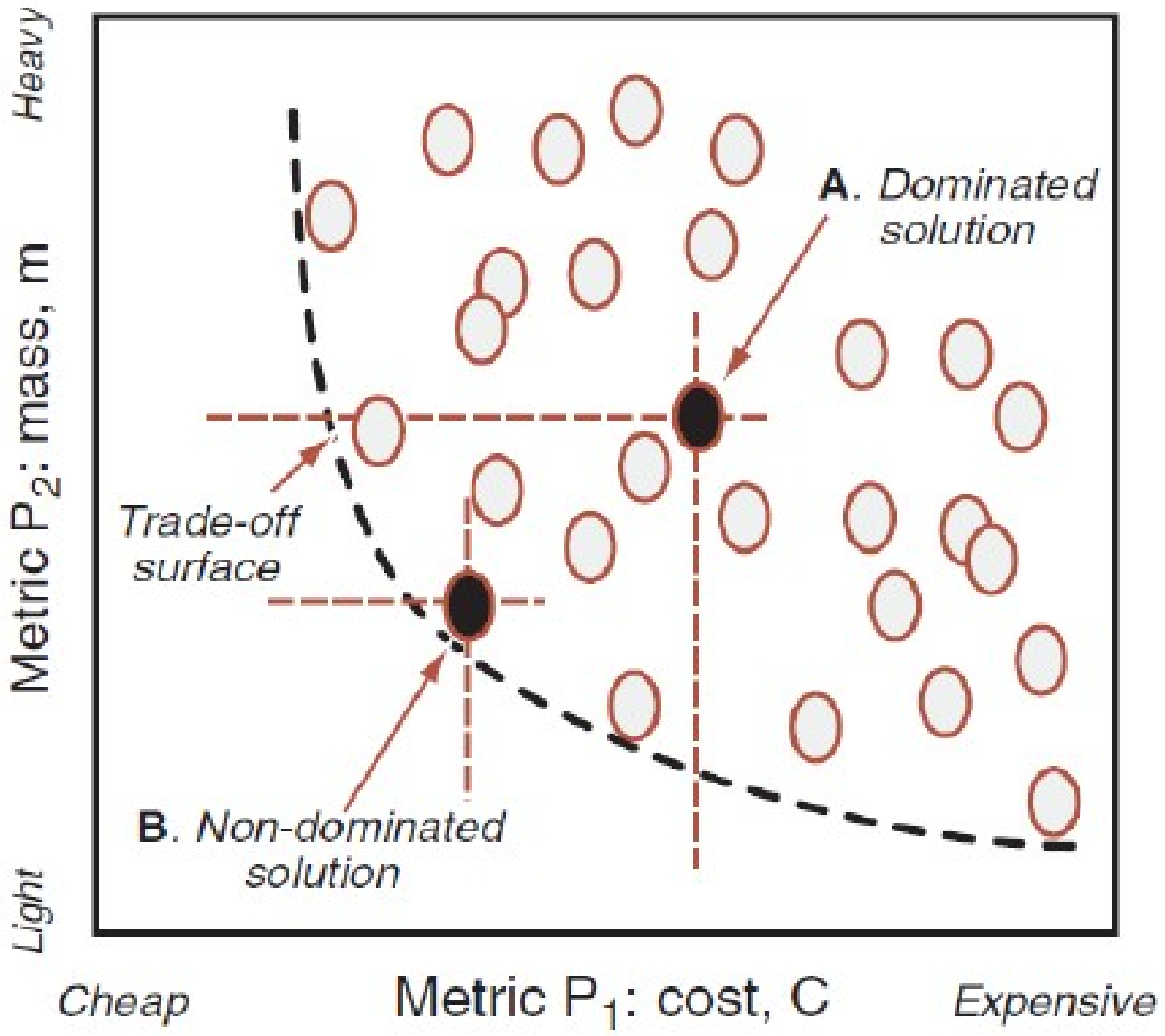
Conflicting objectives

Trade-off analysis

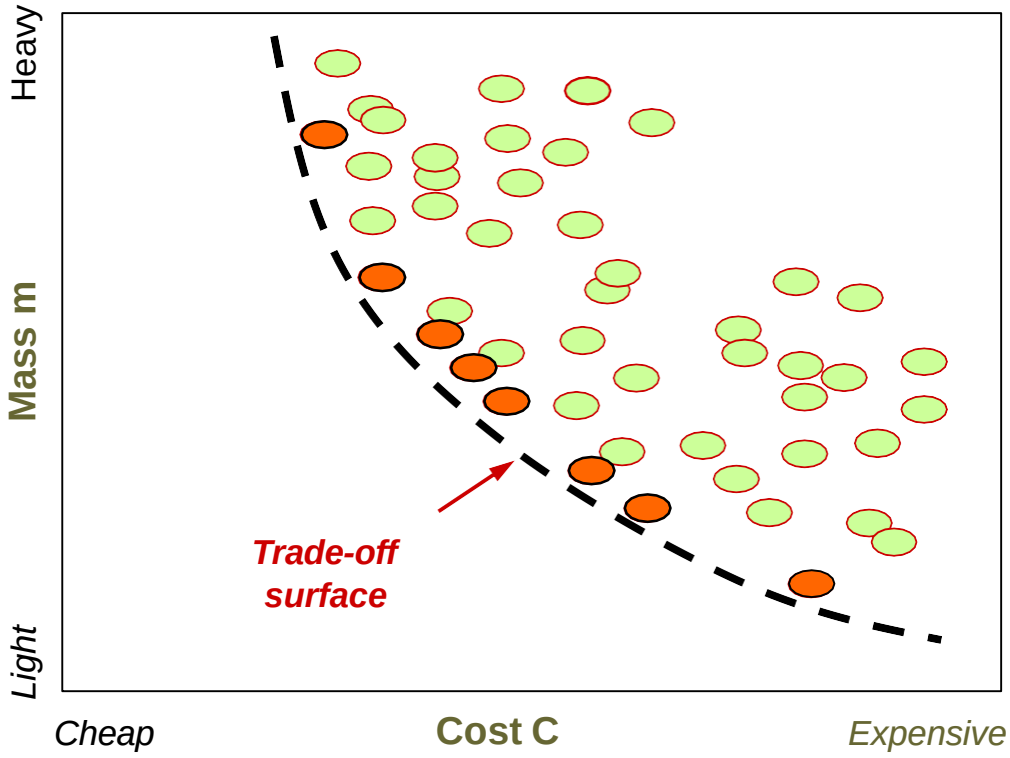


Primary selection: recognition of non-dominated solutions



Finding a compromise: strategy 1

- **Make trade-off plot**
- **Sketch trade-off surface**
- **Use intuition** to select a solution on the trade-off surface

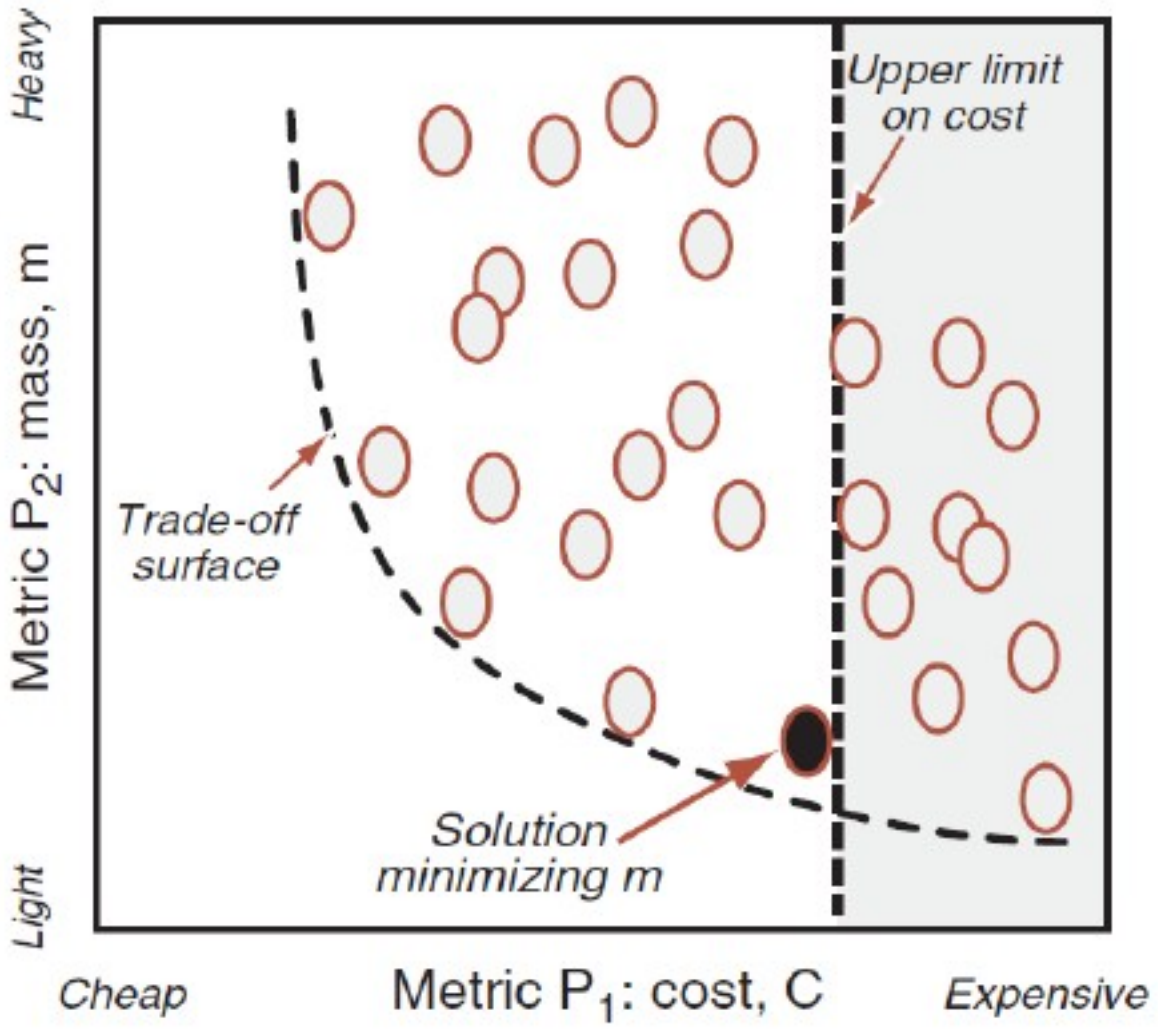


- "Solutions" nearest the surface offer the best **compromise** between mass and cost
- 8 out of 50

