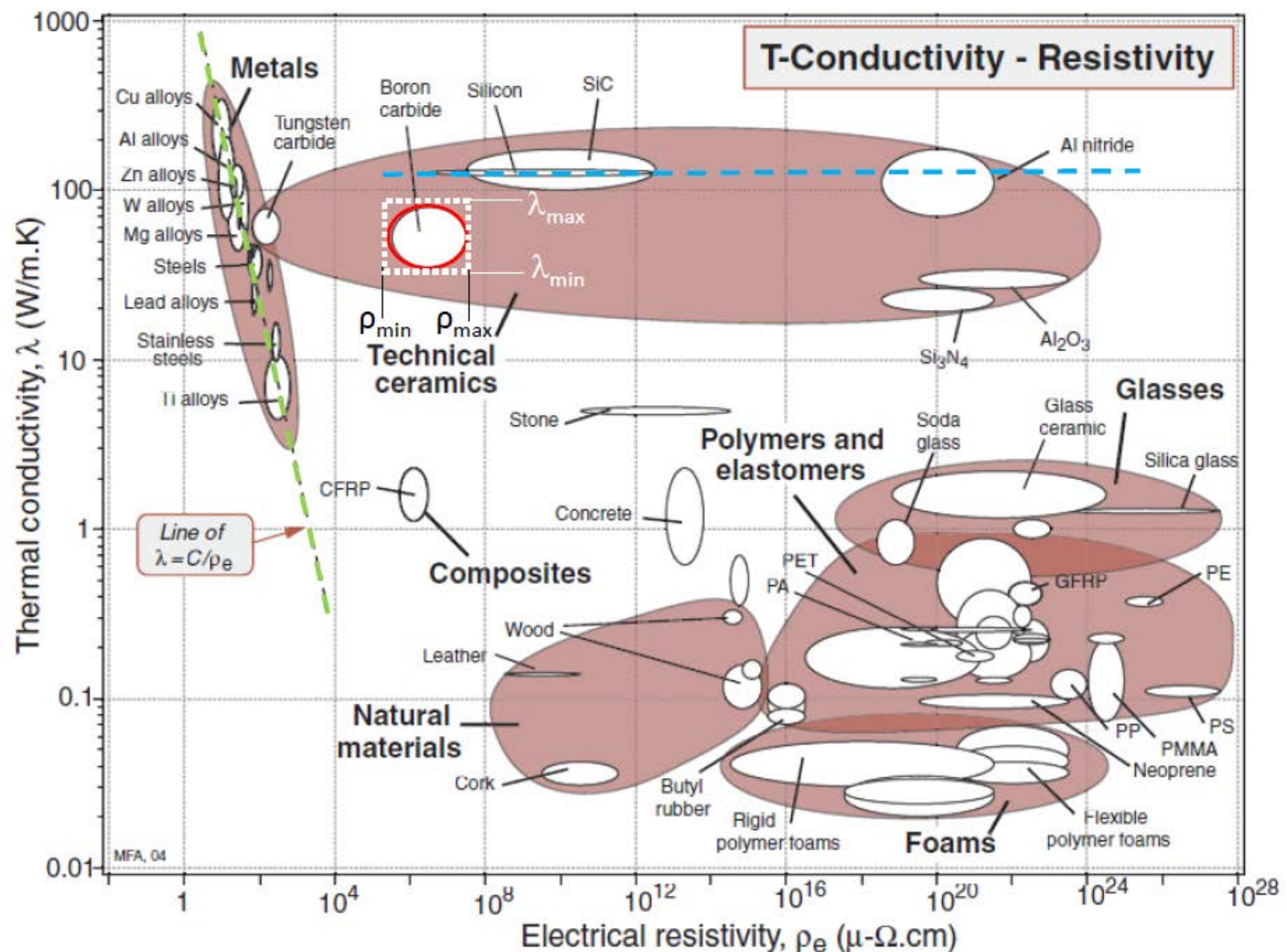




Metals: direct proportionality between thermal and electrical conductivity:

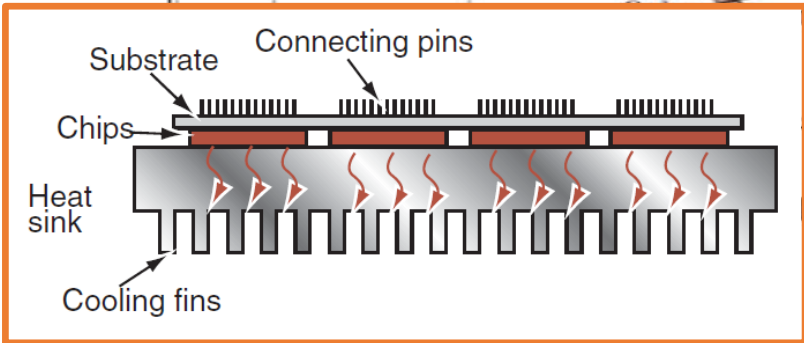
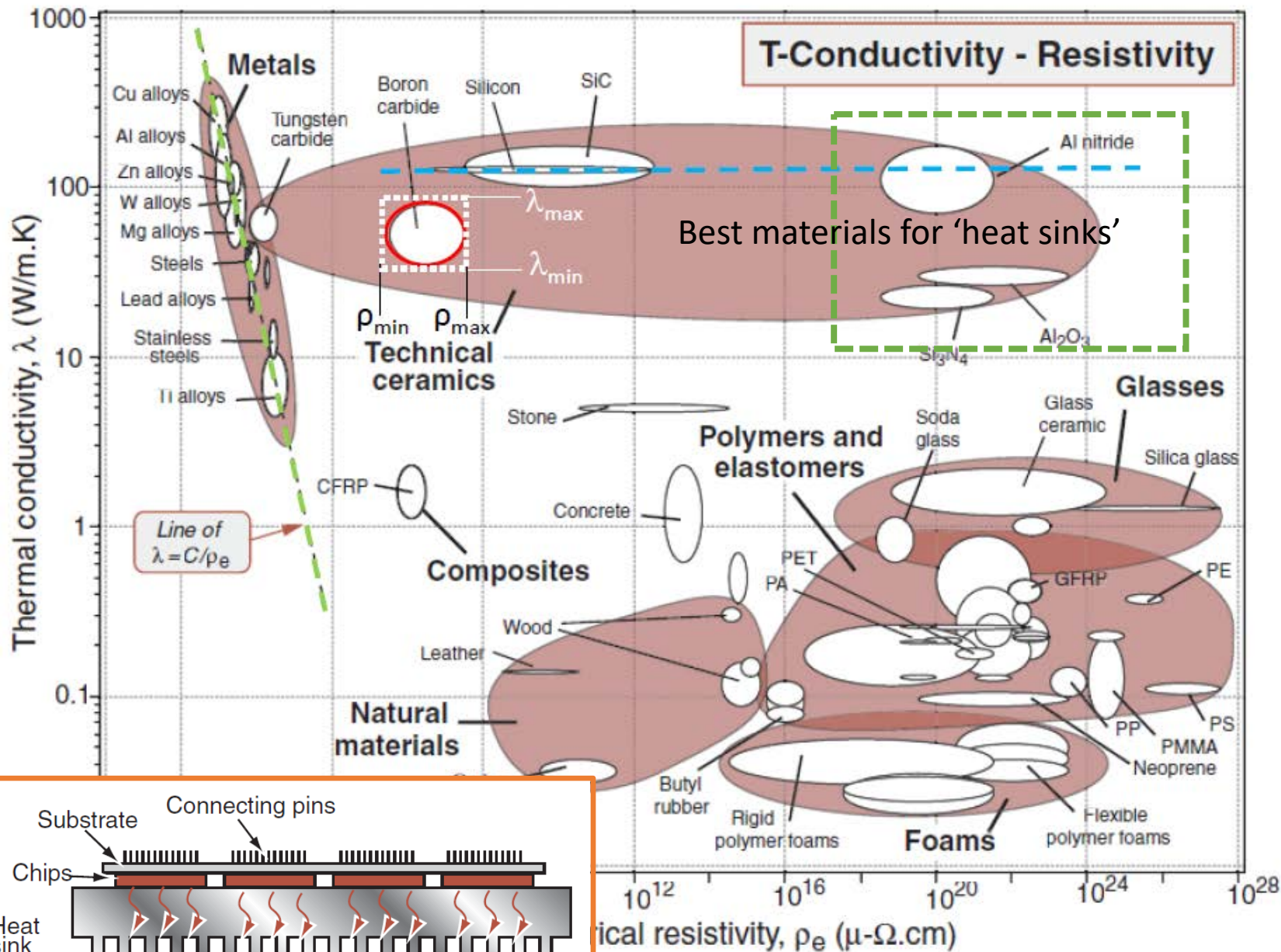
$$\lambda = C \cdot \sigma_e \quad [C=\text{constant}]$$

The equation can be rewritten introducing electrical resistivity:

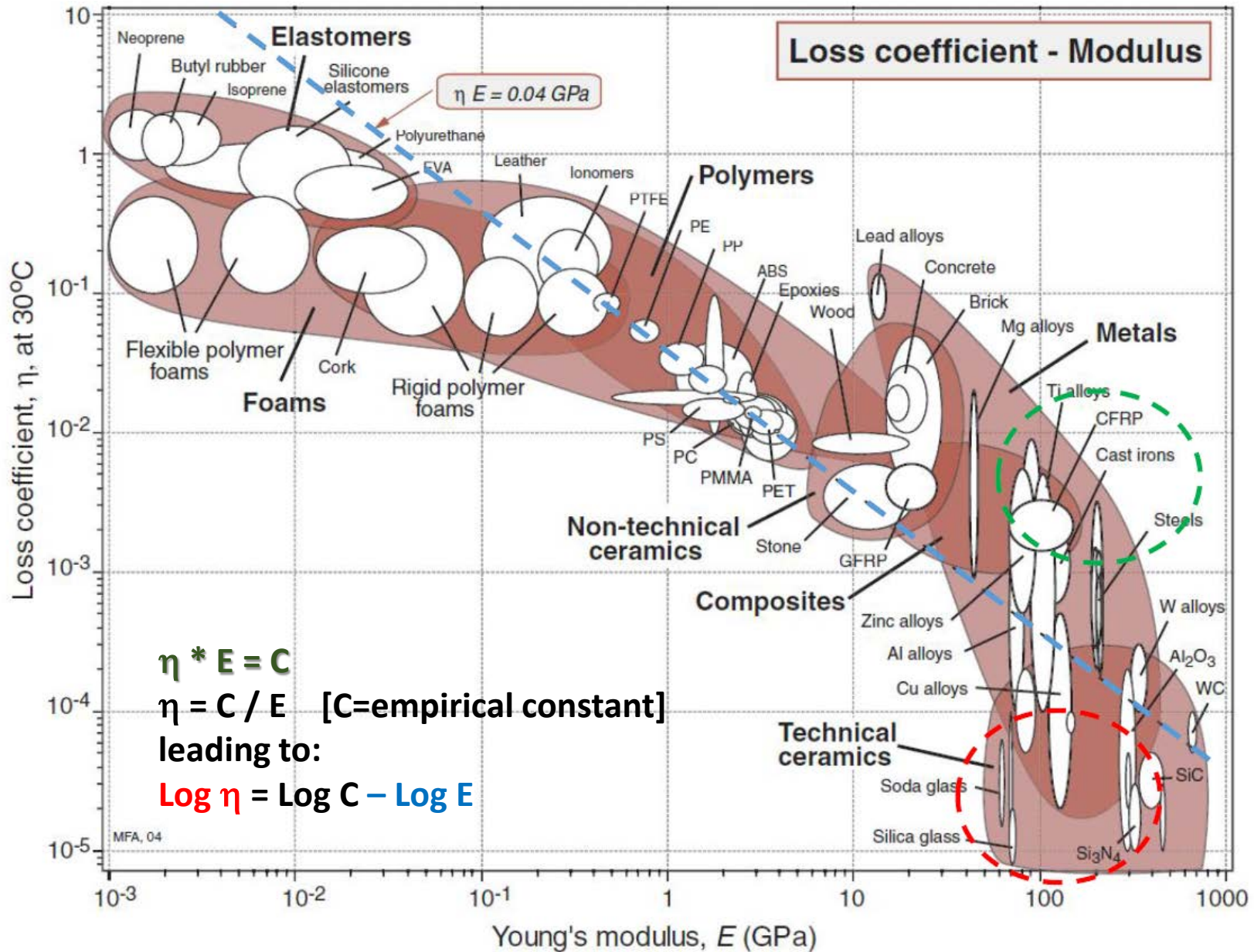
$$\lambda = C / \rho_e$$

Again, by applying logarithms:

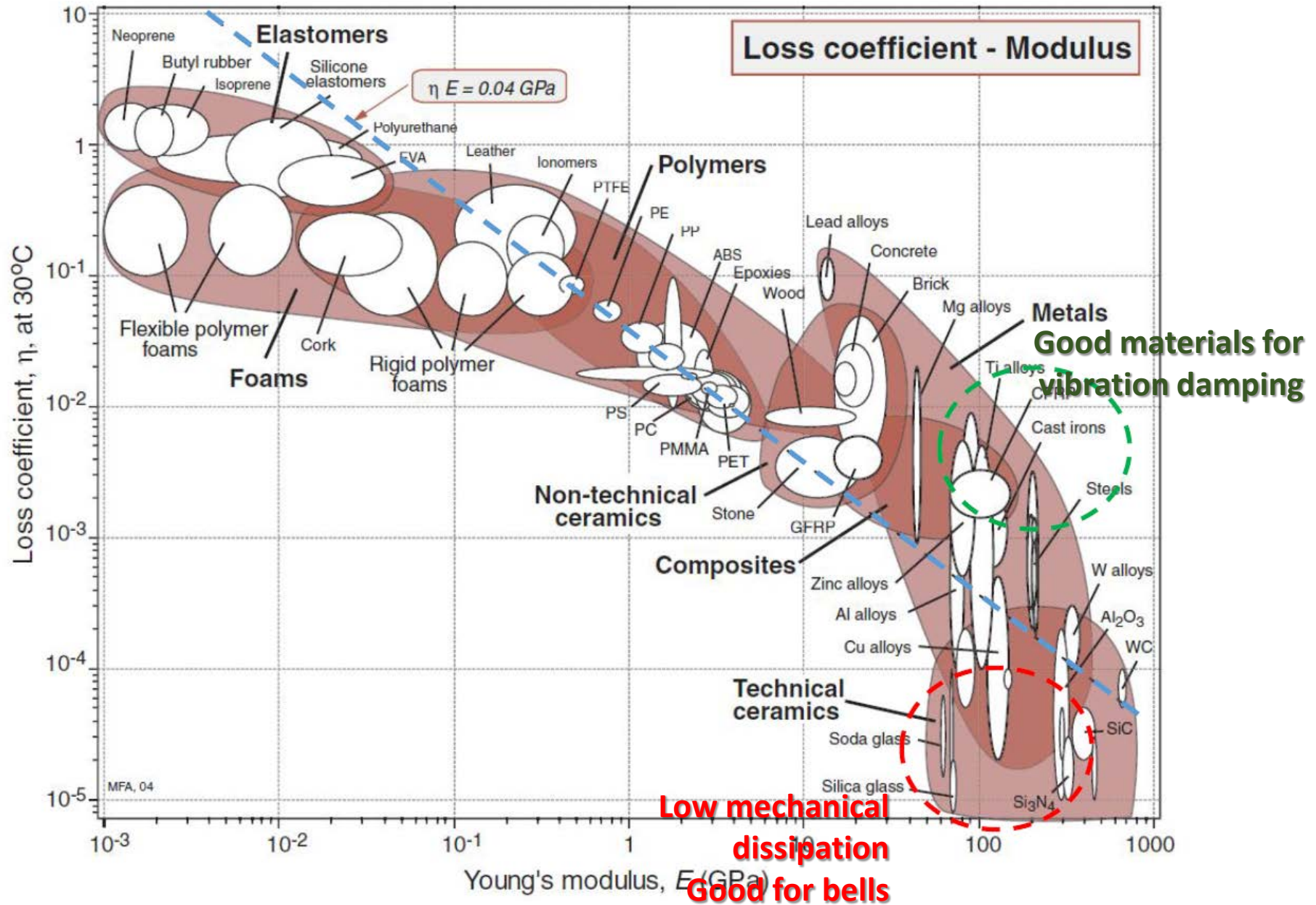
$$\text{Log } \lambda = \text{Log } C - \text{Log } \rho_e$$



Materials for vibration damping or 'bright' sound



Materials for vibration damping or 'bright' sound



Graphical selection (dual-property): materials for high speed of sound

$$v = C \cdot (E / \rho)^{0.5}$$

[C = material-independent constant]

The ratio E/ρ is an obvious **performance index**, expressed as the transmission of mechanical waves:

$$I = E / \rho$$

Leading to:

$$\text{Log } I = \text{Log } E - \text{Log } \rho$$

$$\text{Log } E = \text{Log } \rho + \text{Log } I$$

Dual-property graph

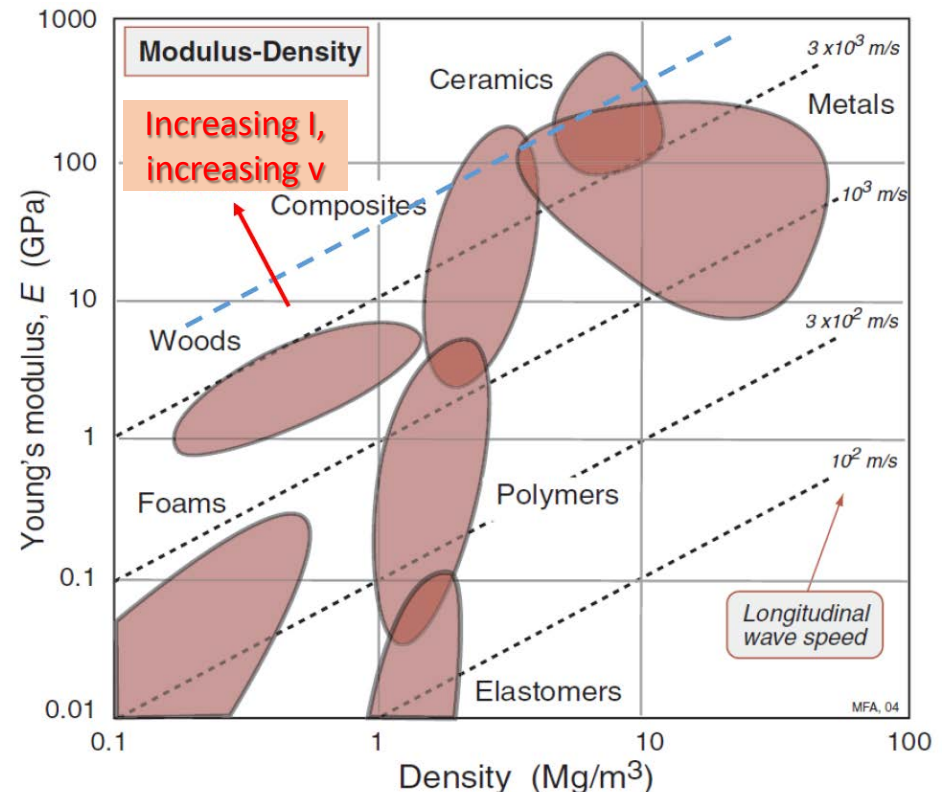
Parallel lines for given values of v

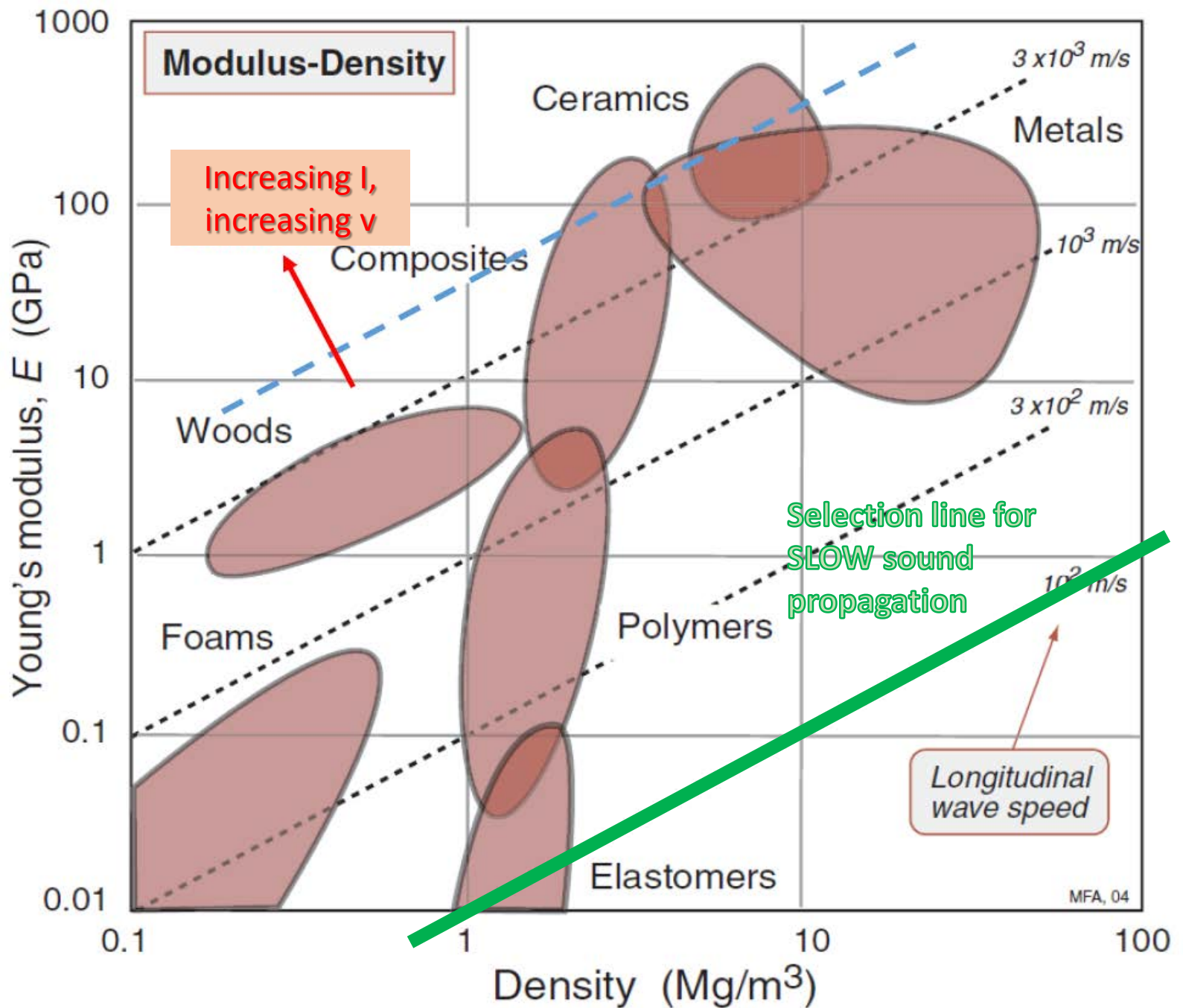
Pay attention to the SIGN (+/-)

