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DI PADOVA

Fundamentals of Materials Selection

Material contribution to STIFFNESS

Materials indices for thickness and mass of TIES



Stiffness of a tie (A=section area, L=length) = ratio between applied force and macroscopic deformation (**extension**)

$$S = \frac{F}{\delta} = \frac{F}{\Delta L} = \frac{\sigma \cdot A}{\varepsilon \cdot L} = E \cdot \frac{A}{L}$$



Stiffness of a tie (A =section area, L =length) = ratio between applied force and macroscopic deformation (extension)

$$S = \frac{F}{\delta} = \frac{F}{\Delta L} = \frac{\sigma \cdot A}{\varepsilon \cdot L} = E \cdot \frac{A}{L}$$

A minimum stiffness implies a maximum allowed extension

$$S \geq S_{min} = \frac{F}{\delta_{max}} \frac{A}{L} = E \cdot \frac{m}{\rho \cdot L^2}$$

$$E \cdot \frac{A}{L} \geq S_{min} \rightarrow A \geq S_{min} \cdot L \cdot \frac{1}{E} \rightarrow A \geq SF_{Area} \cdot \frac{1}{E}$$

$$A = \frac{m}{\rho \cdot L}$$

SF=structural factor (factors not dependent on material)

The section area can be reduced increasing E

$$E \cdot \frac{A}{L} = E \cdot \frac{m}{\rho \cdot L^2} \geq S_{min} \rightarrow m \geq S_{min} \cdot L^2 \cdot \frac{\rho}{E} \rightarrow m \geq SF_m \cdot \frac{\rho}{E}$$

The mass can be reduced increasing E/ρ [ρ =density]



Strength of a tie = applied stress not exceeding the elastic limit

$$\sigma = \frac{F}{A} \leq \sigma_f$$

$$A \geq \frac{F}{\sigma_f} \rightarrow A \geq SF_{Area} \cdot \frac{1}{\sigma_f}$$

SF=structural factor (factors not dependent on material)

The section area can be reduced increasing σ_f

$$\frac{m}{\rho \cdot L} \geq \frac{F}{\sigma_f} \rightarrow m \geq F \cdot L \cdot \frac{\rho}{\sigma_f} \rightarrow m \geq SF_m \cdot \frac{\rho}{\sigma_f}$$

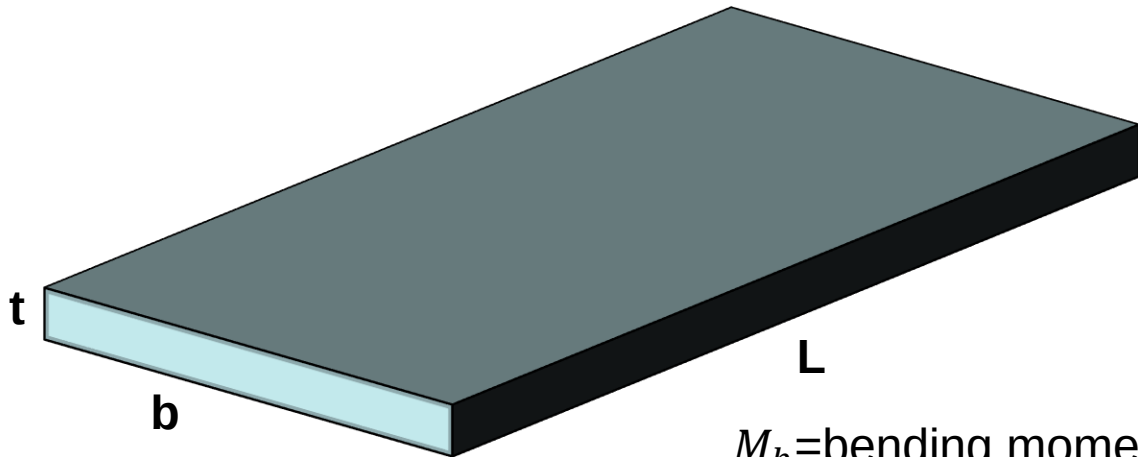
The mass can be reduced increasing σ_f/ρ [ρ =density]



Fundamentals of Materials Selection

Material contribution to STRENGTH

Materials indices for thickness and mass of PANELS



M_b = bending moment

Panels: bidimensional objects with **undetermined thickness t**

Bending → fundamental role of moment of inertia J

$$J = \frac{b \cdot t^3}{12}$$

$$\sigma_{max} = \frac{M_b \cdot \left(\frac{t}{2}\right)}{J} = \frac{M_b \cdot 6}{b \cdot t^2} \leq \sigma_f = M.O.R.$$

Deformation = deflection

$$t \geq \sqrt{\frac{M_b \cdot 6}{b \cdot \sigma_f}} \rightarrow t \geq SF_t \cdot \frac{1}{\sigma_f^{\frac{1}{2}}}$$

$$m = \rho \cdot L \cdot b \cdot t$$
$$\rightarrow m \geq SF_m \cdot \frac{\rho}{\sigma_f^{\frac{1}{2}}}$$

Thickness still controlled by σ_f
Mass still controlled by both σ_f and density
But with different exponents



Overview of material indices

Fundamental equation for thickness and mass
Attention to the exponents of mechanical properties

Reducing thickness t inversely proportional to I (=index) =...