- Choose from among these - depends on how highly you value light weight

Constraints may cut some solutions, dominated and not-dominated

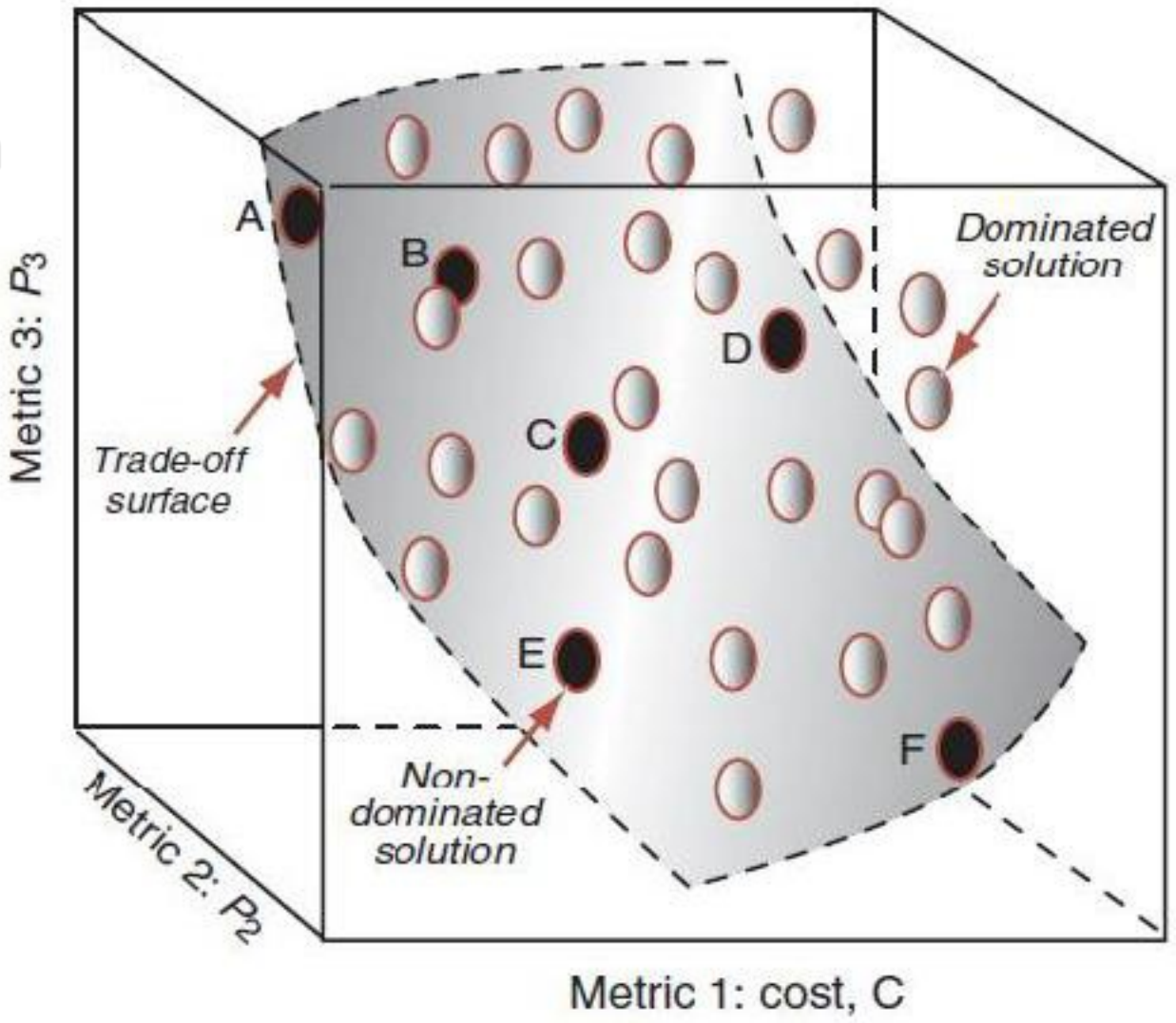
E.g. we do not want to pay more than a threshold value (upper limit on cost)



We could analyse 2 properties combined with cost…

Simple concept

Hard implementation



Penalty function

$$Z = C + \alpha \cdot m$$

Linear combination of characters that should be minimized («decreasing»)

Exchange constant, α, expressing the impact on cost of mass

Easy application to transportation problems

$$C_{\text{service}} = \alpha \cdot m$$

$$C_{\text{total}} = C_{\text{purchase}} + C_{\text{service}} = C_{\text{purchase}} + \alpha \cdot m$$

$$Z = C_{\text{purchase}} + \alpha \cdot m$$

$$\boxed{Z = C_{\text{fixed}} + \alpha \cdot m}$$

Materials for transport systems



Steel



Steel / Alu



Alu / (composite)



Alu / Ti / composites



Composites

Choice of material depends on system

- **Mass**, in transport systems, means **fuel**
- **Life cost** = Initial cost, **C** ($) + Fuel cost over life, scaling with mass **m** (kg)
- **Penalty function** $Z = \mathbf{C} + \alpha m$ (α in $/kg)
- Must first establish **exchange constant,** α

The exchange constant α for transport

Exchange constants for transport systems

Transport system	α ($ per kg)
Family car	3 to 6
Truck	5 to 20
Civil aircraft	100 to 500
Military hardware	500 to 2000
Space vehicle	3000 to 10,000



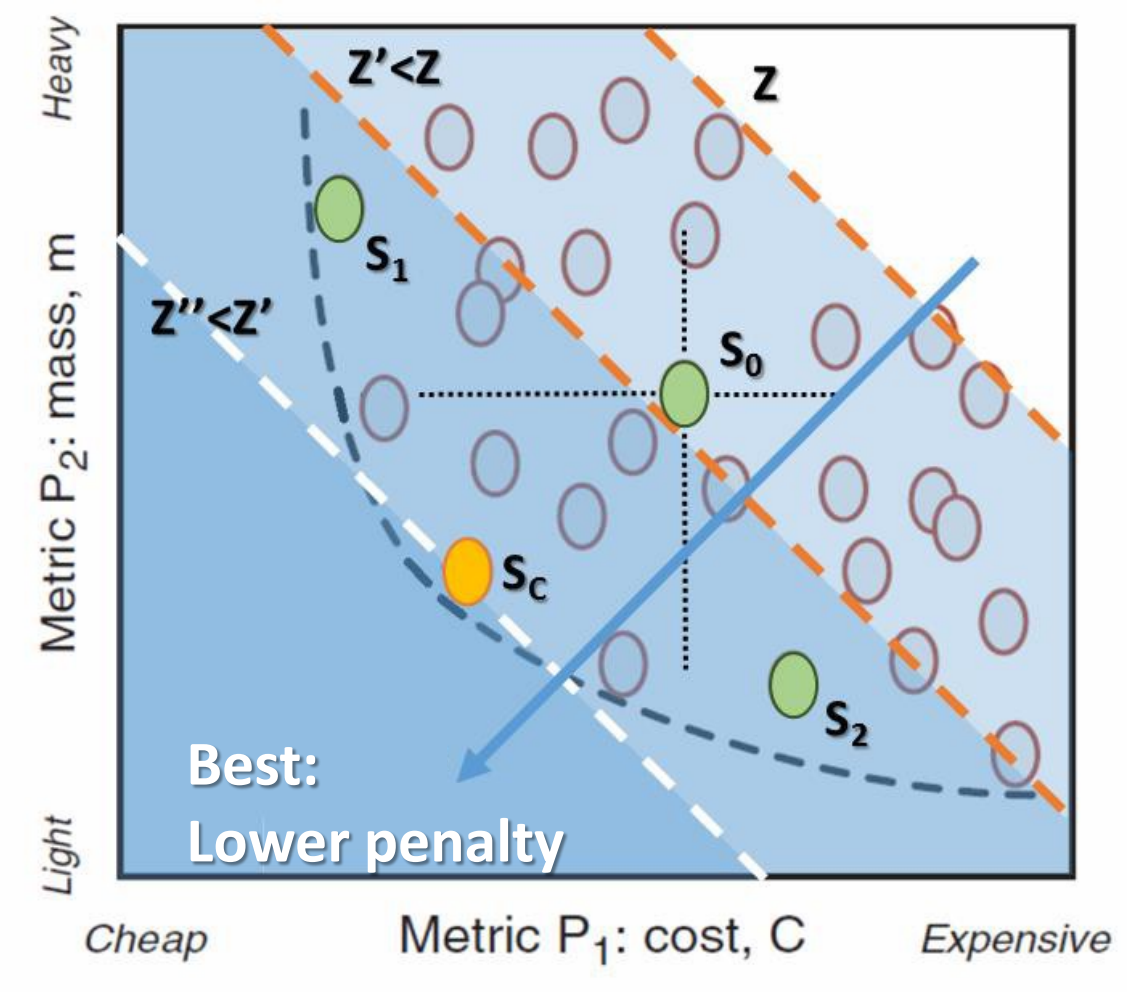
How to get values of α?

- Full life costing: fuel saving
- Value of extra payload
- Extrapolation of historic data

Graphical selection

$$Z = C + \alpha \cdot m \quad \rightarrow \quad m = -\frac{1}{\alpha} \cdot C + \frac{Z}{\alpha}$$

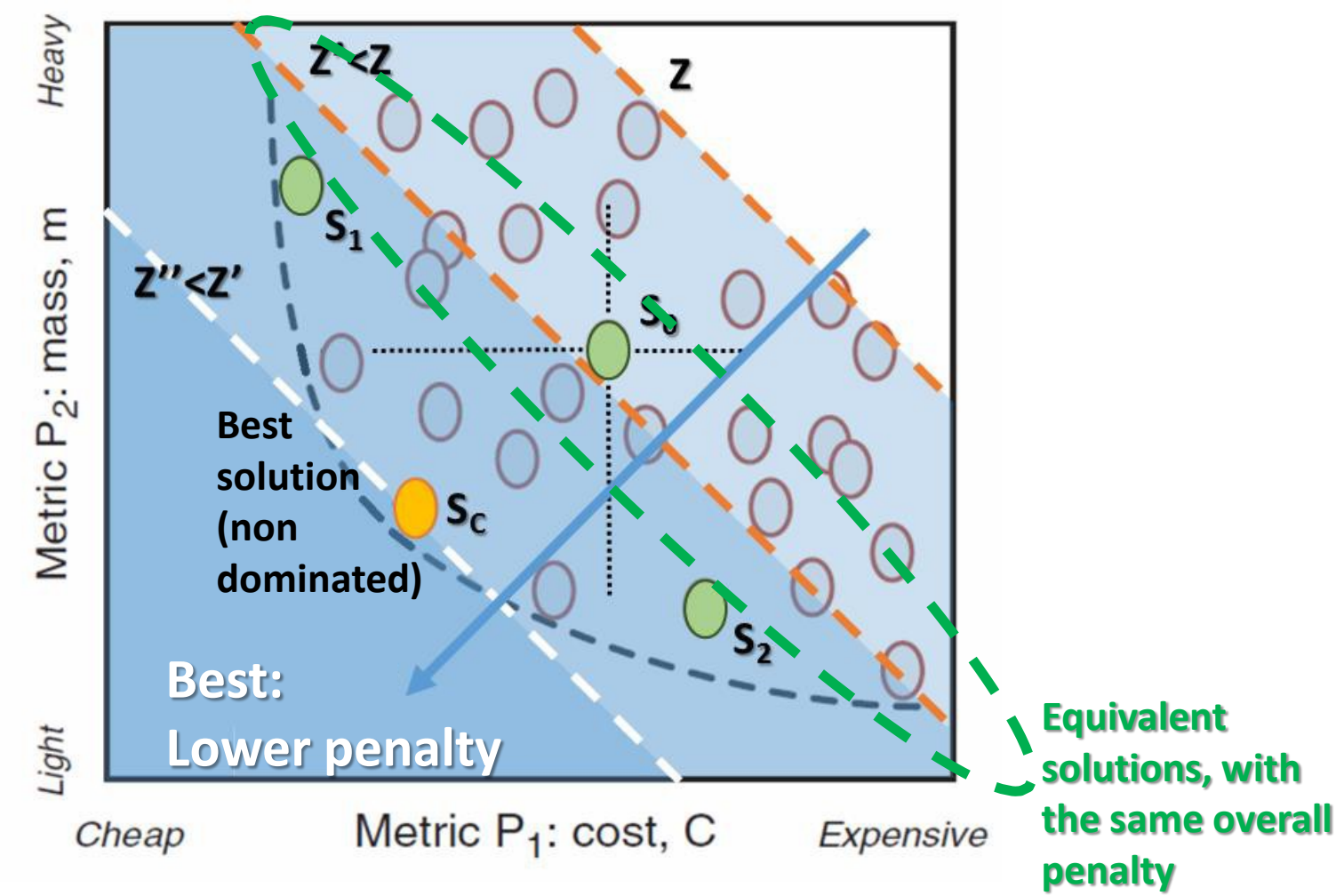
Slope $-1/\alpha$



Graphical selection

$$Z = C + \alpha \cdot m \quad \rightarrow \quad m = -\frac{1}{\alpha} \cdot C + \frac{Z}{\alpha}$$