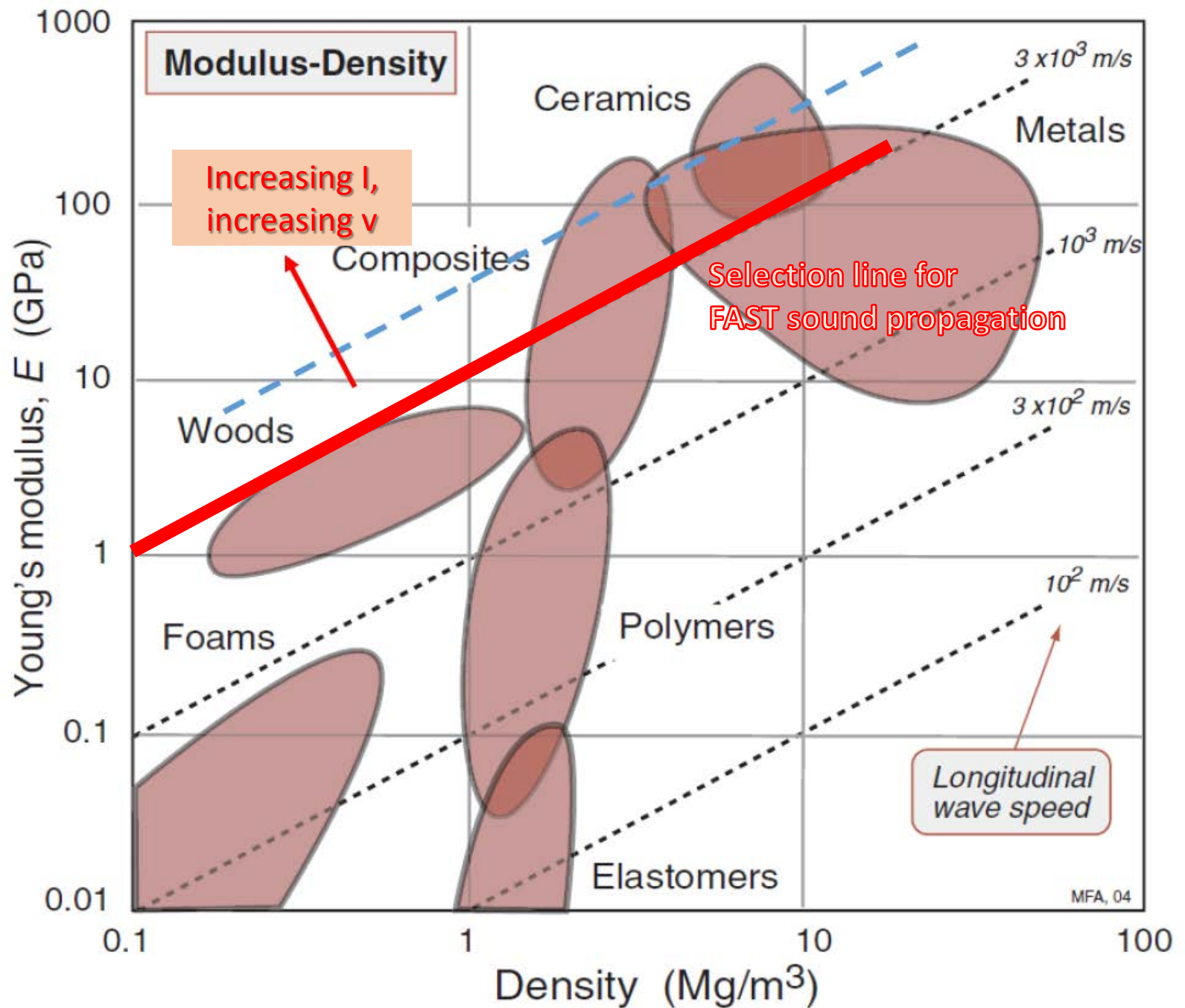
[Index in form of ratio: positive slope;

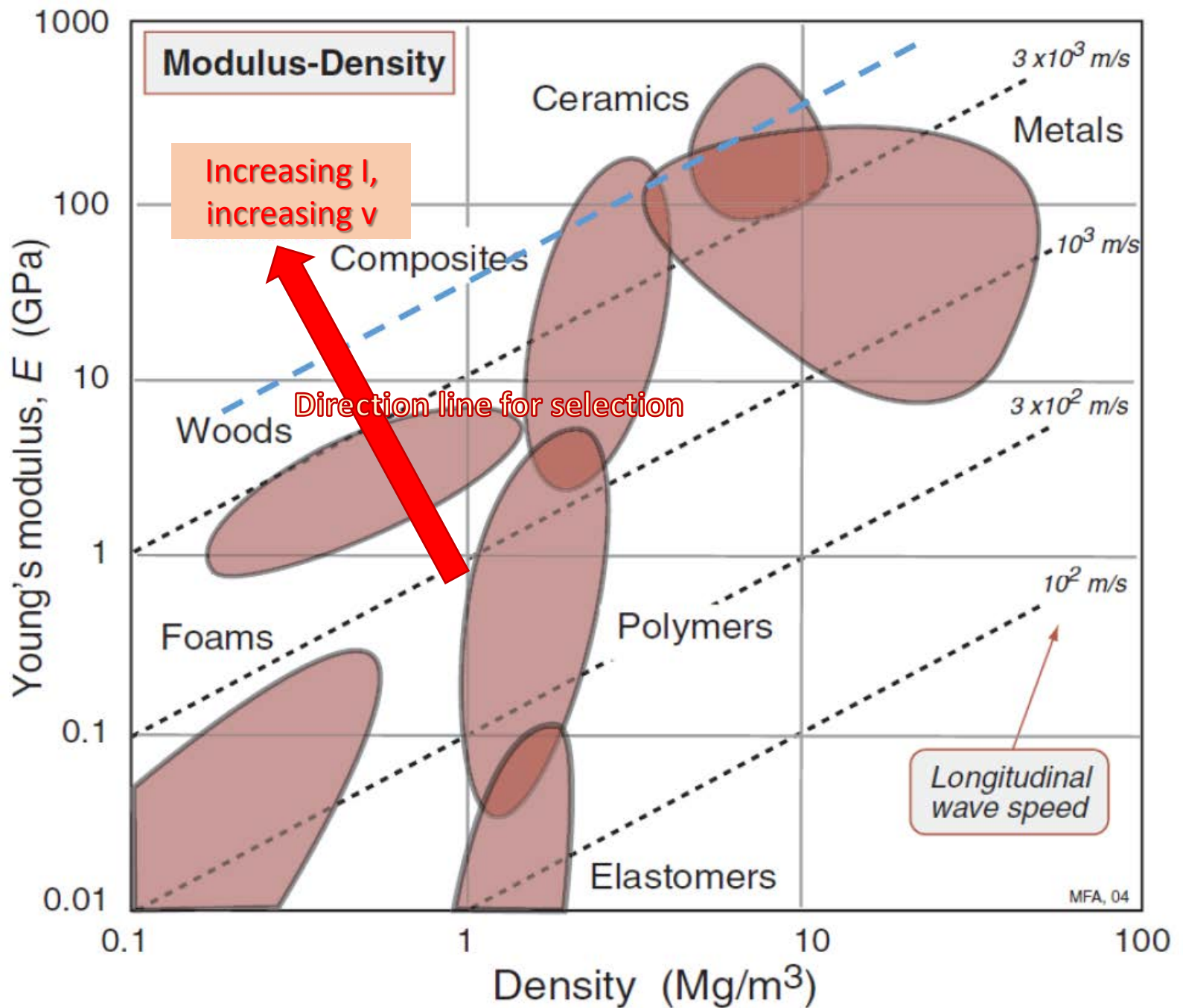
Index in form of product:

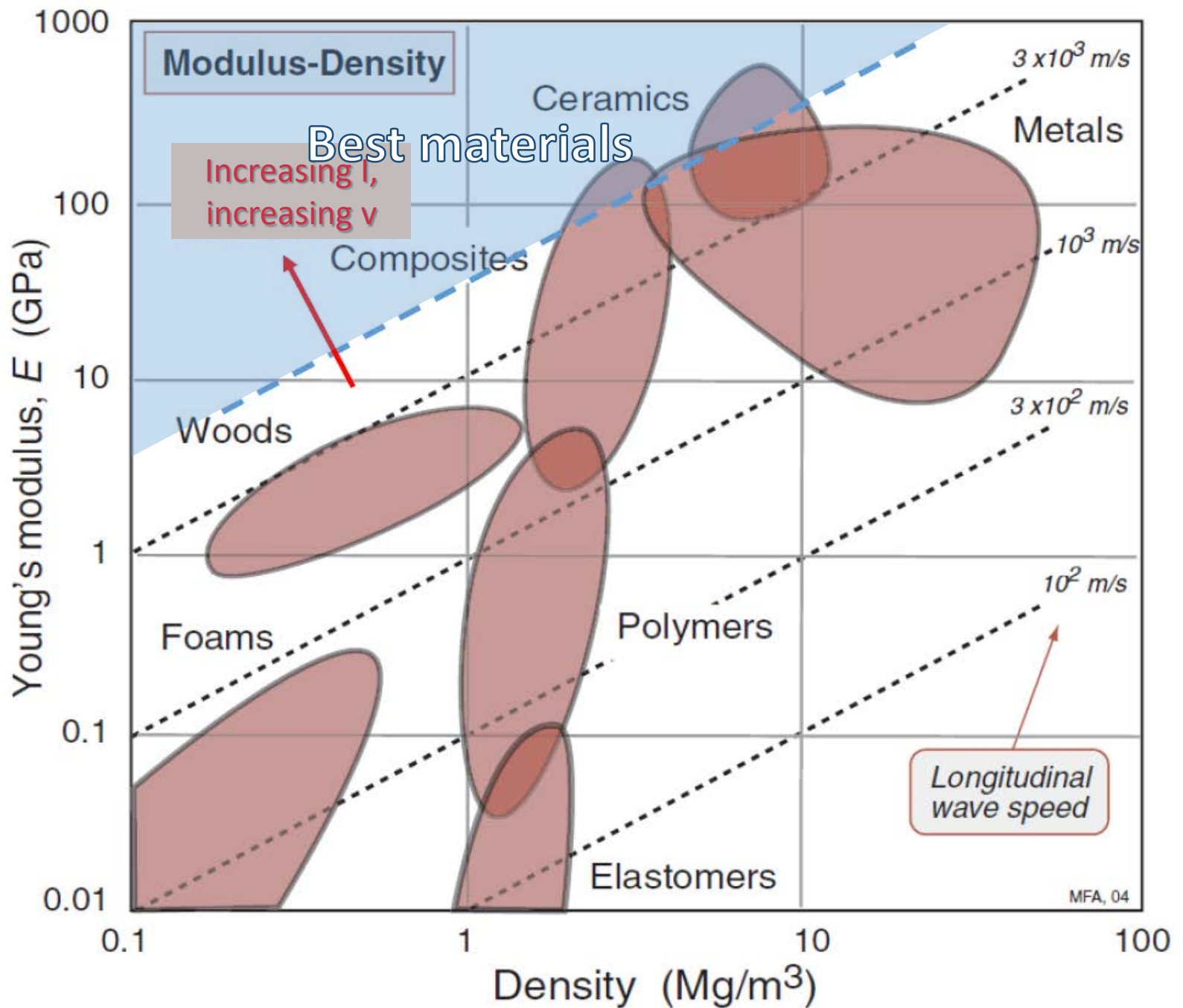
negative slope]