	Stiffness	Strength
Tie (subjected to axial force)	E	σ_f
Bar (subjected to bending)	$E^{1/2}$	$\sigma_f^{2/3}$
Panel (subjected to bending)	$E^{1/3}$	$\sigma_f^{1/2}$

Optimization of thickness

Up-scaling of mechanical properties **undermined** by the exponents, in ***bending configuration***

Optimization of mass

Attention to the **down-scaling** of mechanical properties (more significant in bending) operated by density
Density as additional mechanical property

Reducing mass

m inversely proportional to I (=index) =...

	Stiffness	Strength
Tie (subjected to axial force)	E/ρ	σ_f/ρ
Bar (subjected to bending)	$(E^{1/2})/\rho$	$(\sigma_f^{2/3})/\rho$
Panel (subjected to bending)	$(E^{1/3})/\rho$	$(\sigma_f^{1/2})/\rho$

ENVIRONMENTAL INDICES FOR STRENGTH PROBLEMS

= AVOIDING FAILURE

$$I = \frac{\sigma_f}{\rho}$$

SIMPLE
TENSION
→
TIES

$$I' = \frac{\sigma_f}{\rho \cdot H_m}$$

↑ I' ↓ 'REAL' ENERGY

$$I = \frac{\sigma_f^{2/3}}{\rho}$$

BENDING
OF
BARS

$$I' = \frac{\sigma_f^{2/3}}{\rho \cdot H_m}$$

$$I = \frac{\sigma_f^{1/2}}{\rho}$$

PANELS

$$I' = \frac{\sigma_f^{1/2}}{\rho \cdot H_m}$$

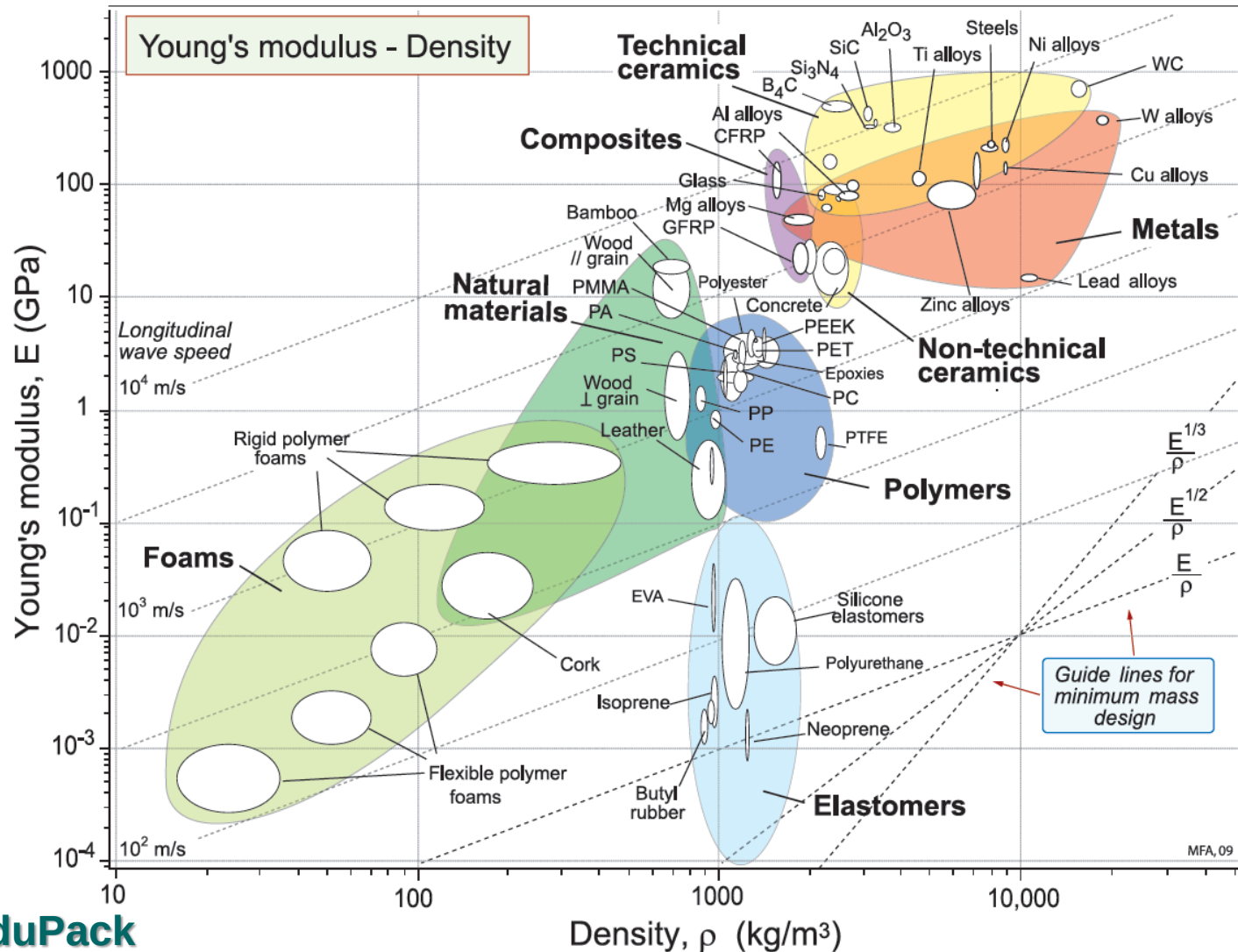
MIND:

- MANY 'NATURAL-BASED' MATERIALS, WITH LOW H_m , ARE WEAK (LOW σ_f)
- GOOD TO KEEP OPTIMIZED σ_f/ρ OF SYNTHETIC MATERIALS BY IMPLEMENTING RECYCLING (AFFECTING H_m)