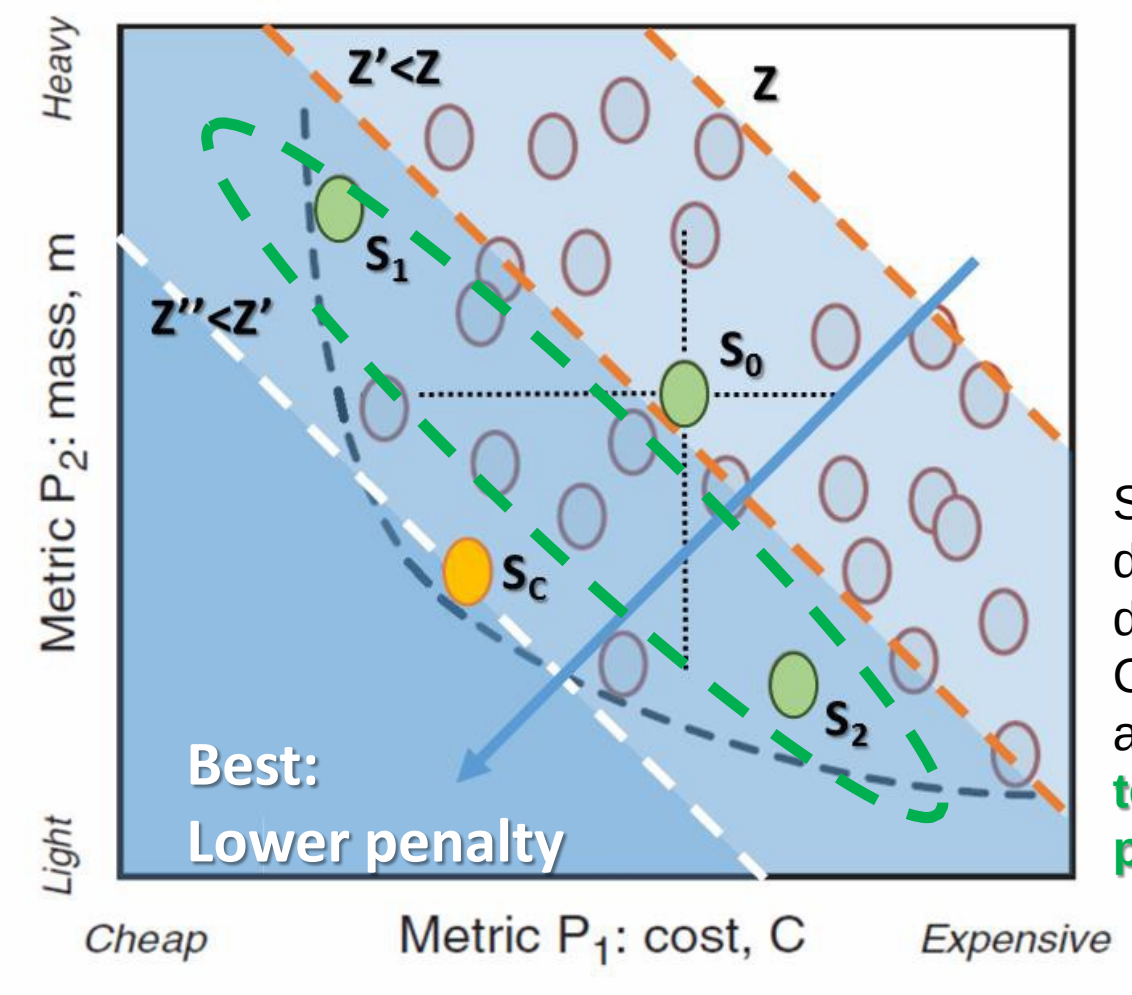
Slope $-1/\alpha$



Graphical selection

$$Z = C + \alpha \cdot m \quad \rightarrow \quad m = -\frac{1}{\alpha} \cdot C + \frac{Z}{\alpha}$$

Slope $-1/\alpha$

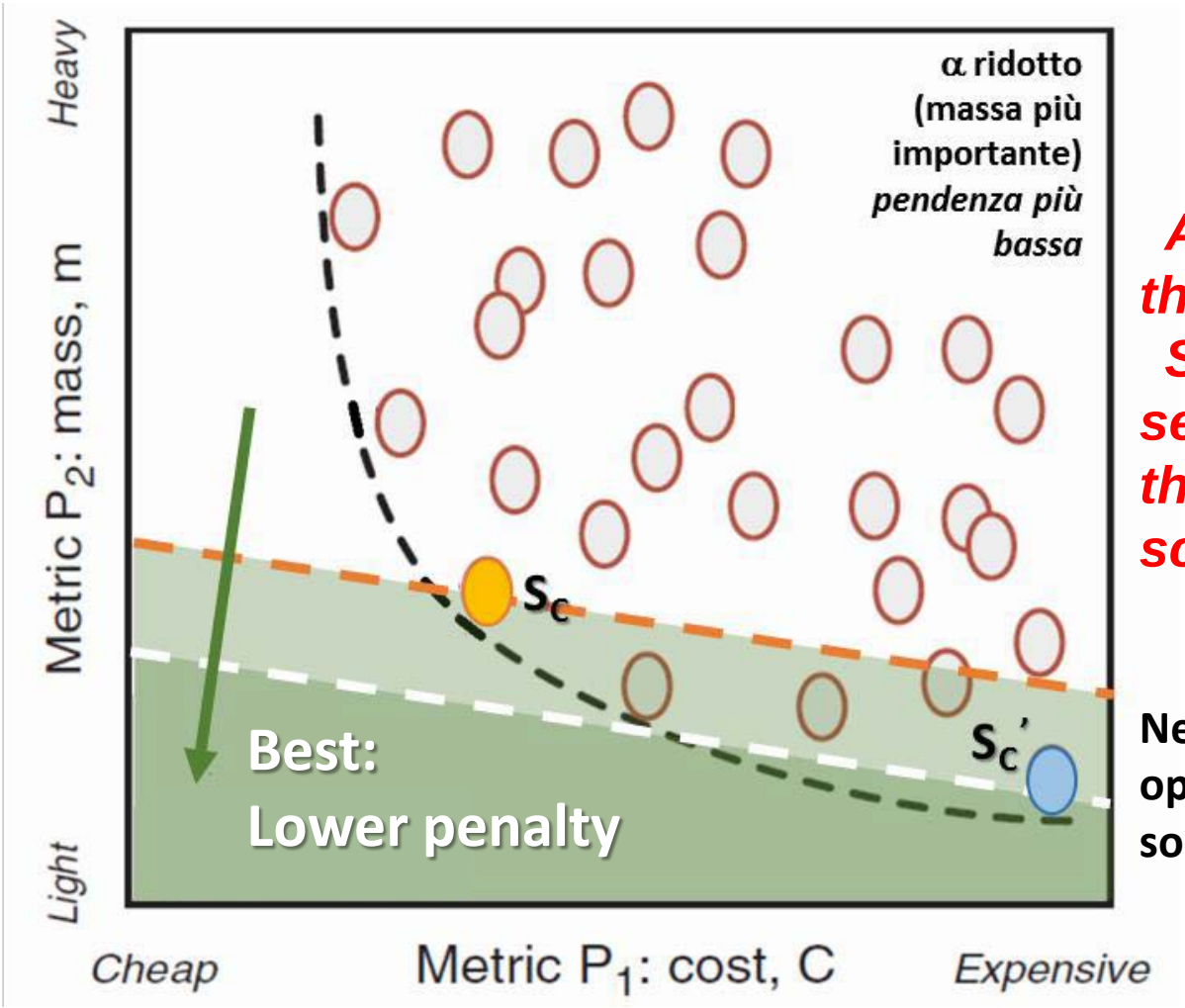


S_1 and S_2 are dramatically different in terms of C and m, but they are **equivalent in terms of overall penalty**

Graphical selection

$$Z = C + \alpha \cdot m \quad \rightarrow \quad m = -\frac{1}{\alpha} \cdot C + \frac{Z}{\alpha}$$