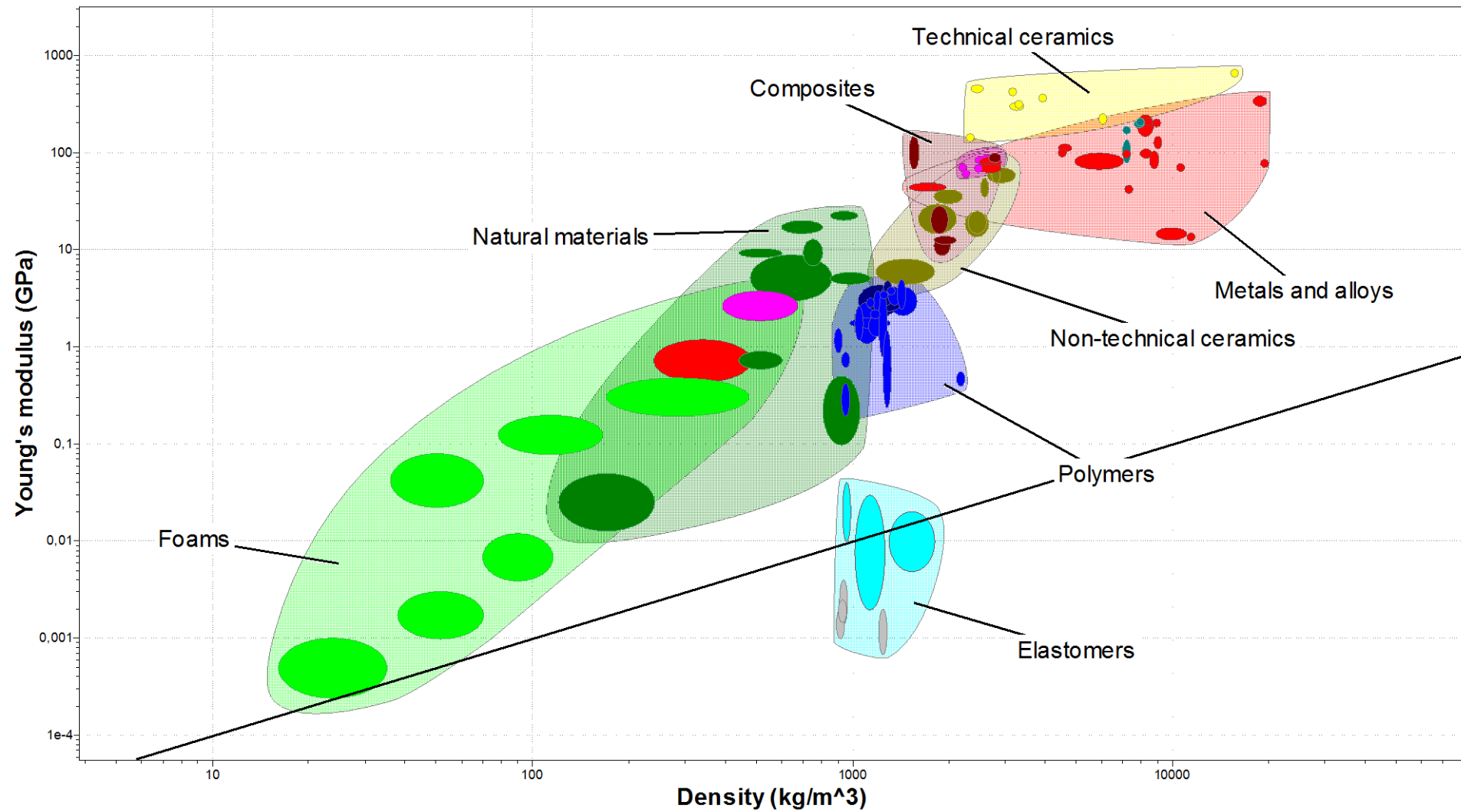




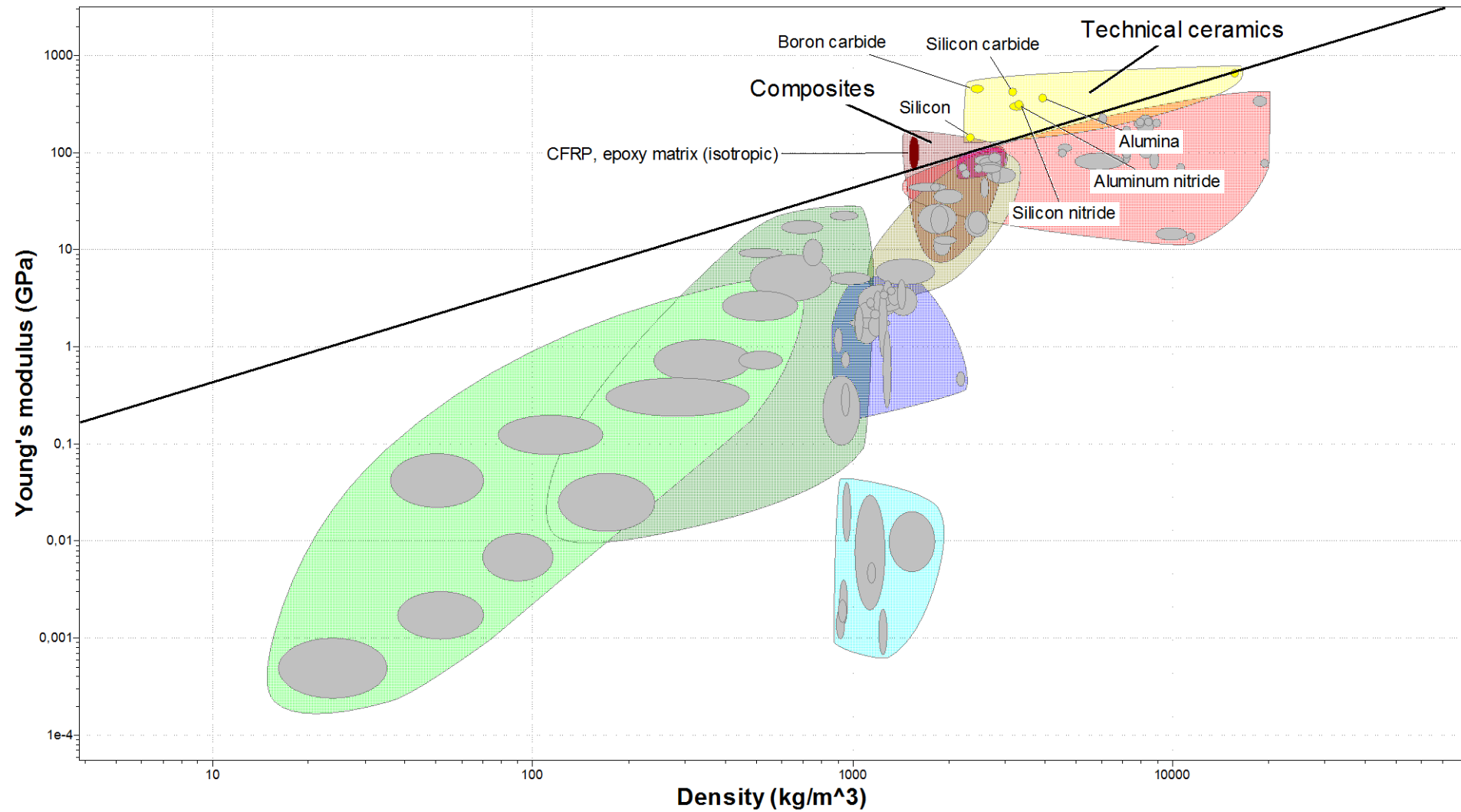




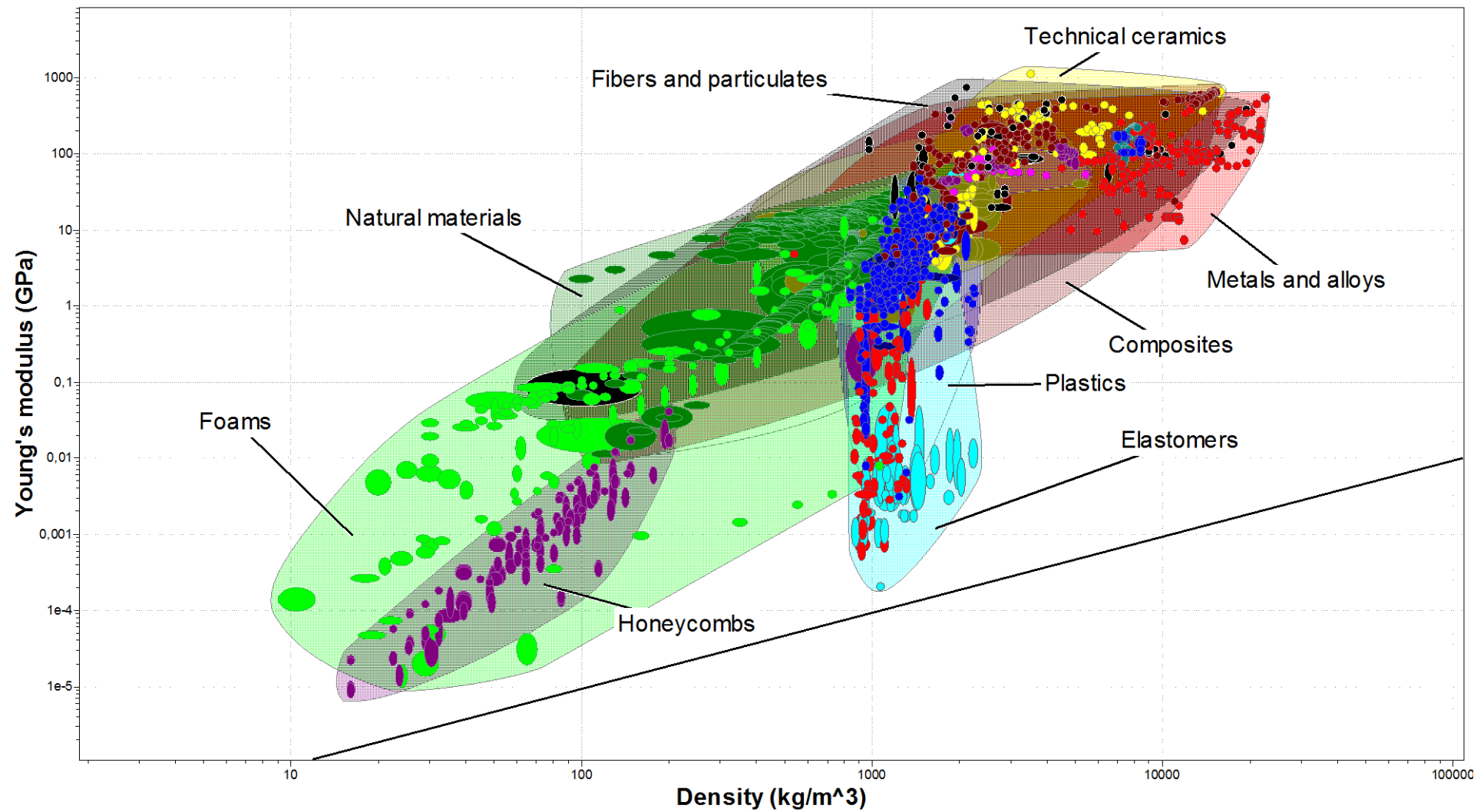
CES (Cambridge Engineering Selector) level 2