Application: Ashby's plots

Indices represented in material property graphs
Logarithmic scales

**Materials
selection for
light stiff ties**
**Reduction of
mass**



Graphical selection

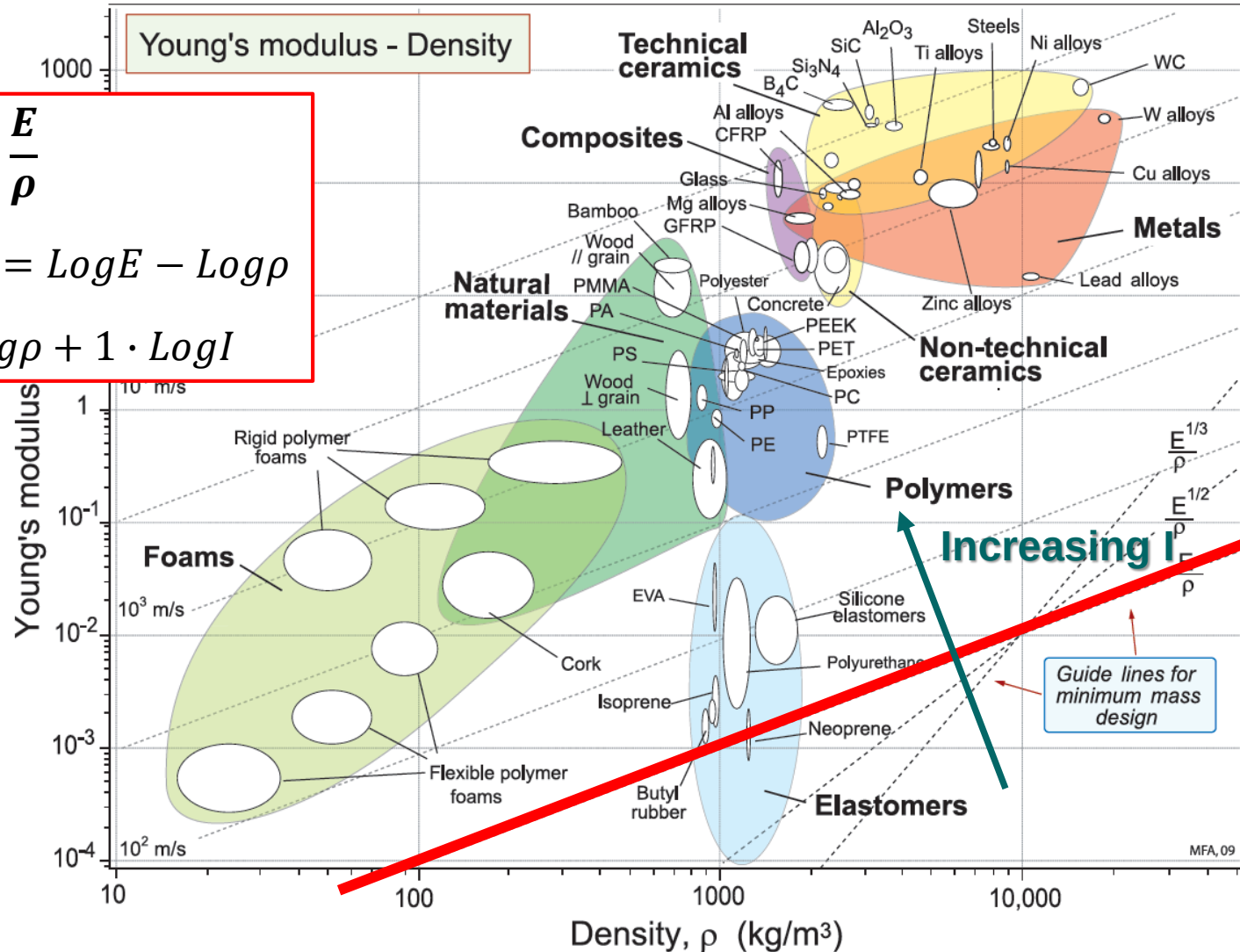
Optimization of stiffness
Selection lines and directions

Young's modulus - Density

$$I = \frac{E}{\rho}$$

$$\text{Log} I = \text{Log} \left(\frac{E}{\rho} \right) = \text{Log} E - \text{Log} \rho$$