Slope $-1/\alpha$



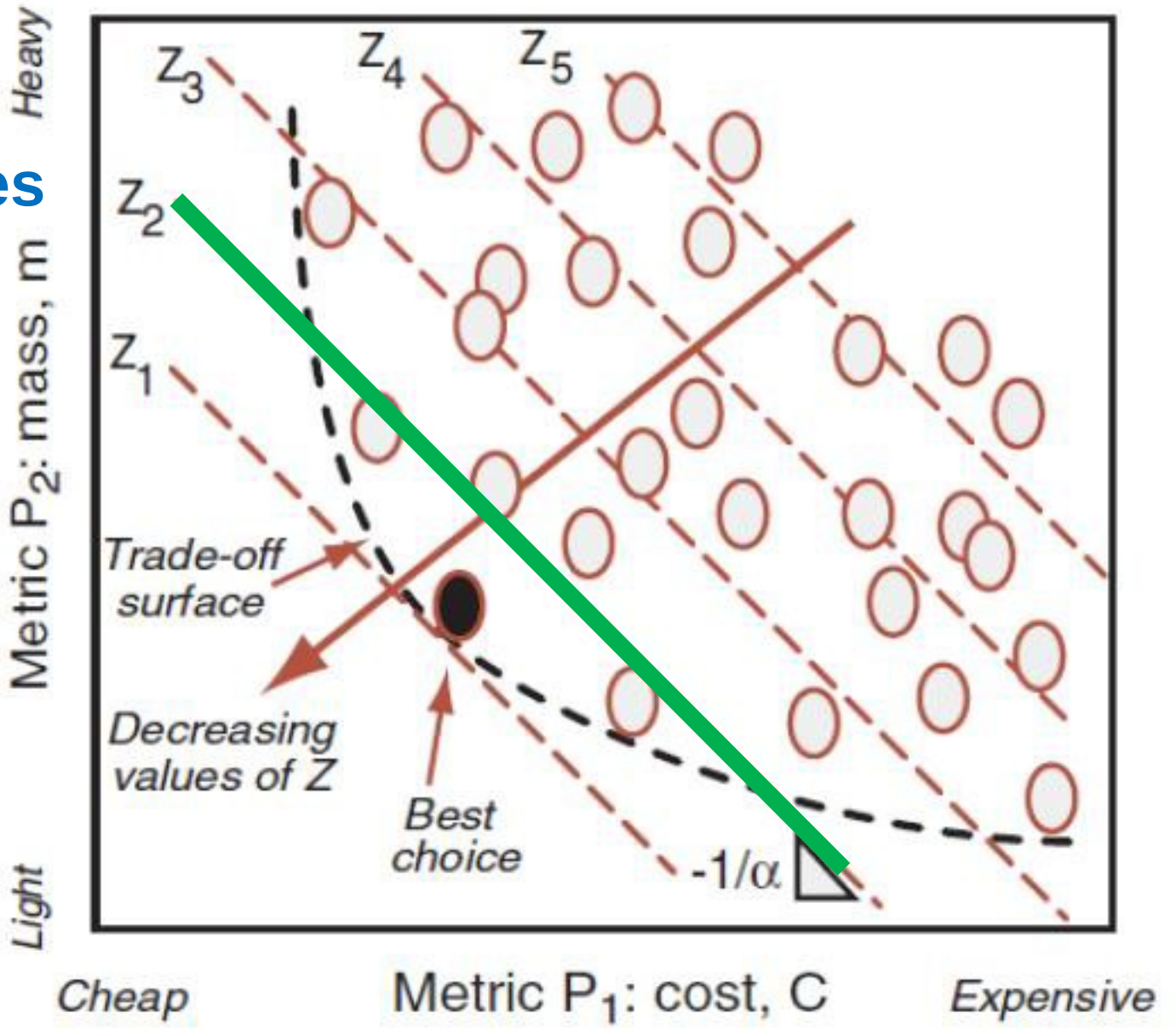
Enhancing α
Lower slope

Attention to the conditions Selection sensitive to the particular scenario

New optimum solution

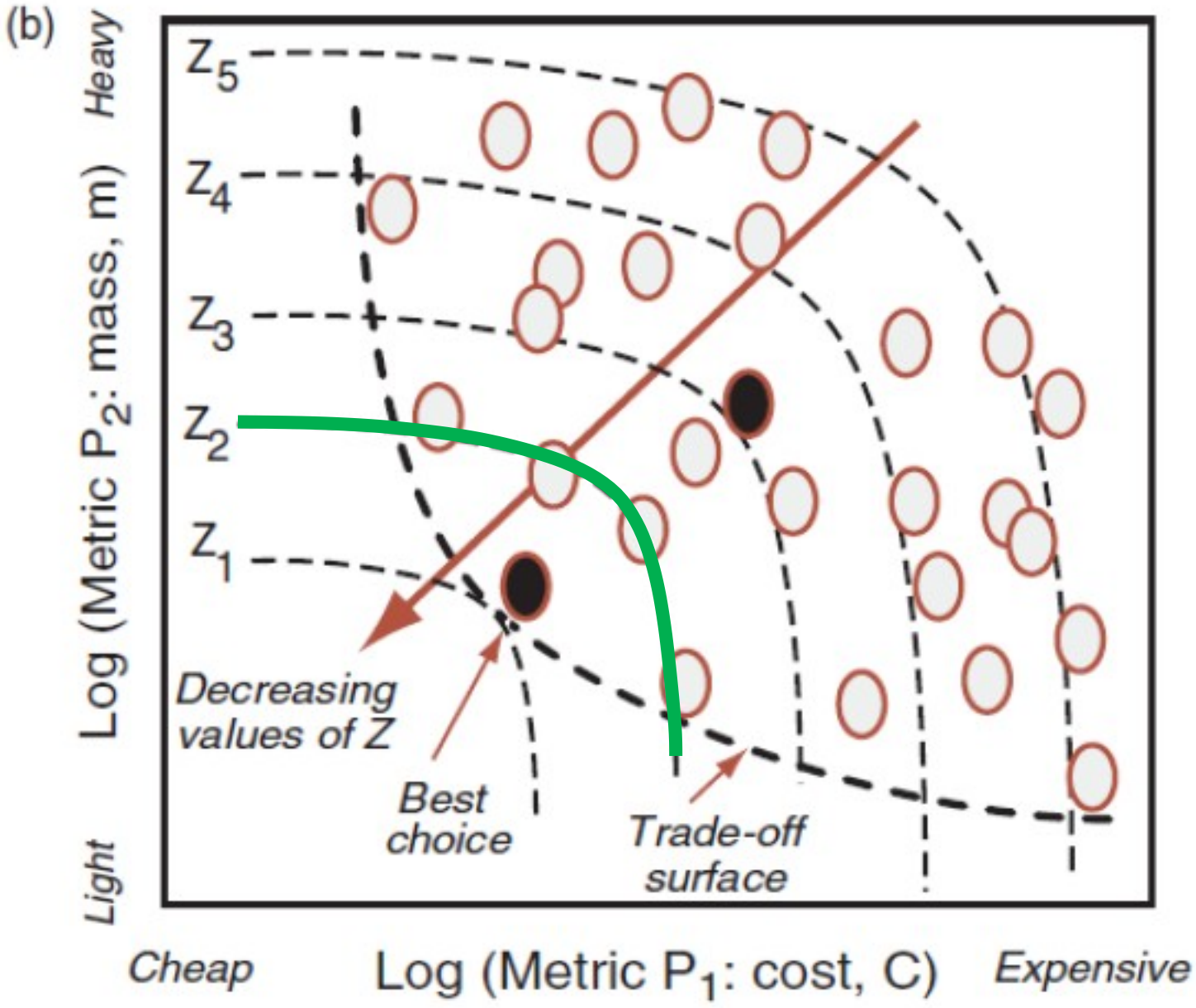
Graphical selection

Linear scales



Graphical selection

Log scales



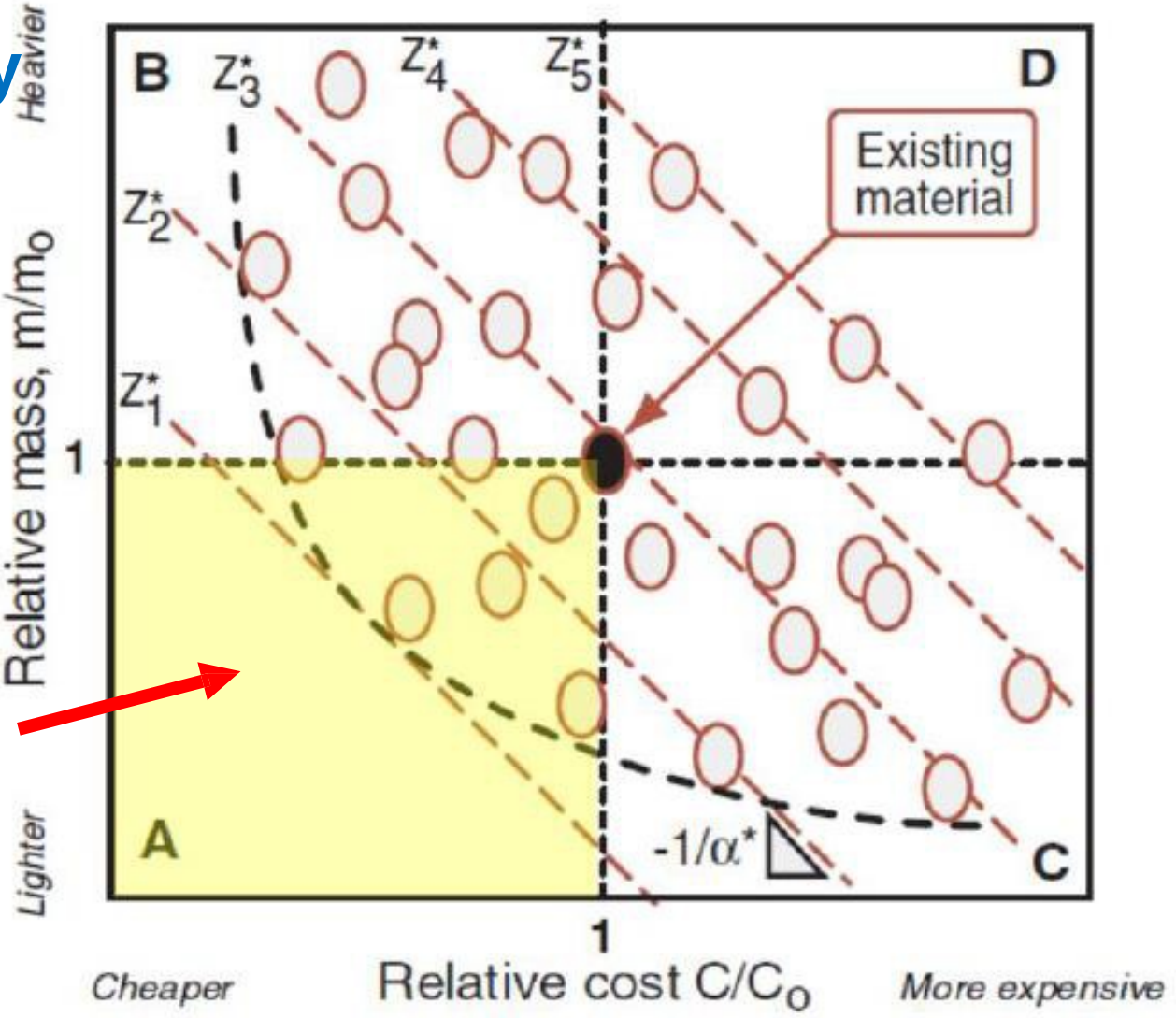
Graphical selection

$$Z = \frac{C}{C_0} + \alpha \cdot \frac{m}{m_0} \rightarrow \frac{m}{m_0} = -\frac{1}{\alpha} \cdot \frac{C}{C_0} + \frac{Z}{\alpha}$$