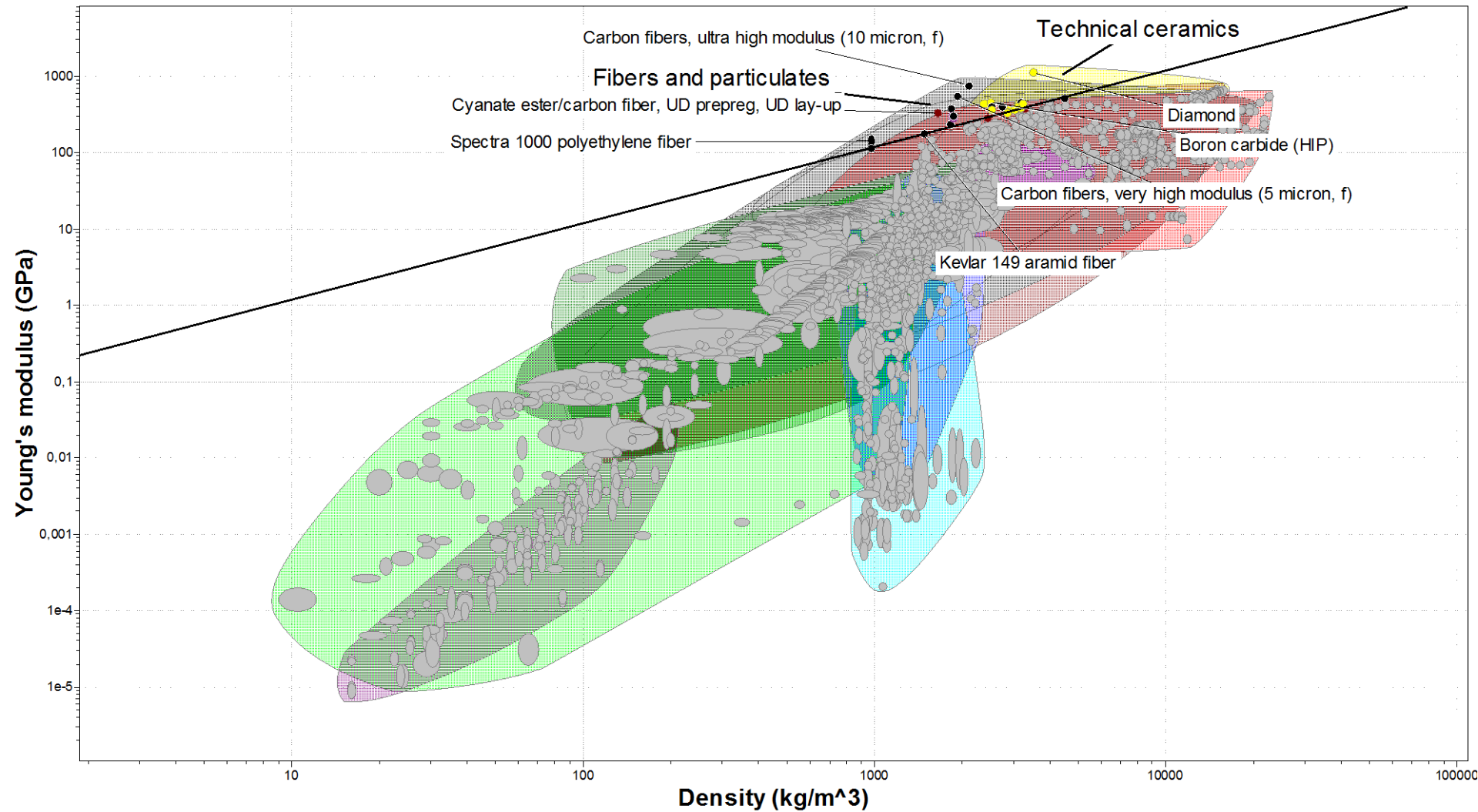
CES (Cambridge Engineering Selector) level 2



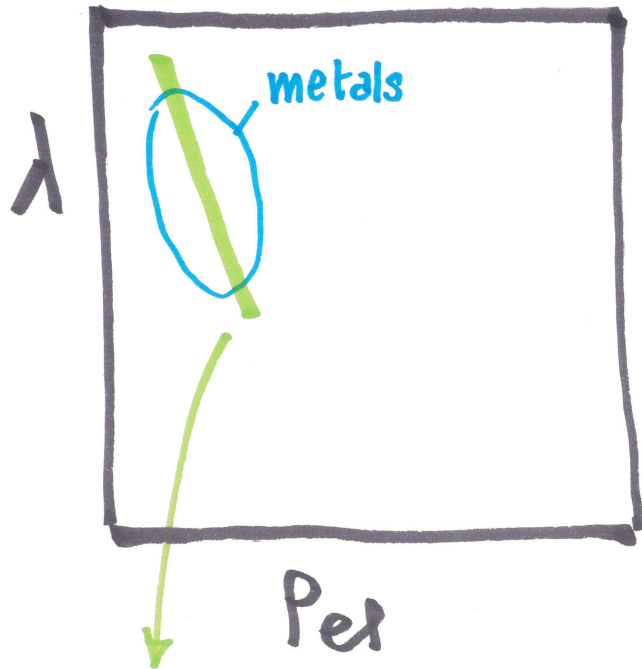
CES (Cambridge Engineering Selector) level 3