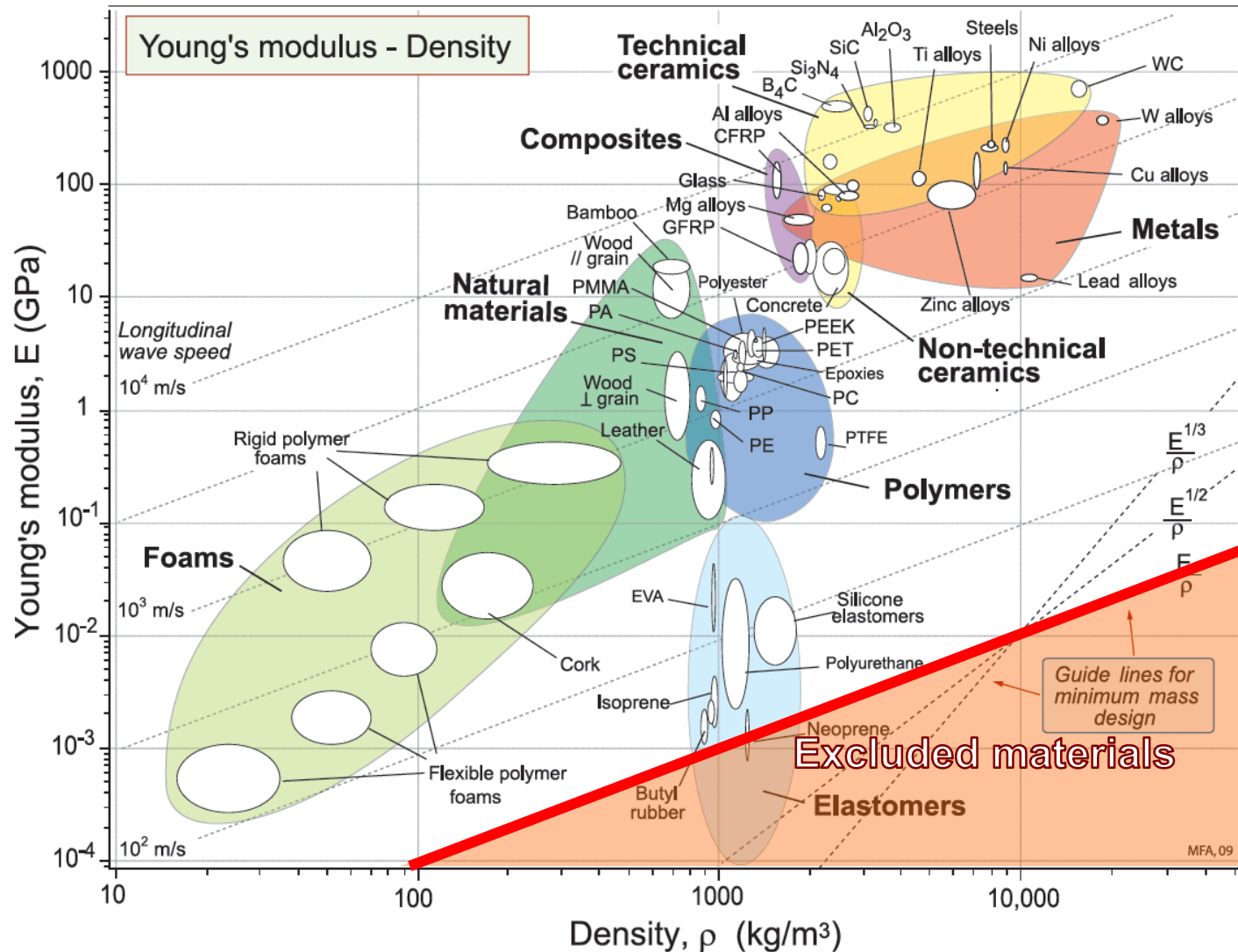
$$\text{Log} E = 1 \cdot \text{Log} \rho + 1 \cdot \text{Log} I$$



Graphical selection

Optimization of stiffness
Selection lines and directions

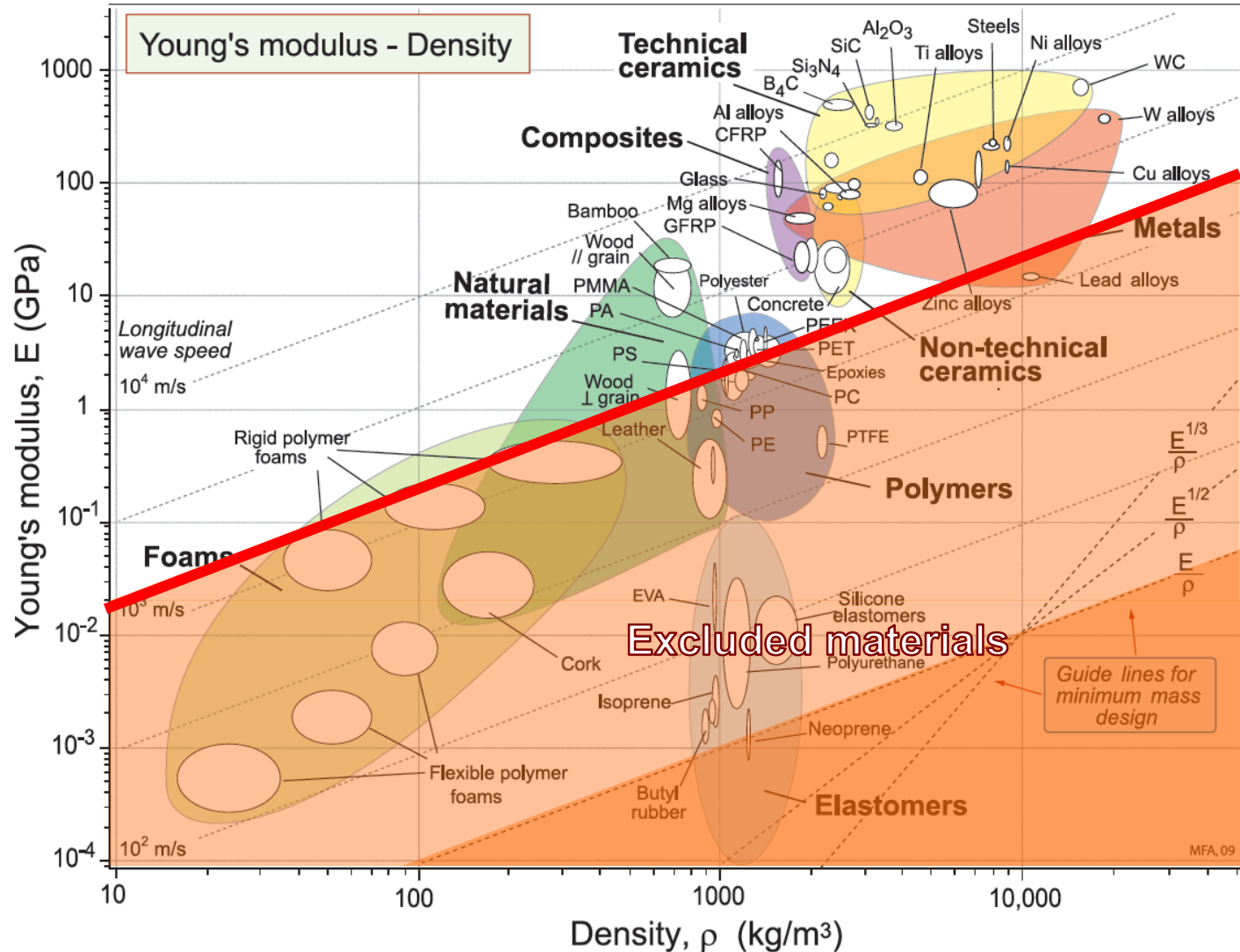
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Graphical selection