Relative penalty function

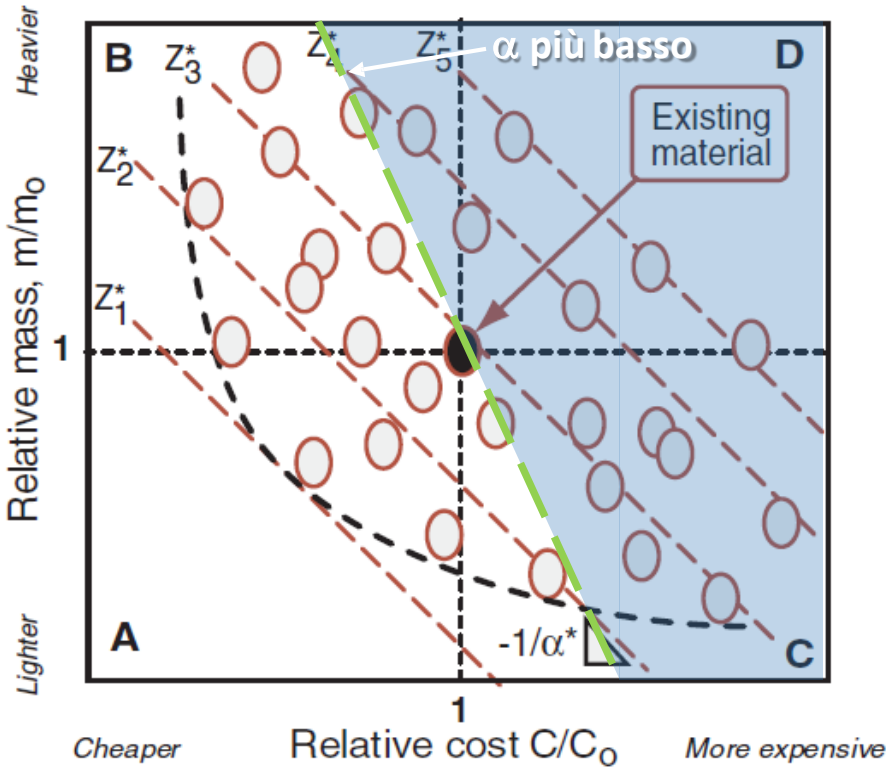
Existing material as standard

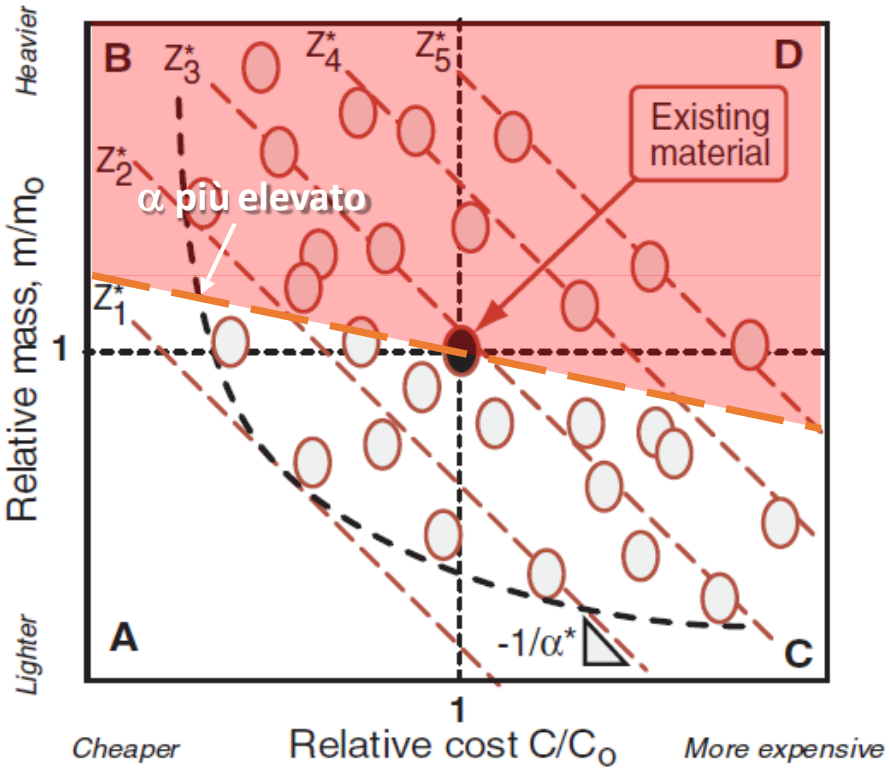
Others compared



WIN-WIN Quadrant: solutions always offering an improvement

B & C quadrants: constrained solutions





Lower α

Higher slope

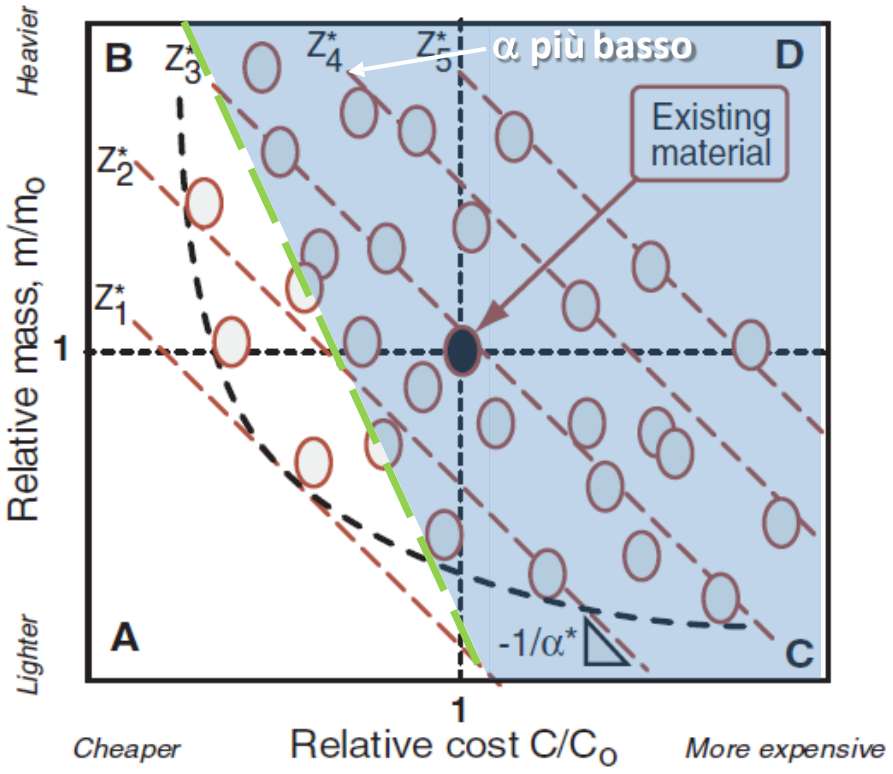
B quadrant offers improvements

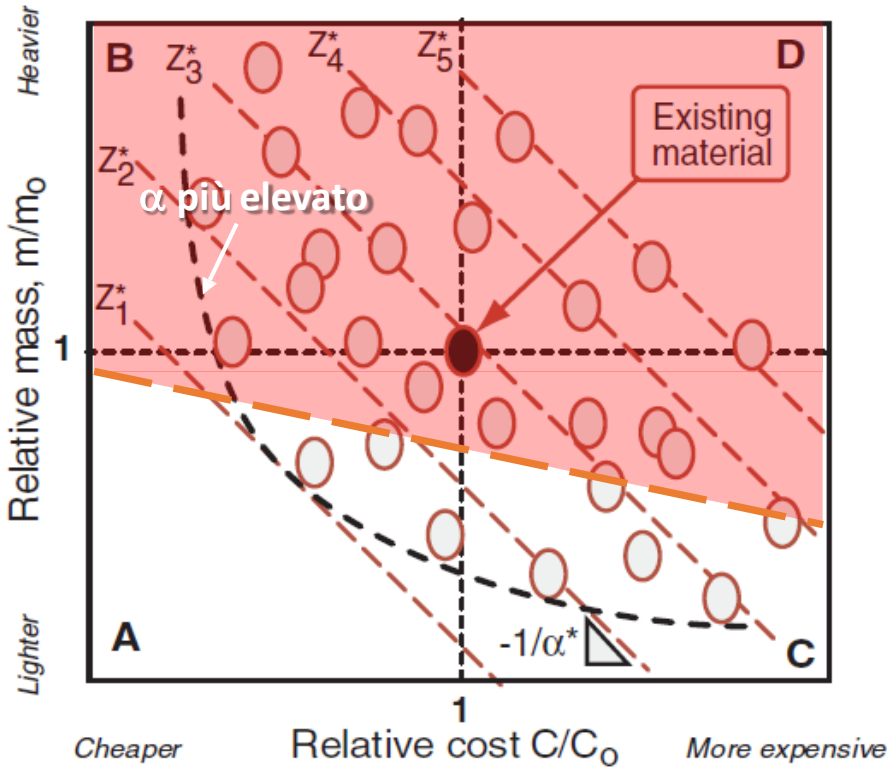
Higher α

Lower slope

C quadrant offers improvements

B & C quadrants: constrained solutions





Lower α

Higher slope

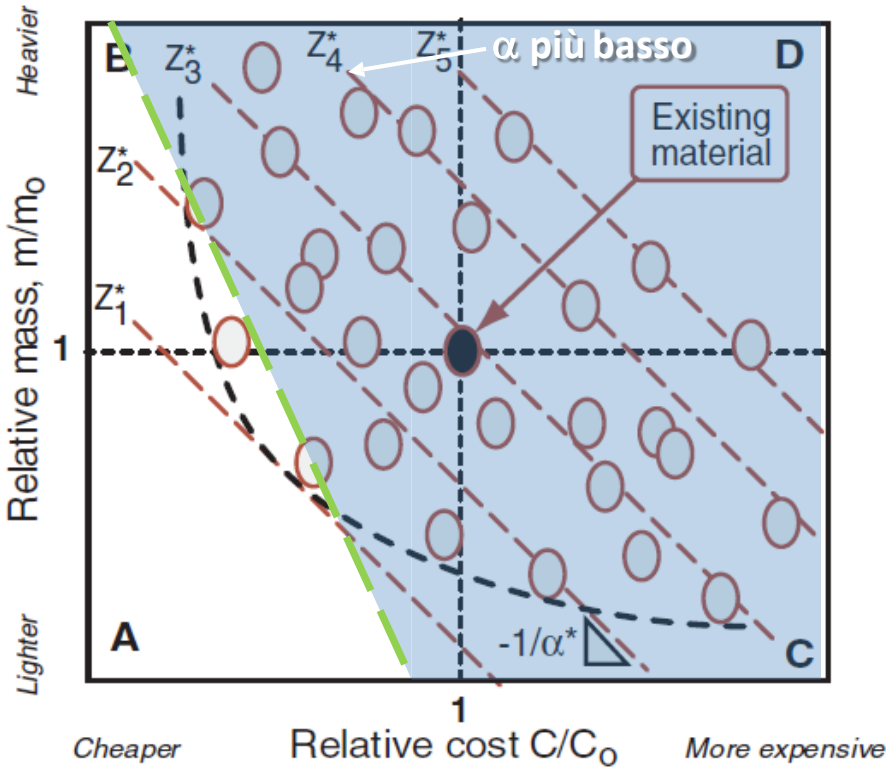
B quadrant offers improvements

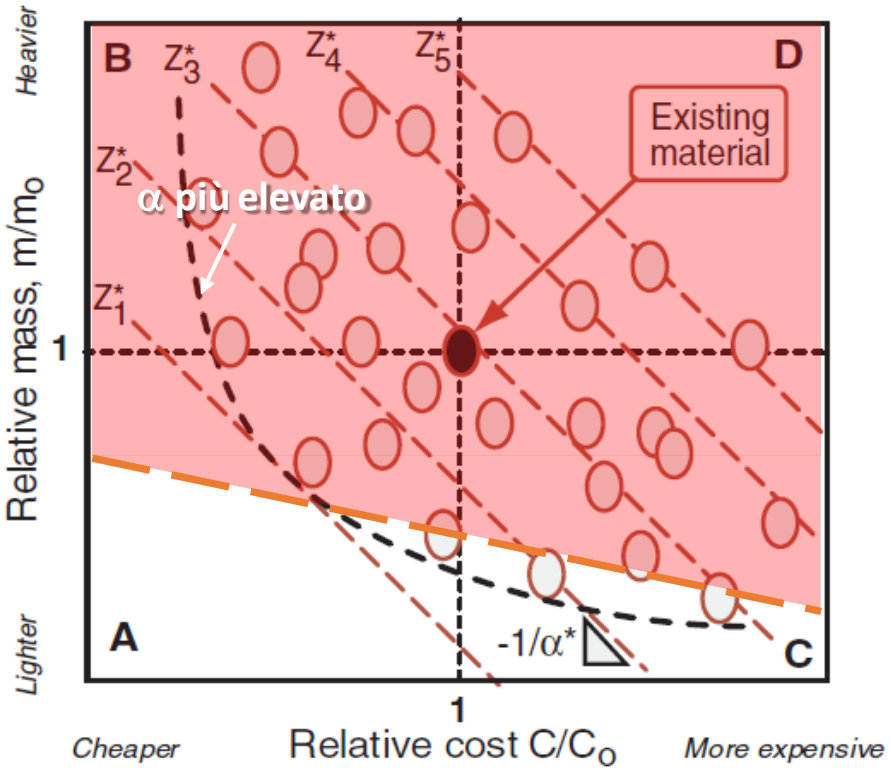
Higher α

Lower slope

C quadrant offers improvements

B & C quadrants: constrained solutions





Lower α

Higher slope

B quadrant offers improvements

Higher α

Lower slope

C quadrant offers improvements

Example: STRONG pull-rod, both LIGHT and INEXPENSIVE

$$m = \mathrm{F} \cdot \mathrm{L} \cdot \frac{\rho}{\sigma_f} = FS_m \cdot \frac{\rho}{\sigma_f}$$

$$m_0 = FS_m \cdot \frac{\rho_0}{\sigma_{f,0}}$$

$$C = C_m \cdot \mathrm{m} = \mathrm{F} \cdot \mathrm{L} \cdot \frac{\rho \cdot C_m}{\sigma_f} = FS_m \cdot \frac{\rho \cdot C_m}{\sigma_f}$$

$$C_0 = FS_m \cdot \frac{\rho_0 \cdot C_{m,0}}{\sigma_{f,0}}$$

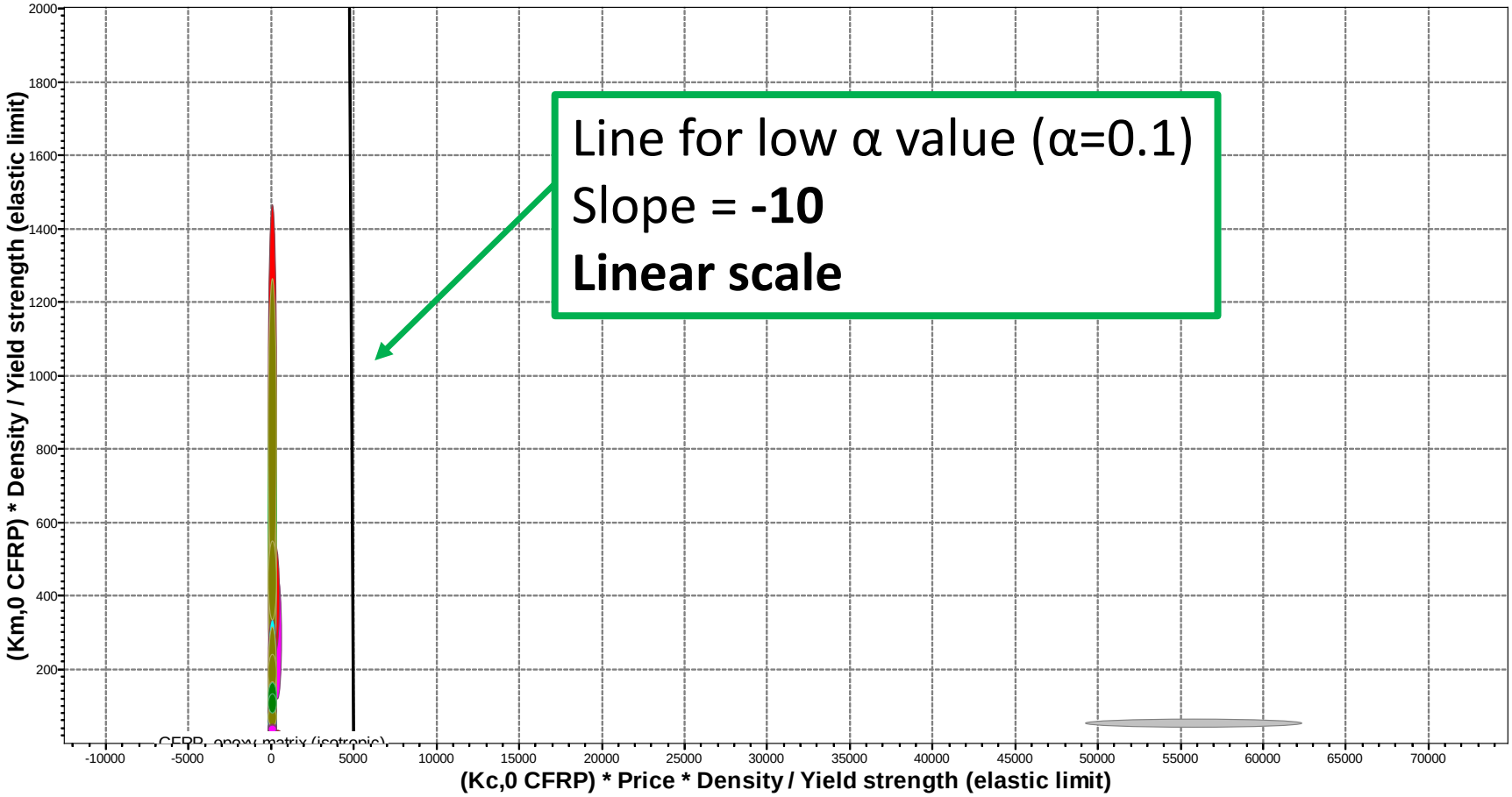
$$\frac{m}{m_0} = \frac{FS_m \cdot \frac{\rho}{\sigma_f}}{FS_m \cdot \frac{\rho_0}{\sigma_{f,0}}} = \frac{1}{\frac{\rho_0}{\sigma_{f,0}}} \cdot \frac{\boldsymbol{\rho}}{\boldsymbol{\sigma_f}} = K_{m,0} \cdot \frac{\boldsymbol{\rho}}{\boldsymbol{\sigma_f}}$$