CES (Cambridge Engineering Selector) level 3



MIND THE DIFFERENCE

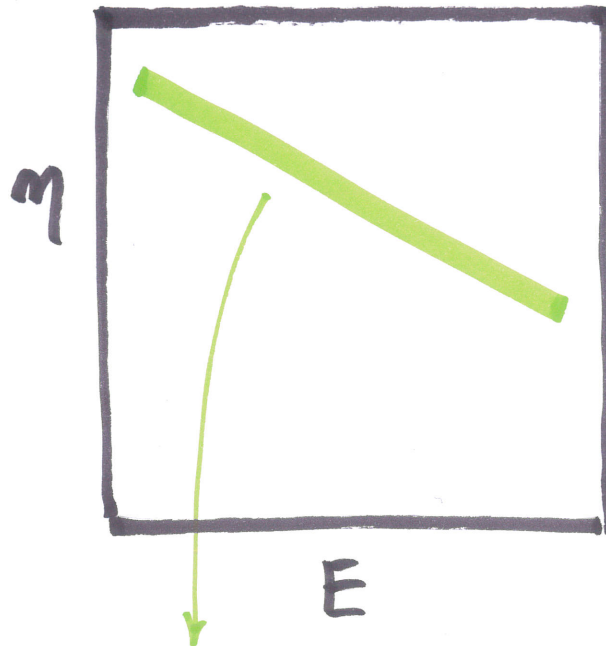


LINE EXPRESSING
 λ - P_{el} CORRELATION
FOR METALS

$$\lambda = \frac{\text{constant}}{P_{el}} = \frac{C}{P_{el}}$$

$$\lambda \cdot P_{el} = C$$

$$\log \lambda = -\log P_{el} + \log C$$

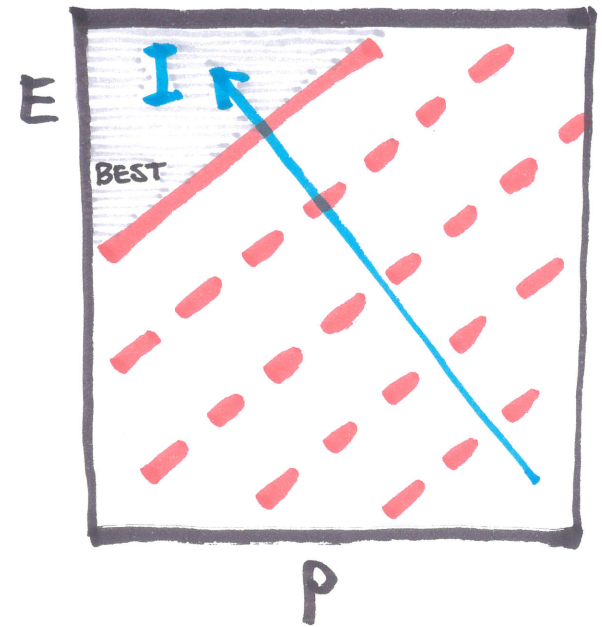


LINE EXPRESSING
 η - E CORRELATION
FOR MOST MATERIALS

$$\eta \approx \frac{C}{E}$$

$$\log \eta = -\log E + \log C$$

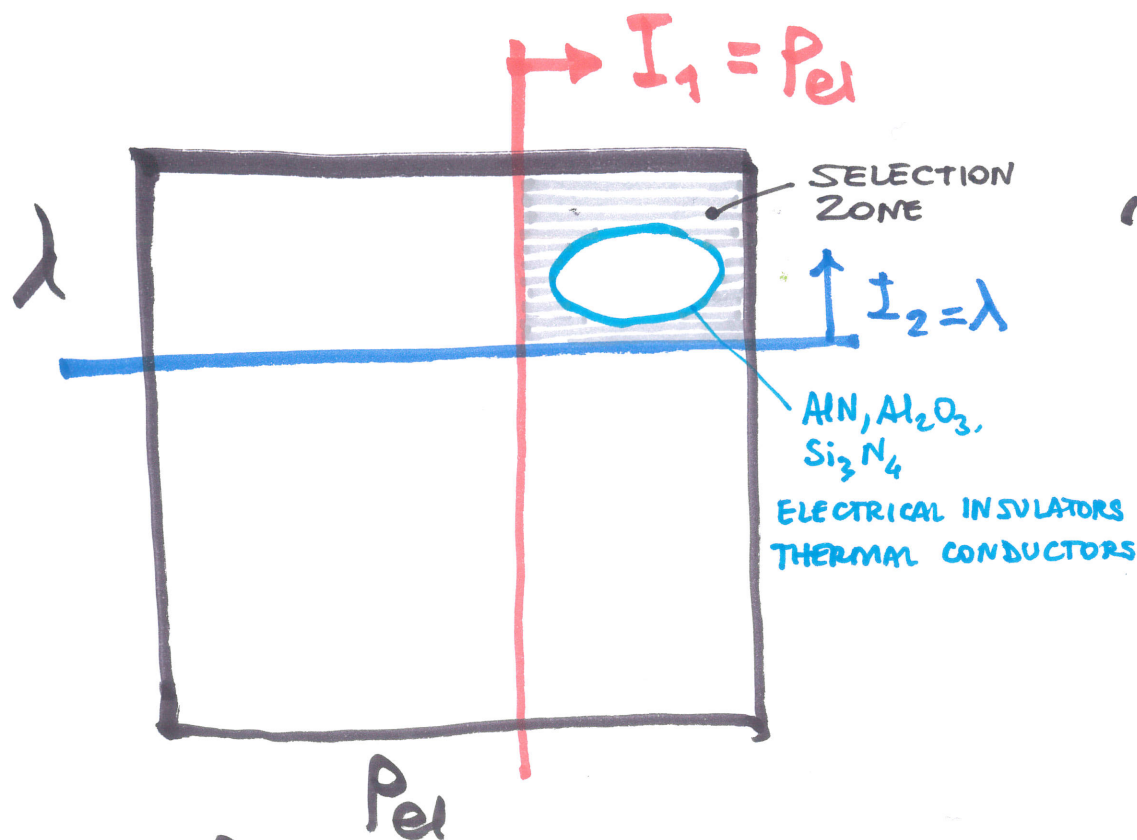
$$\log E = \log P + \log I$$



LINES EXPRESSING
VALUES OF INDEX

$$I = \frac{E}{P}$$

PERFORMANCE
INDEX

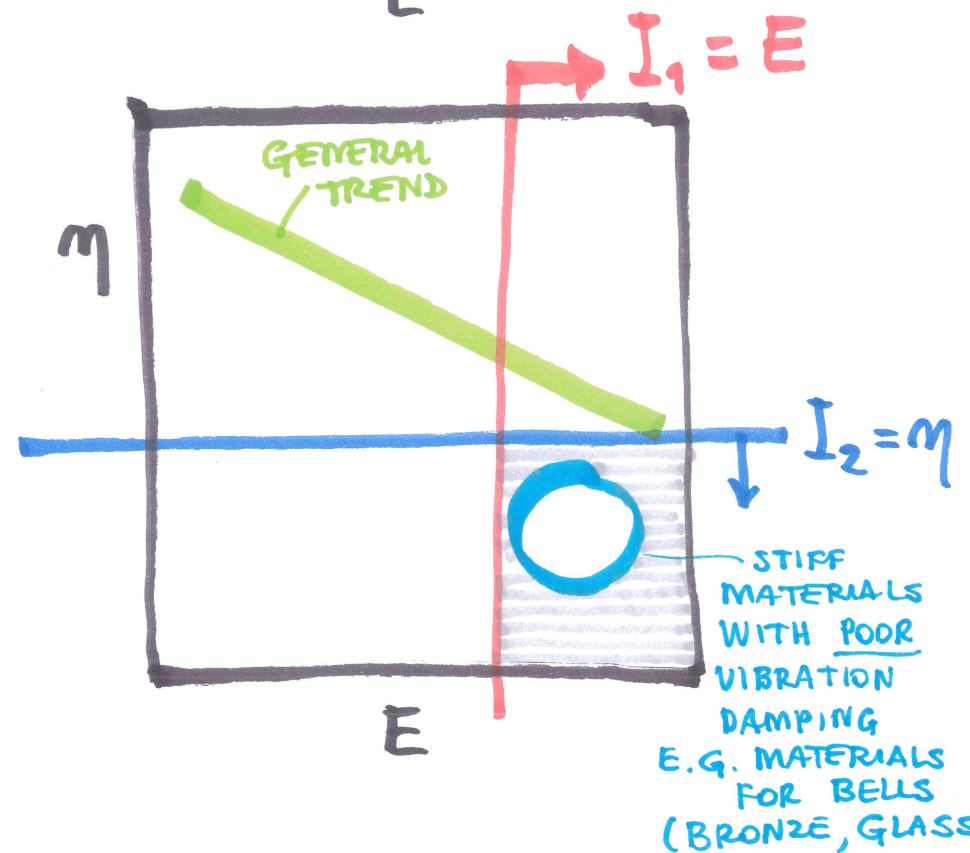
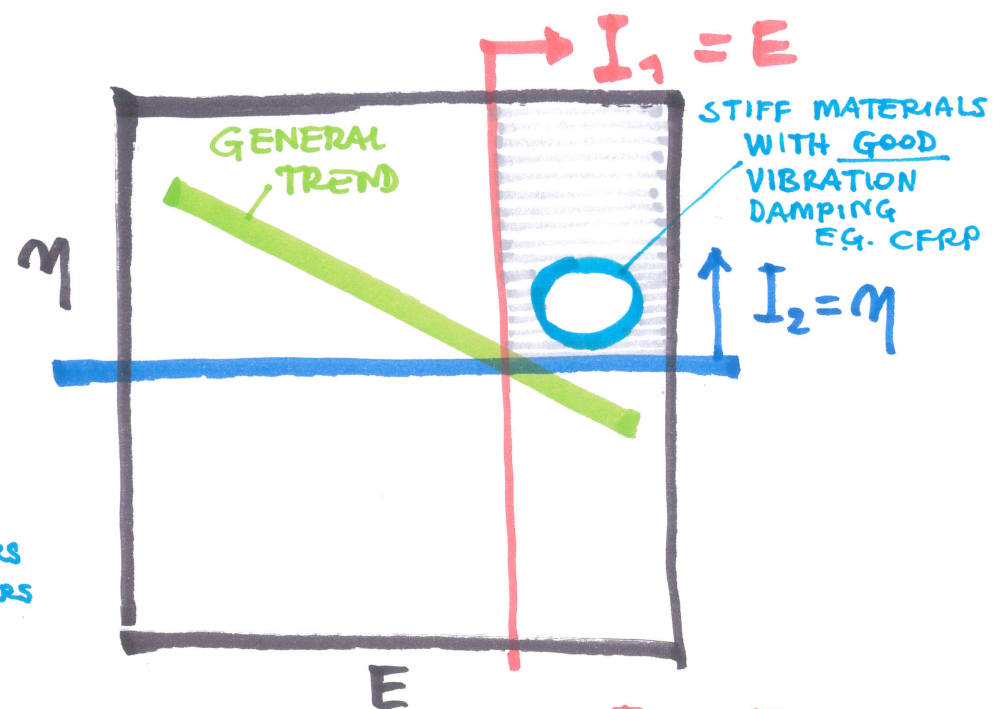


λ - P_{el} & η - E CHARTS
 ARE ACTUALLY USEFUL WHEN
 APPLYING 2 MATERIAL INDICES
 SIMULTANEOUSLY (VERTICAL AND
 HORIZONTAL LINES)

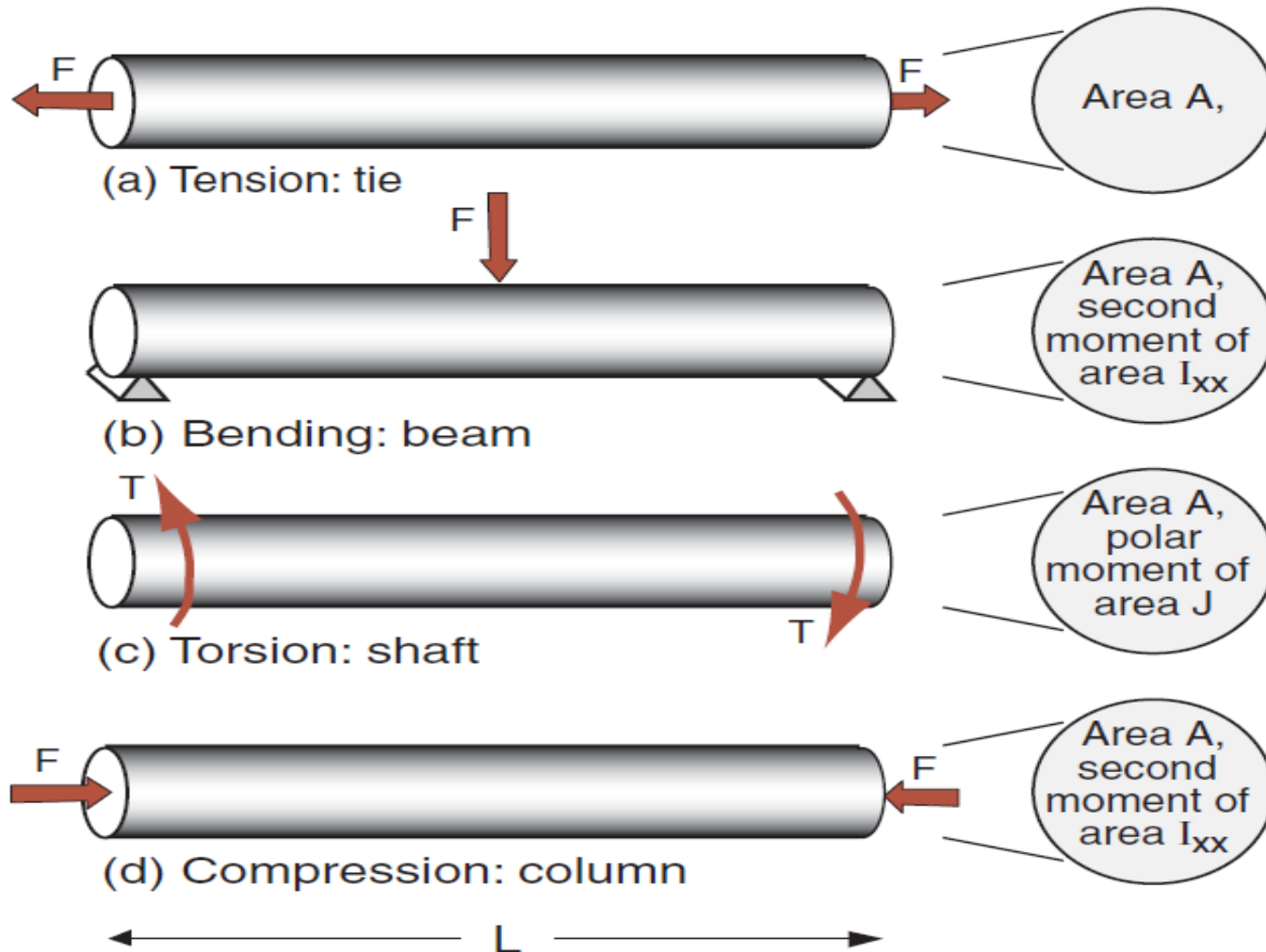
$I_1 = \text{PROPERTY 1}$
 $I_2 = \text{PROPERTY 2}$

$I_1 \rightarrow$ SELECTION
 $I_2 \rightarrow$ ZONE

ATTENTION: GRAPHICAL APPROACH, WITH MORE
 THAN ONE INDEX, IS NOT ALWAYS RELIABLE



Optimizing stiffness (of structures)



Tie rods (**pull-rods**), #1: control of cross-section area

- macroscopic deformation, δ , defined as elongation ΔL , by definition equal to $\varepsilon \cdot L$ (L is the length);
- '*Elongational*' stiffness (measuring the resistance of the **structure** against elongation) easily correlated to elastic modulus, according to Hooke's law:

$$S = \frac{F}{\delta} = \frac{F}{\Delta L} = \frac{\sigma \cdot A}{\varepsilon \cdot L} = E \cdot \frac{A}{L}$$

Let's fix *a minimum value of stiffness* (obviously related to a maximum elongation, for a given applied force F):

$$S \geq S_{min} = \frac{F}{\delta_{max}}$$

$$E \cdot \frac{A}{L} \geq S_{min} \rightarrow A \geq S_{min} \cdot L \cdot \frac{1}{E} \rightarrow A \geq F S_{Area} \cdot \frac{1}{E}$$

At a given threshold stiffness value (S_{min}), or maximum deformation (δ_{max}), the cross-section area may be reduced adopting materials with high elastic modulus. **Elastic modulus** is an **index of section area**

Tie rods (**pull-rods**), #2: control of mass

$$S \geq S_{min} = \frac{F}{\delta_{max}}$$

Let's express the equation according to the mass:

$$E \cdot \frac{A}{L} = E \cdot \frac{m}{\rho \cdot L^2} \geq S_{min} \rightarrow m \geq S_{min} \cdot L^2 \cdot \frac{\rho}{E} \rightarrow \mathbf{m} \geq \mathbf{FS_m} \cdot \frac{\rho}{E}$$

At a given minimum stiffness, or maximum sustainable deformation, mass may be **minimized** adopting materials with **high E/ρ ratio**. This ratio is an **index of mass**.

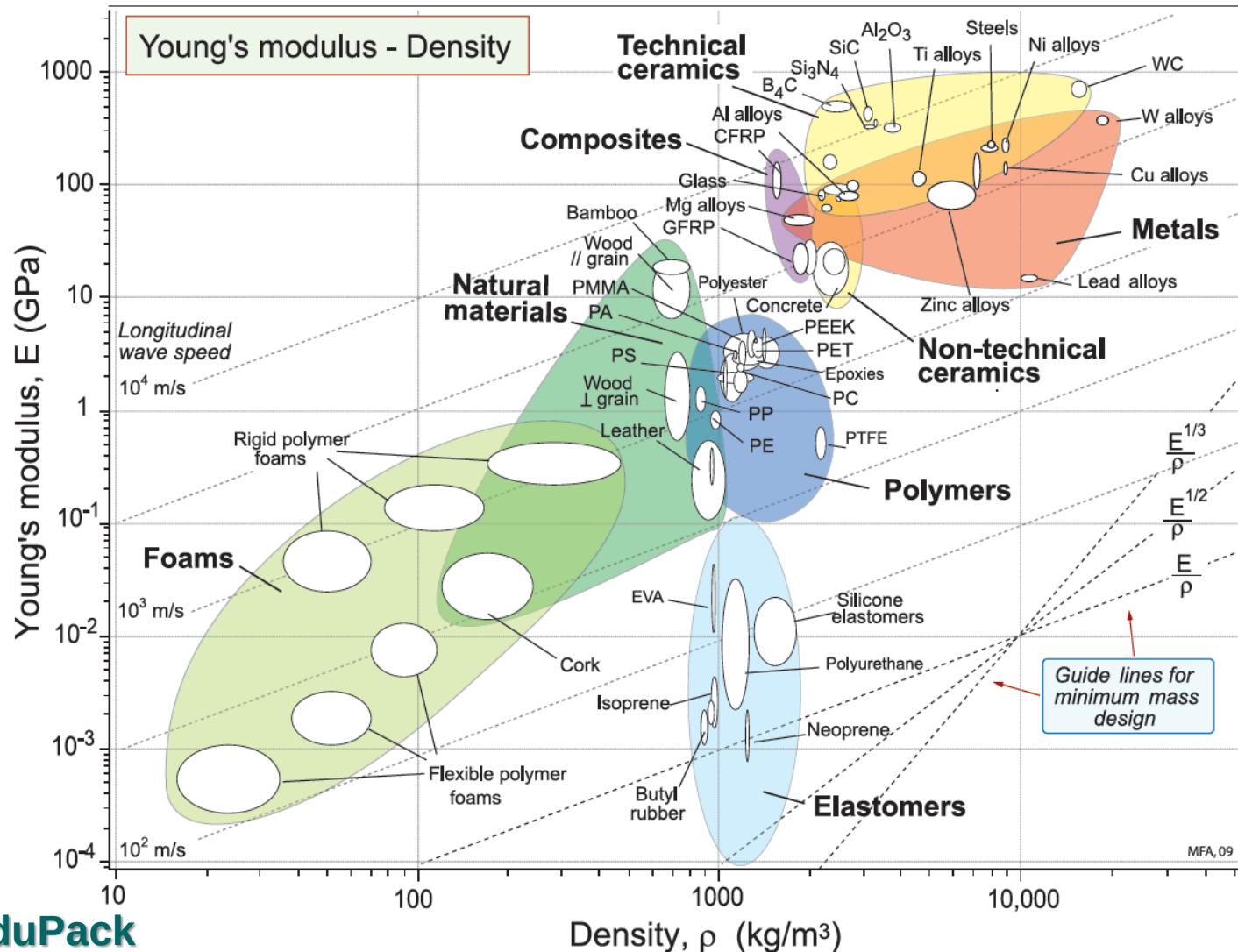
$$S_{min} \leq \frac{m}{L^2} \cdot \frac{E}{\rho} \rightarrow S_{min} \leq \mathbf{FS_{Smin}} \cdot \frac{E}{\rho}$$

At a given mass the **minimum stiffness** may be **increased**, i.e. the **maximum deformation** may be **decreased**, by increasing **E/ρ**.