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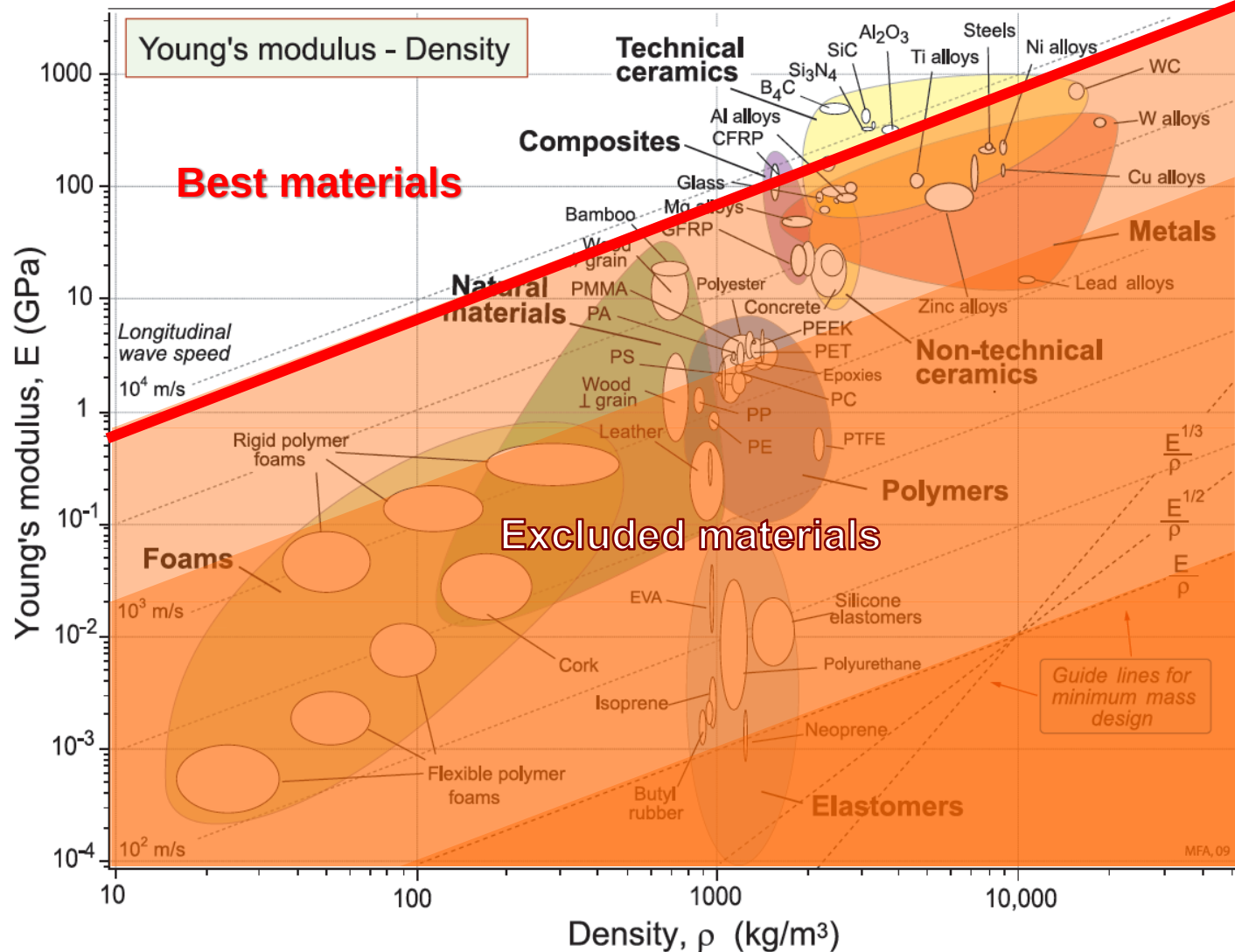


Graphical selection

Optimization of stiffness
Selection lines and directions

Materials selection for light stiff ties
Reduction of mass

...some candidates can be excluded for excessive cost or brittleness (refined selection on 'winners')