Relative mass and relative cost ARE material properties

The constants ($K_{m,0}$, $K_{C,0}$) refer to the standard

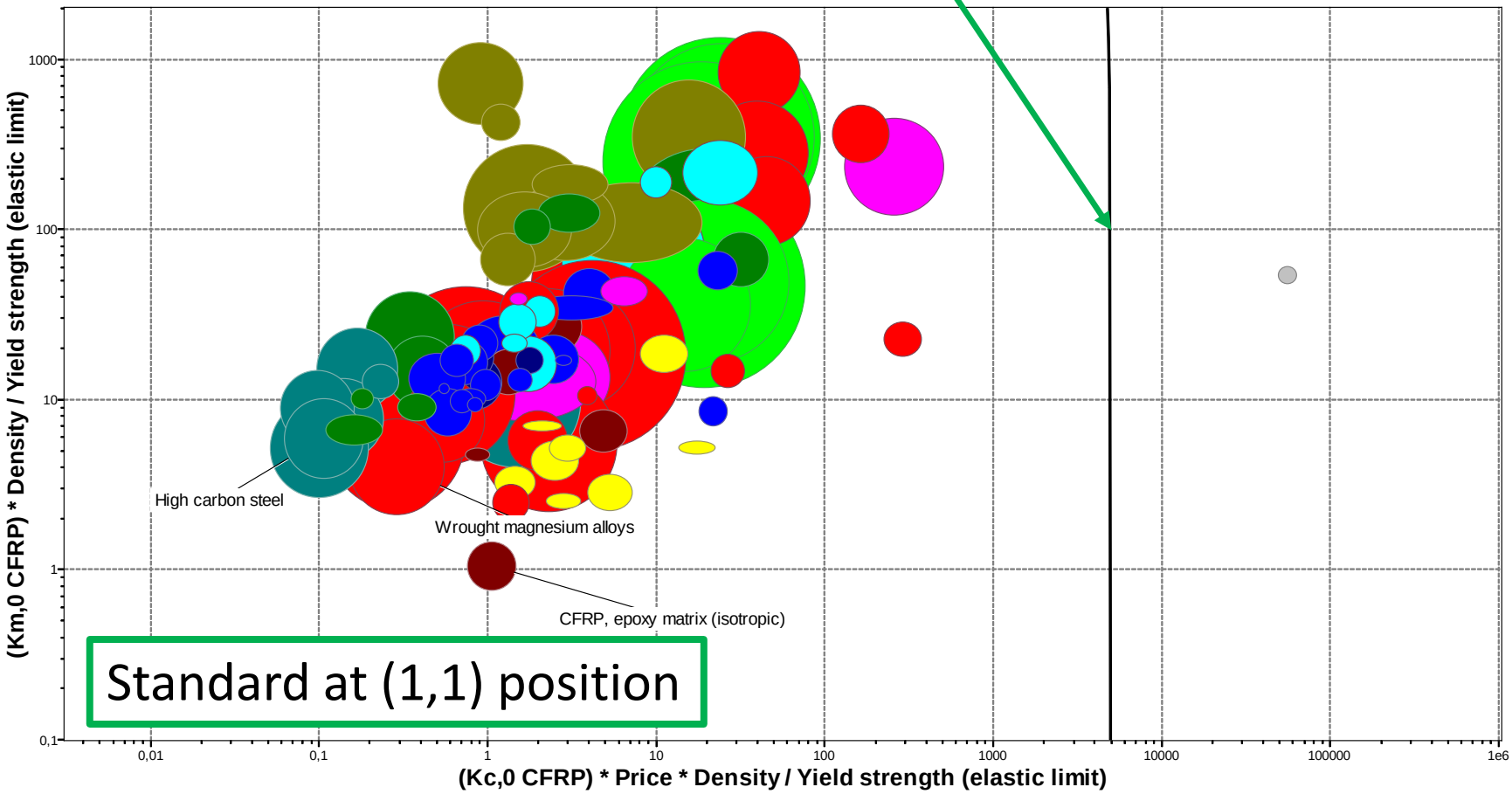
$$\frac{C}{C_0} = \frac{FS_m \cdot \frac{\rho \cdot C_m}{\sigma_f}}{FS_m \cdot \frac{\rho_0 \cdot C_{m,0}}{\sigma_{f,0}}} = \frac{1}{\frac{\rho_0 \cdot C_{m,0}}{\sigma_{f,0}}} \cdot \frac{\boldsymbol{\rho \cdot C_m}}{\boldsymbol{\sigma_f}} = K_{C,0} \cdot \frac{\boldsymbol{\rho \cdot C_m}}{\boldsymbol{\sigma_f}}$$

$$Z = \frac{C}{C_0} + \alpha \cdot \frac{m}{m_0} \quad \rightarrow \quad \frac{m}{m_0} = -\frac{1}{\alpha} \cdot \frac{C}{C_0} + \frac{Z}{\alpha}$$