Application: Ashby's plots

Indices represented in material property graphs
Logarithmic scales

**Materials
selection for
light stiff ties**
**Reduction of
mass**



Graphical selection

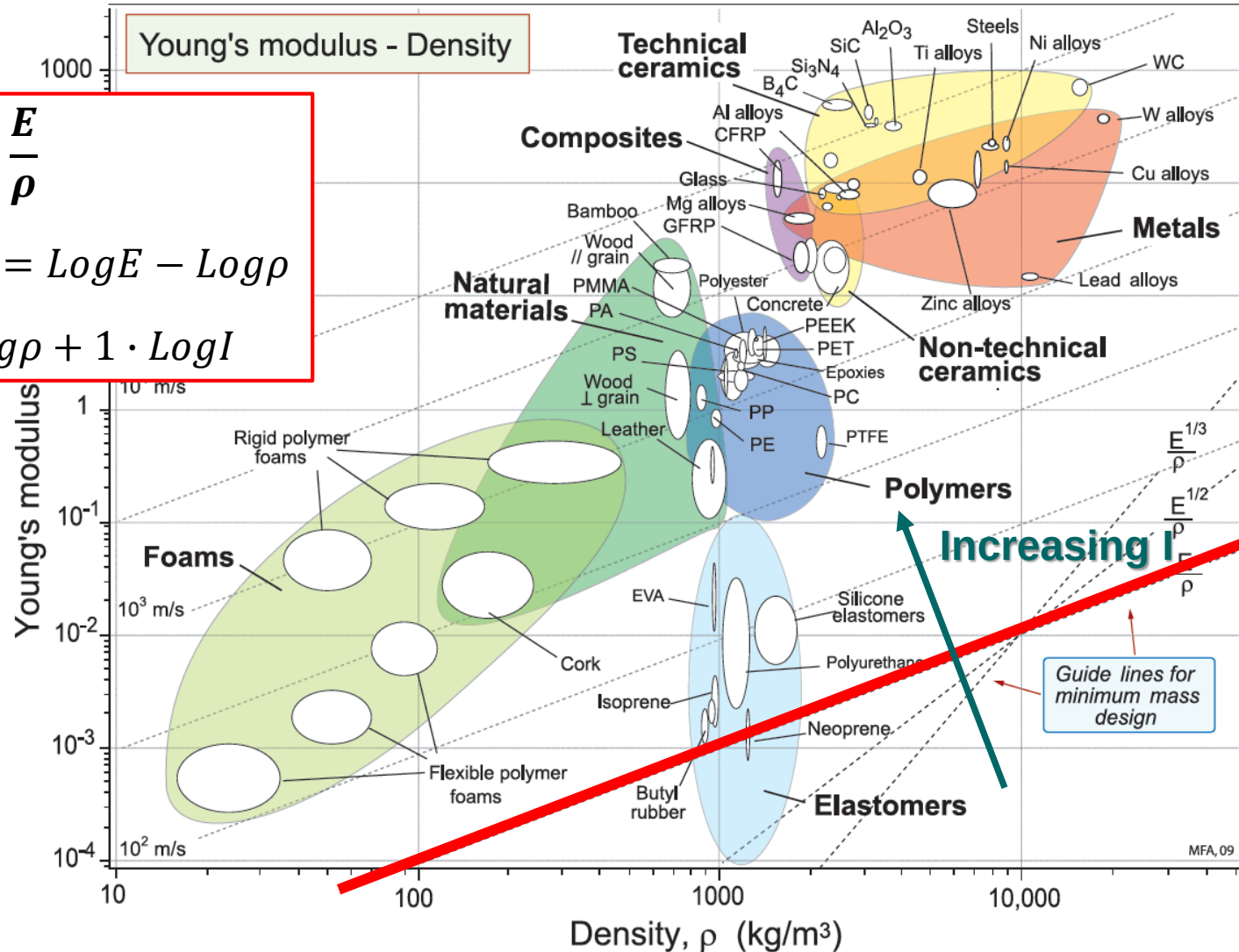
Optimization of stiffness
Selection lines and directions

Young's modulus - Density

$$I = \frac{E}{\rho}$$

$$\text{Log} I = \text{Log} \left(\frac{E}{\rho} \right) = \text{Log} E - \text{Log} \rho$$

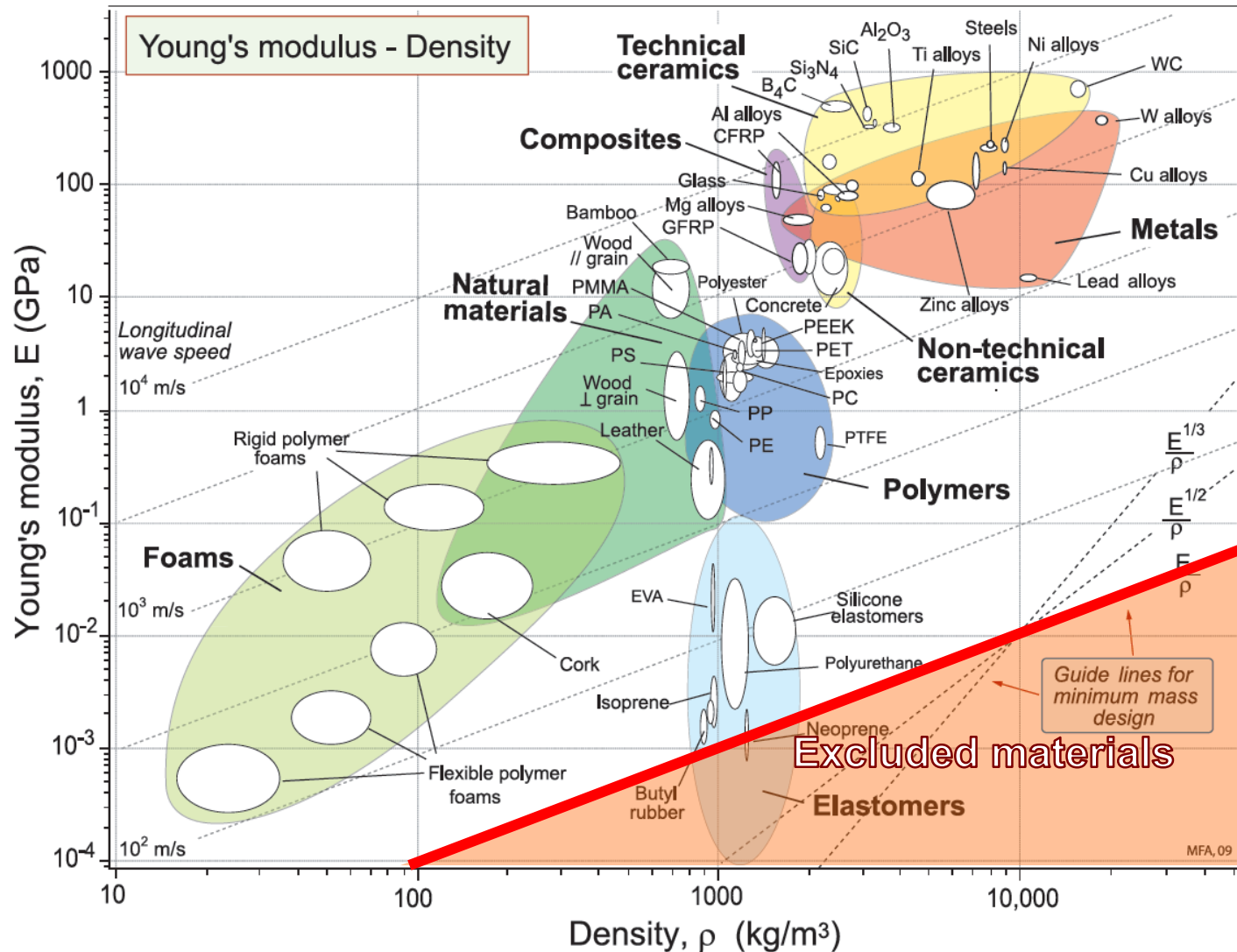
$$\text{Log} E = 1 \cdot \text{Log} \rho + 1 \cdot \text{Log} I$$



Graphical selection

Optimization of stiffness
Selection lines and directions

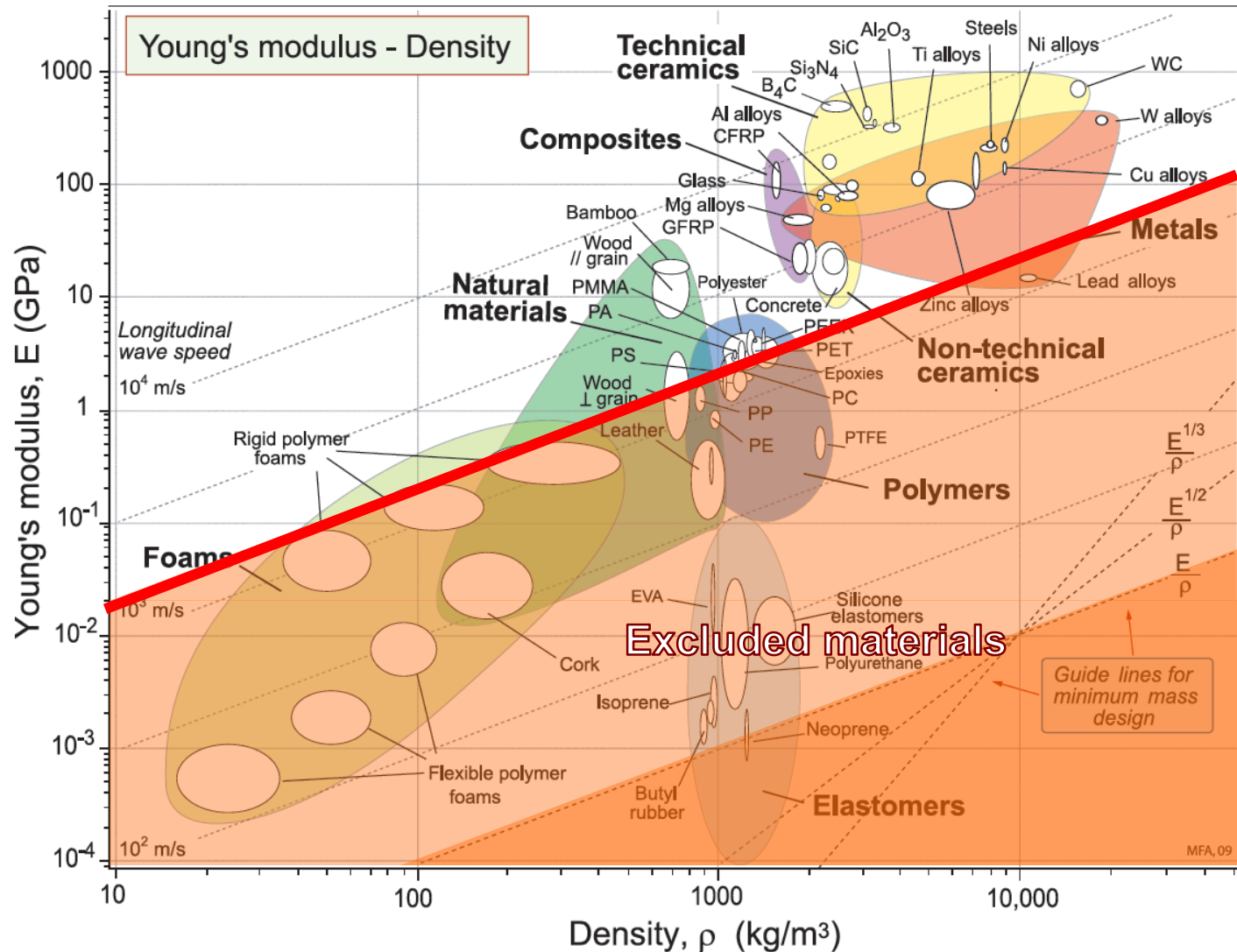
**Materials
selection for
light stiff ties
Reduction of
mass**