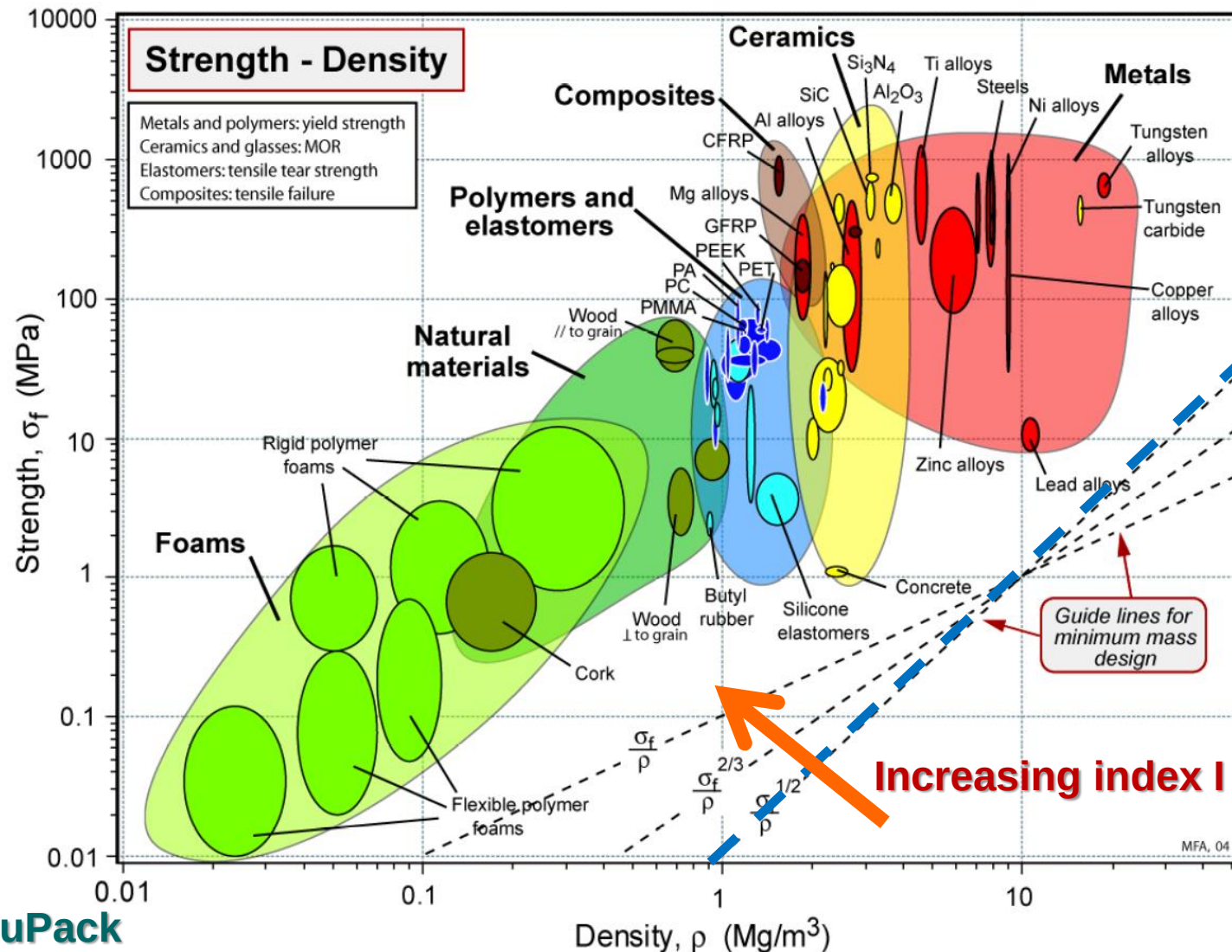


Panels and strength

Rapid evaluation of the structural efficiency

Panels: $I = \sigma^{1/2} / \rho$

Lines in
logarithmic
graph

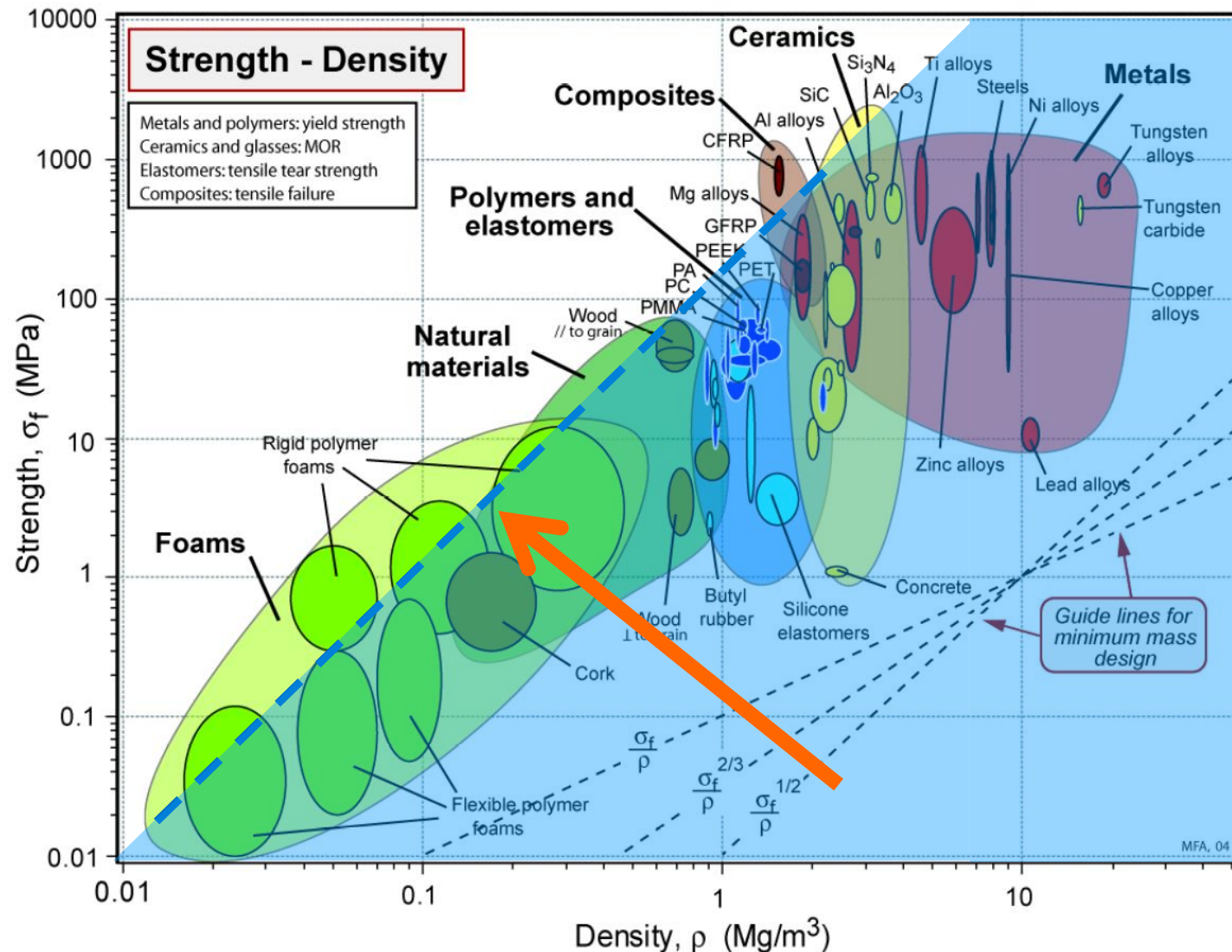


Panels and strength

Are cellular materials actually weak?
NO, as shown by Ashby's plots

Panels: $I = \sigma^{1/2} / \rho$
Lines in
logarithmic
graph

Cellular materials
as **out-**
performing
solutions





Other graphs: conflicting objectives

Applications may require multiple properties to be optimized
Key idea: focus on the most significant ones, refine later