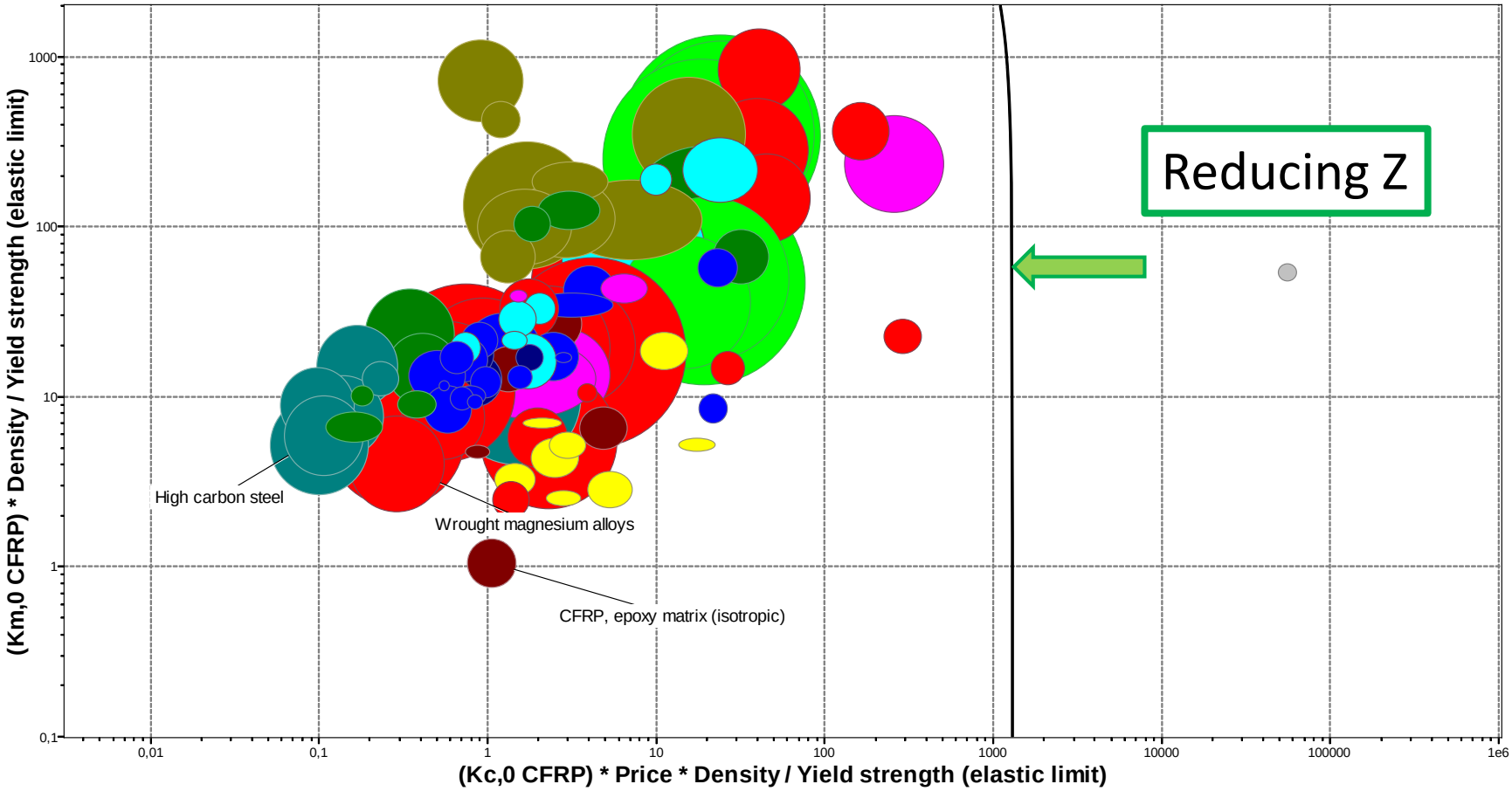


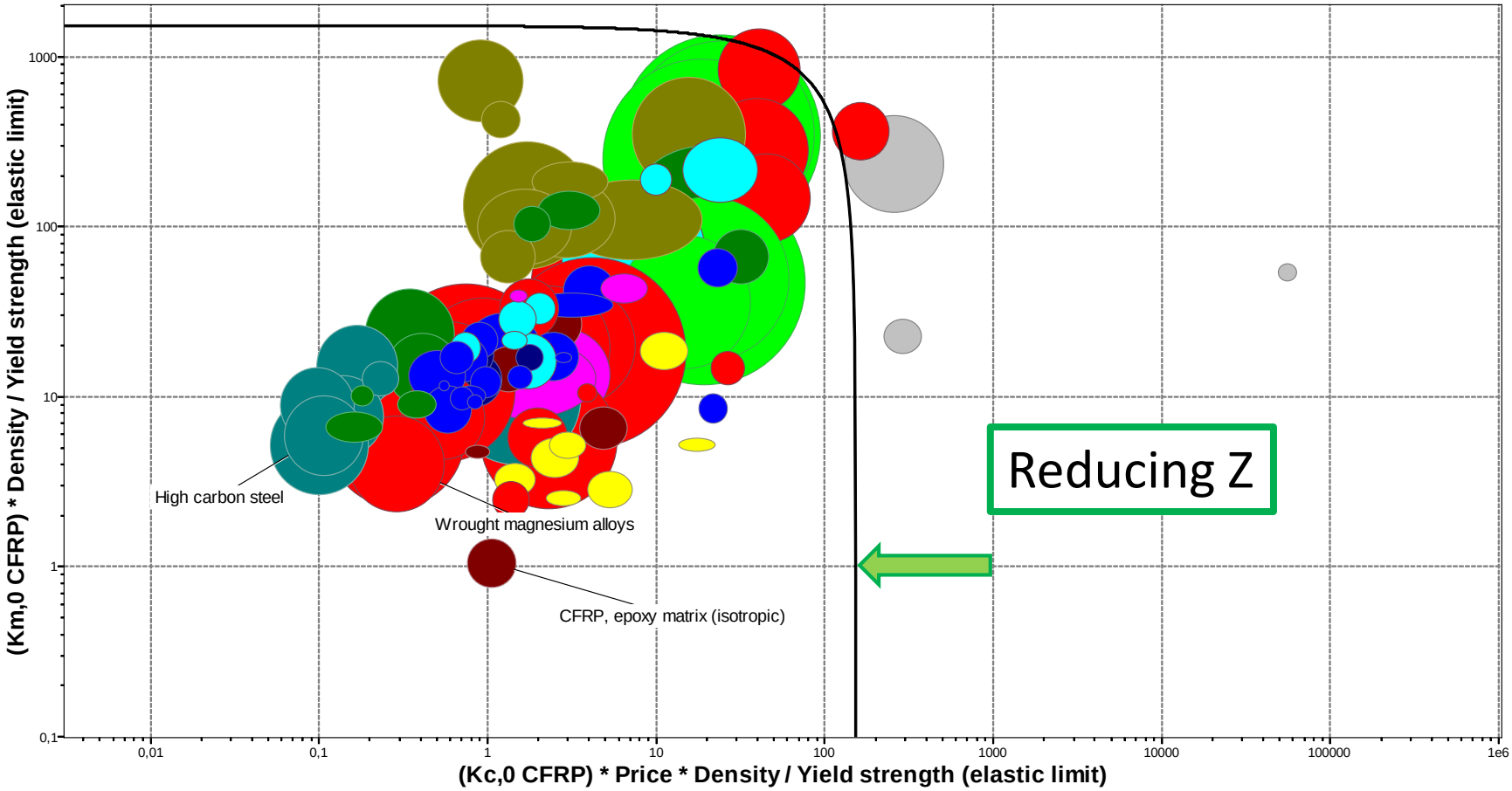
Line for low α value (α=0.1) Slope = **-10**

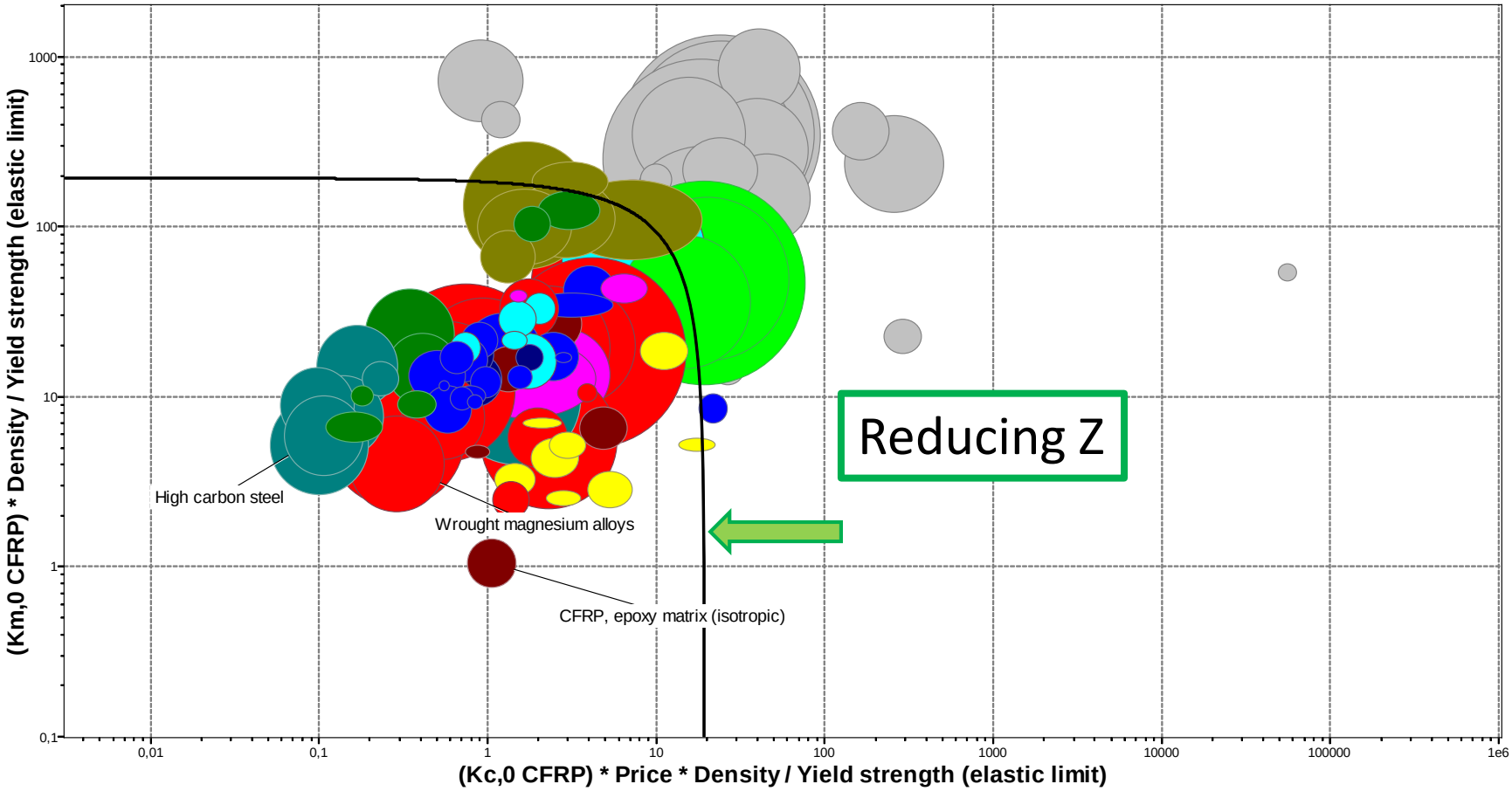
Transformed into a curve (Log scale)

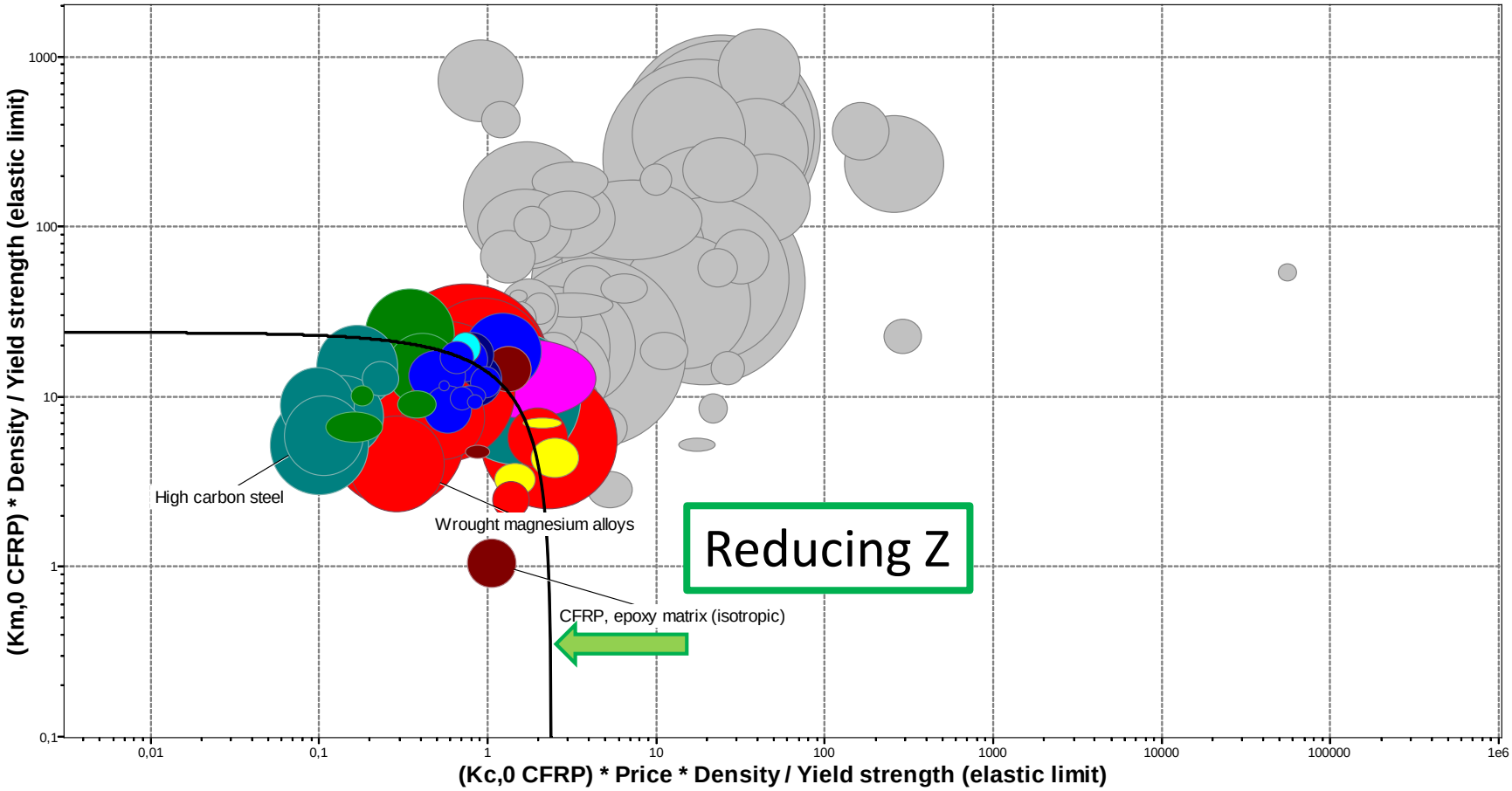


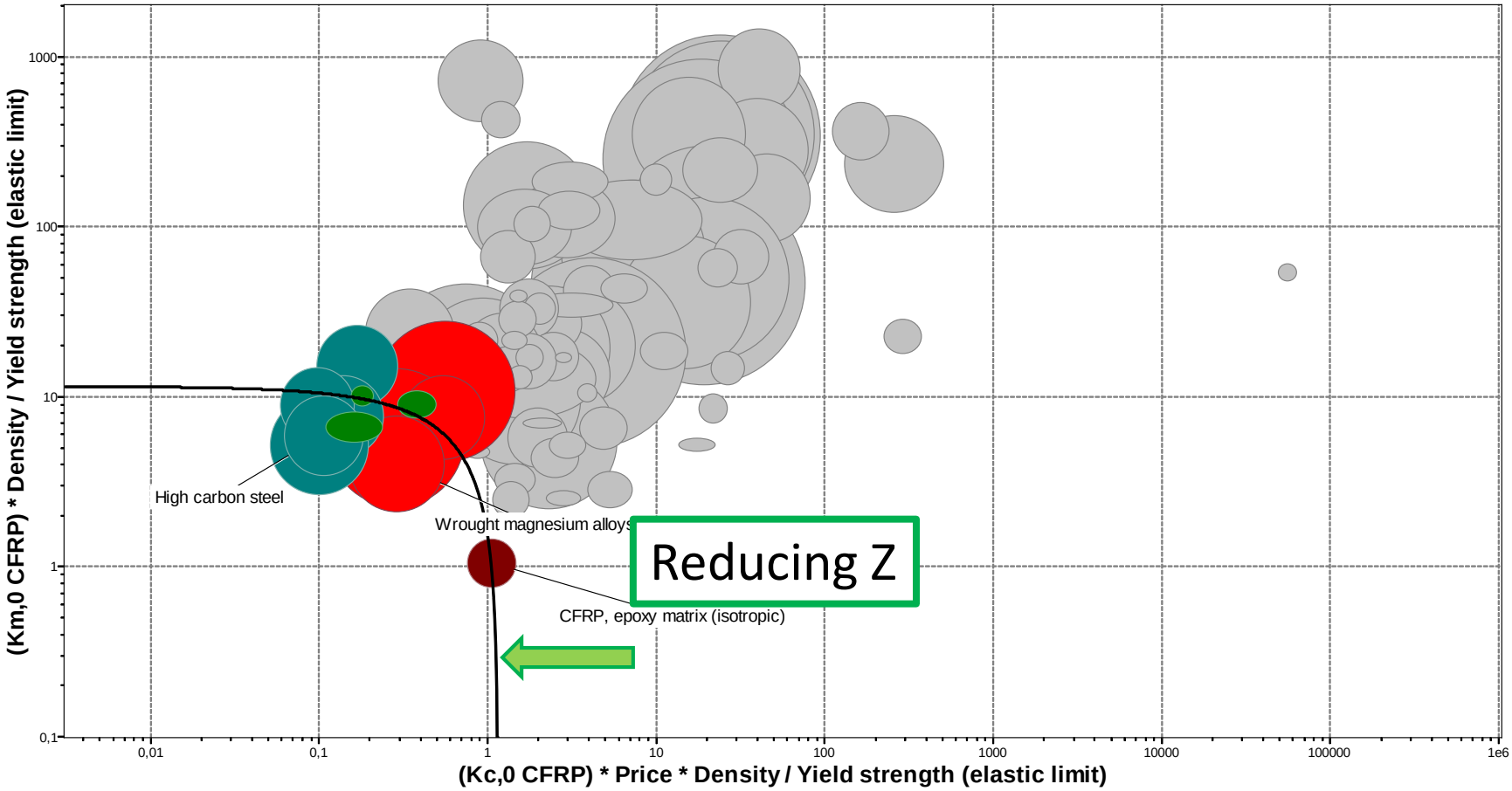
Standard at (1,1) position

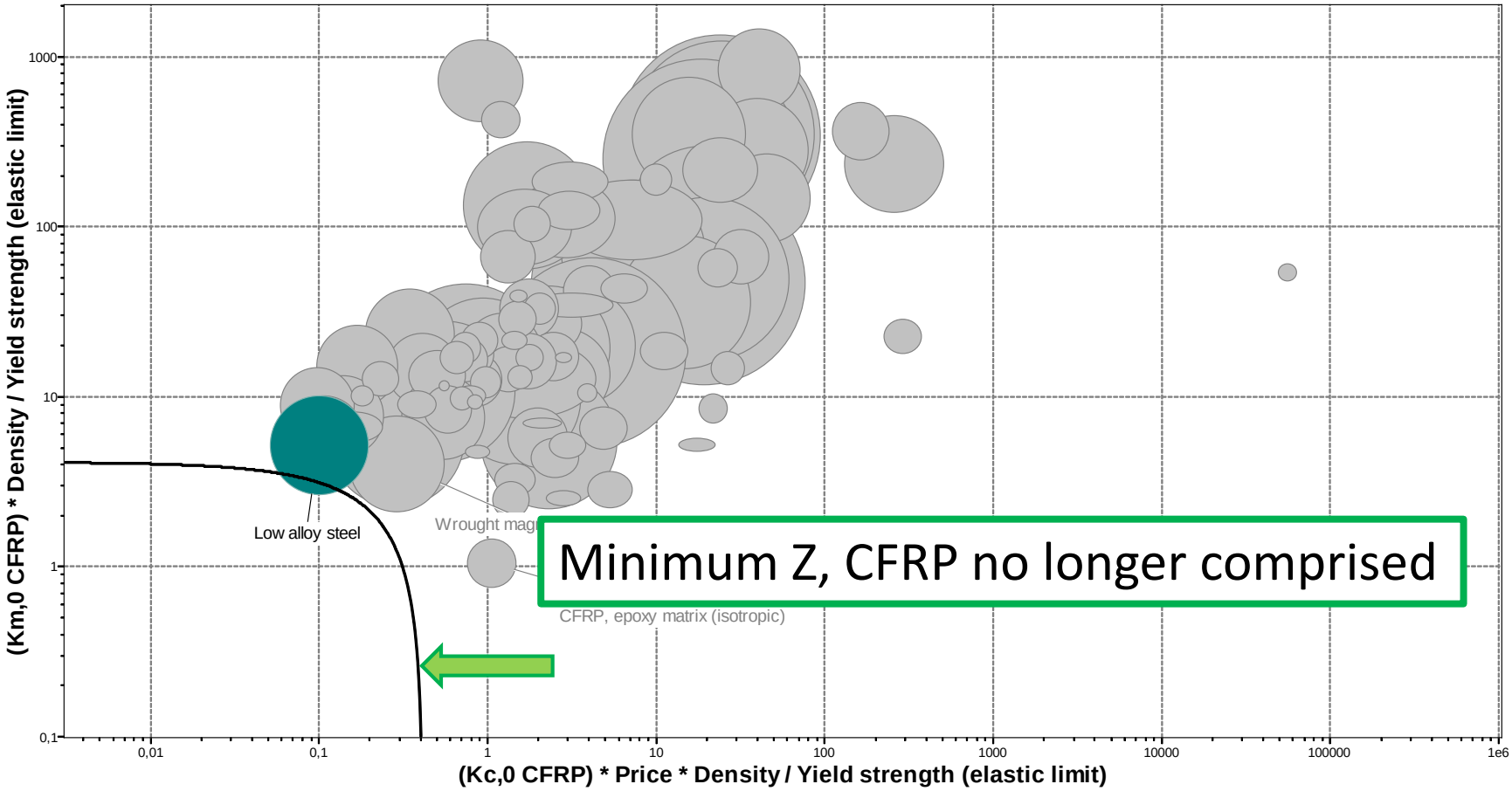

Reducing Z
High carbon steel
Wrought magnesium alloys
CFRP, epoxy matrix (isotropic)
(Km,0 CFRP) * Density / Yield strength (elastic limit)
(Kc,0 CFRP) * Price * Density / Yield strength (elastic limit)
1000
100
10
1
0,1
0,01
0,1
1
10
100
1000
10000
100000
1e6

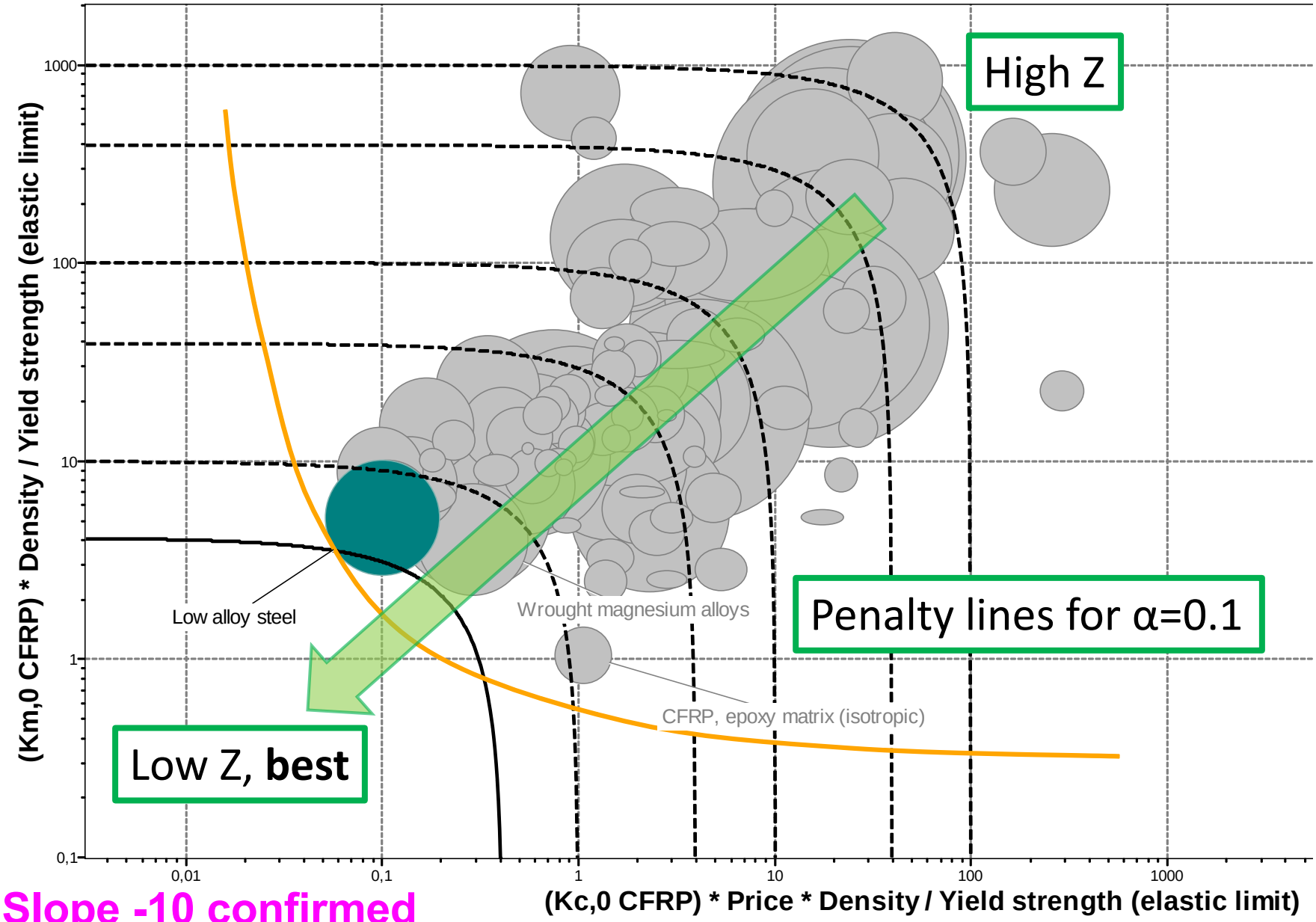

(Km,0 CFRP) * Density / Yield strength (elastic limit)
1000
100
10
1
0,1
0,01
0,1
1
10
100
1000
10000
100000
1e6
(Kc,0 CFRP) * Price * Density / Yield strength (elastic limit)
High carbon steel
Wrought magnesium alloys
CFRP, epoxy matrix (isotropic)
Reducing Z


(Km,0 CFRP) * Density / Yield strength (elastic limit)
1000
100
10
1
0,1
0,01
0,1
1
10
100
1000
10000
100000
1e6
(Kc,0 CFRP) * Price * Density / Yield strength (elastic limit)
Reducing Z
High carbon steel
Wrought magnesium alloys
CFRP, epoxy matrix (isotropic)


(Km,0 CFRP) * Density / Yield strength (elastic limit)
1000
100
10
1
0,1
0,01
0,1
1
10
100
1000
10000
100000
1e6
(Kc,0 CFRP) * Price * Density / Yield strength (elastic limit)
High carbon steel
Wrought magnesium alloys
CFRP, epoxy matrix (isotropic)
Reducing Z


(Km,0 CFRP) * Density / Yield strength (elastic limit)
1000
100
10
1
0,1
0,01
0,1
1
10
100
1000
10000
100000
1e6
(Kc,0 CFRP) * Price * Density / Yield strength (elastic limit)
High carbon steel
Wrought magnesium alloys
CFRP, epoxy matrix (isotropic)
Reducing Z


(Km,0 CFRP) * Density / Yield strength (elastic limit)
1000
100
10
1
0,1
0,01
0,1
1
10
100
1000
10000
100000
1e6
(Kc,0 CFRP) * Price * Density / Yield strength (elastic limit)
Low alloy steel
Wrought magn
CFRP, epoxy matrix (isotropic)
Minimum Z, CFRP no longer comprised



Slope -10 confirmed

See $m/m_0=1$ connected to $C/C_0=10$