- Material indices (E^n/ρ , σ_f^n/ρ) already express combinations of properties
- **Mechanical performance may be coupled with thermal performance, e.g.**
 1. The solution should be efficient in terms of **specific strength (σ_f/ρ), to be enhanced**
 2. The solution should be efficient in terms of **thermal conductivity (λ), to be limited** [to enhance thermal insulation]

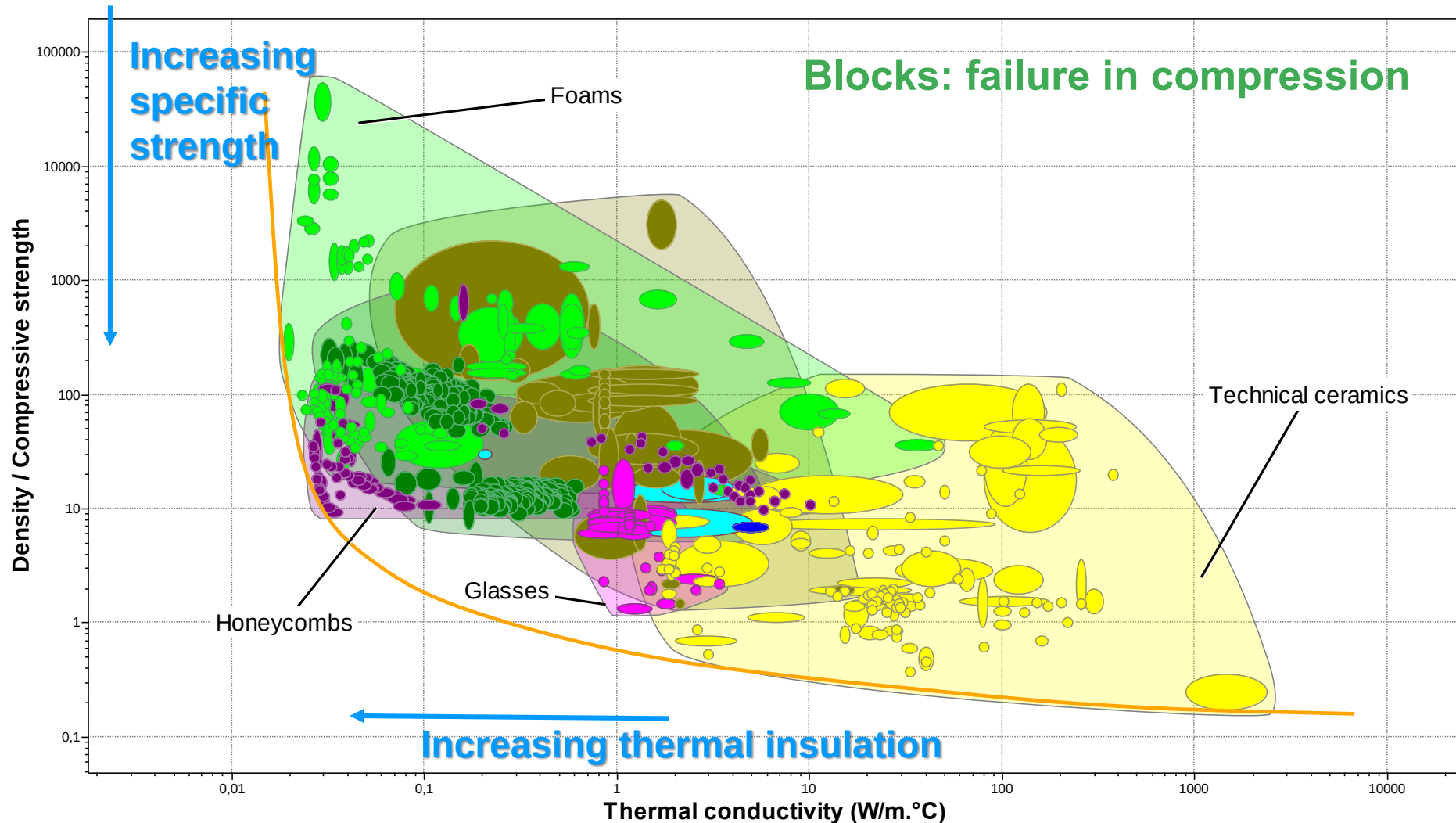
Trade-off approach: a solution is valid when *it minimizes all 'penalties'*, i.e. properties that negatively affect the acceptability

- Thermal conductivity (λ), to be limited
- Reverse specific strength (ρ/σ_f), to be limited



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Graphical representation
Trade-off curve as a limit to all solutions
Graph deriving from the two “penalties”



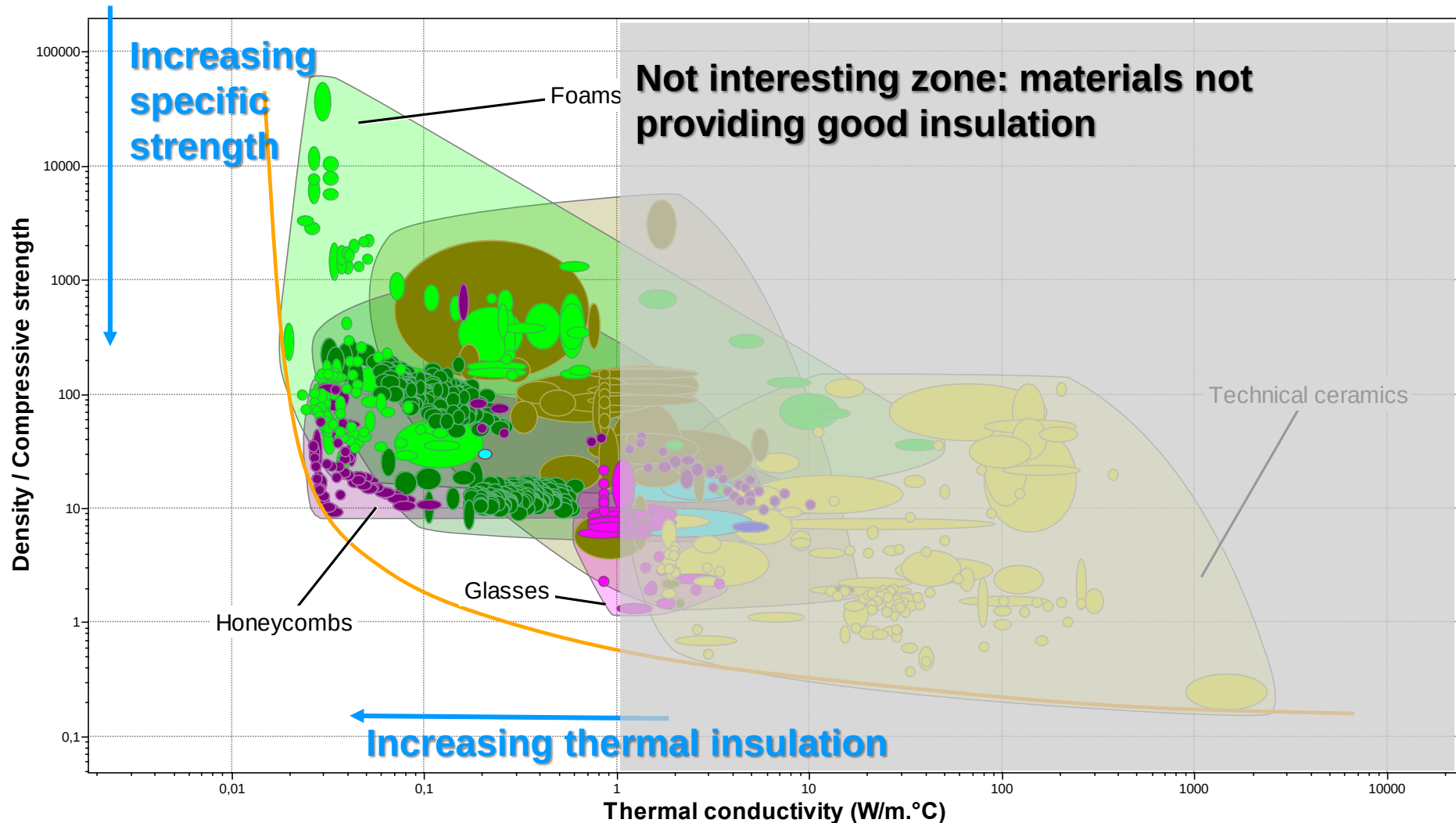


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Graphical representation

Trade-off curve as a limit to all solutions

Some solutions can be excluded applying constraints





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Graphical representation

Trade-off curve as a limit to all solutions
Some solutions are appropriate since they offer balanced properties