Graphical selection

Optimization of stiffness
Selection lines and directions

**Materials
selection for
light stiff ties**
**Reduction of
mass**

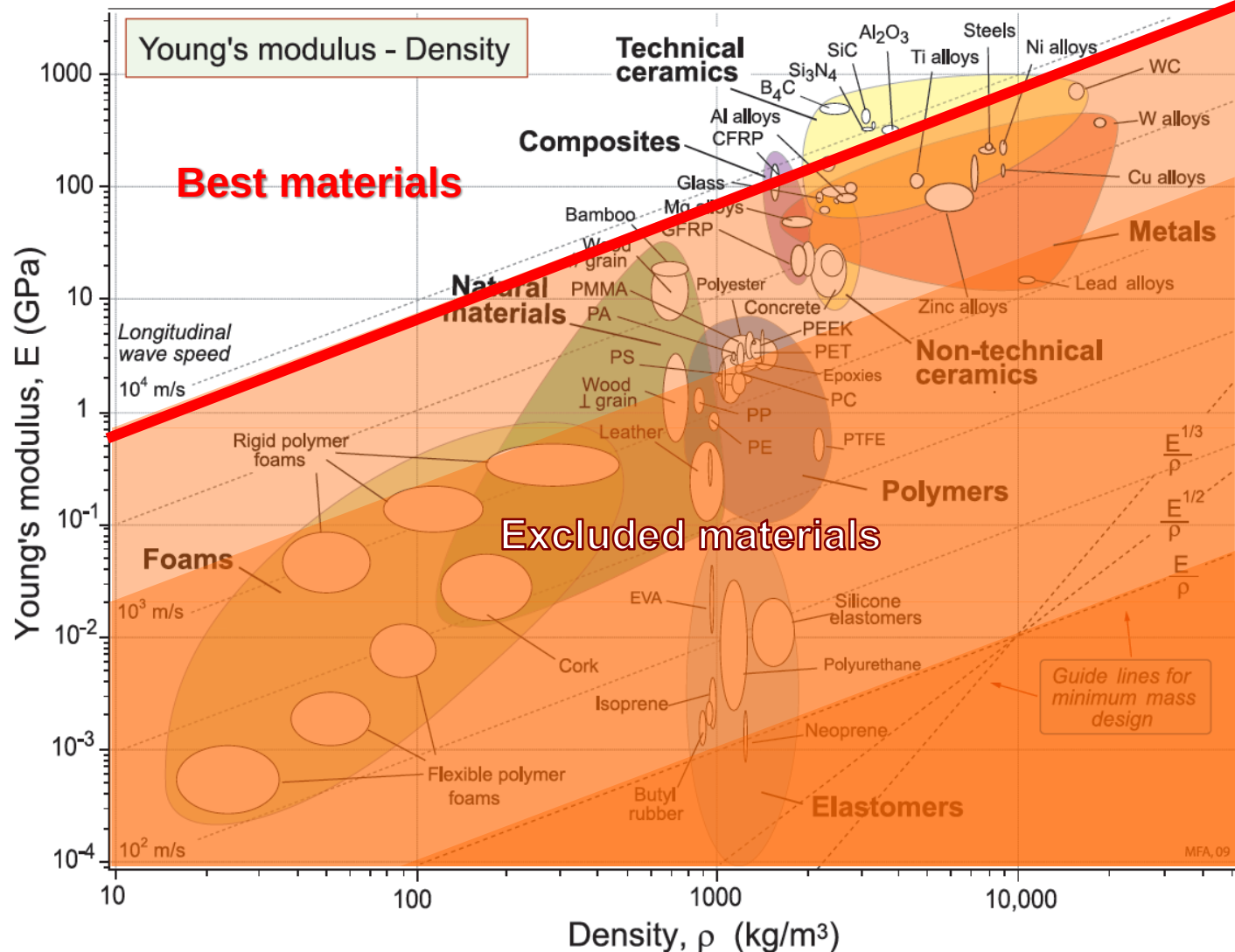


Graphical selection

Optimization of stiffness
Selection lines and directions

**Materials
selection for
light stiff ties
Reduction of
mass**

**...some
candidates can
be excluded for
excessive cost or
brittleness
(refined selection
on 'winners')**



Graphical selection

Optimization of stiffness
Selection lines and directions

Young's modulus - Density

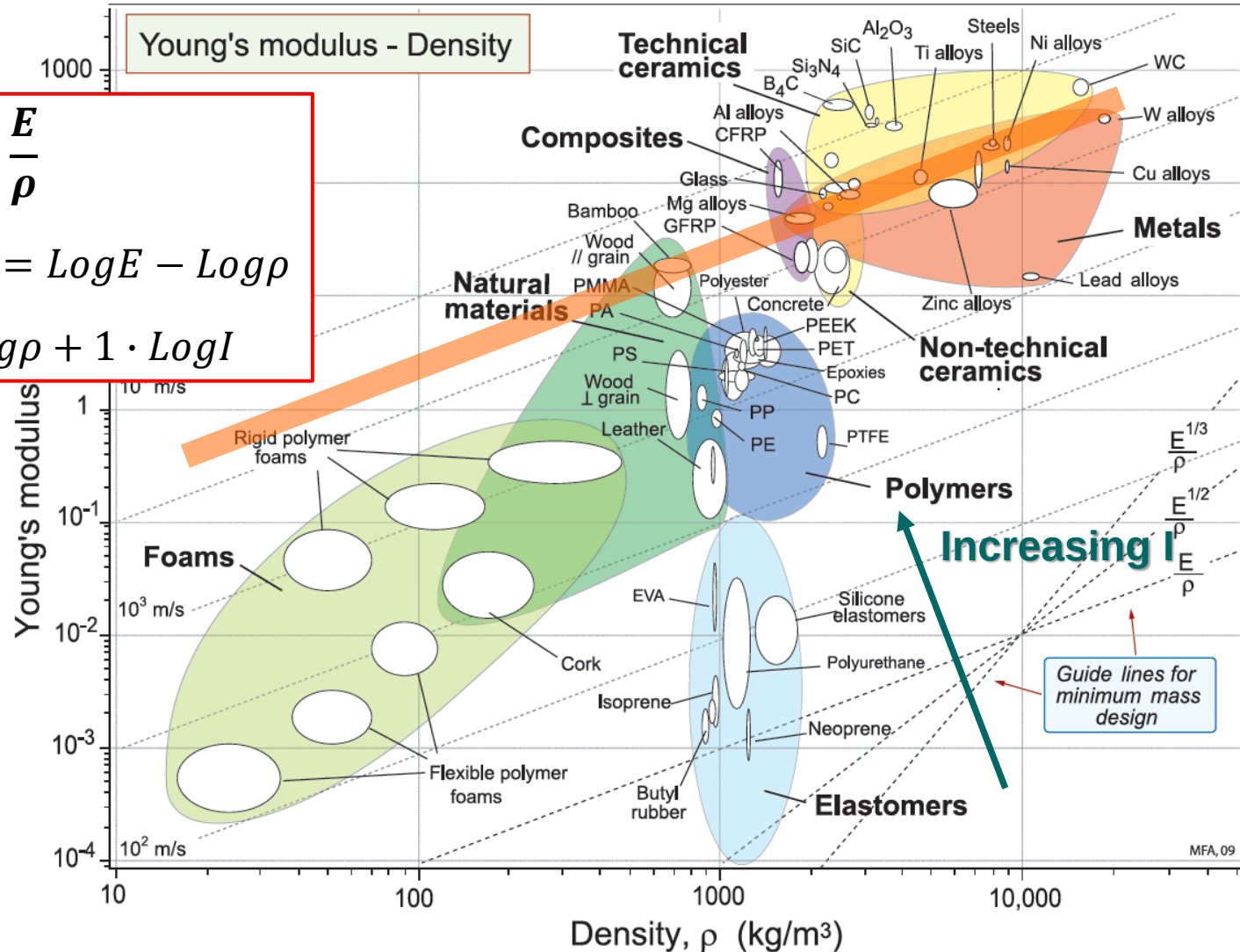
$$I = \frac{E}{\rho}$$

$$\text{Log} I = \text{Log} \left(\frac{E}{\rho} \right) = \text{Log} E - \text{Log} \rho$$

$$\text{Log} E = 1 \cdot \text{Log} \rho + 1 \cdot \text{Log} I$$

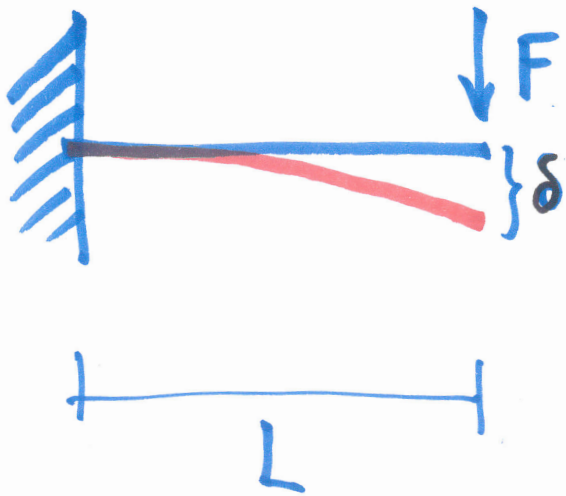
Lightweight
alloys
equivalent
to Ti alloys
and steels

Steel offers
area
reductions



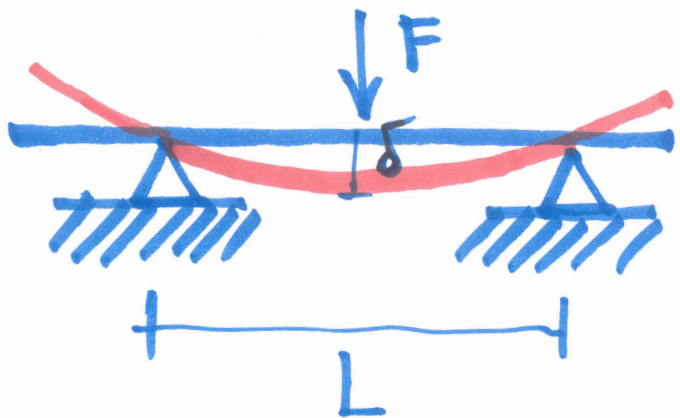
Some mechanics...

STIFFNESS OF BARS



δ = DEFLECTION

$$S = \frac{F}{\delta}$$



$$S = C' \cdot \frac{EJ}{L^3}$$

MOMENT OF INERTIA

1ST OF FUNDAMENTAL EQUATIONS (NOT SO MANY...)

DESIGN FOR STIFFNESS : MINIMIZE DEFLECTION

$$\delta \leq \delta_{\max} = \text{MAXIMUM ALLOWED DEFLECTION}$$



AT A GIVEN APPLIED FORCE

$$\frac{F}{\delta} = S \geq S_{\min} = \frac{F}{\delta_{\max}}$$

$$C \cdot \frac{EJ}{L^3} \geq S_{\min}$$

BARS

$$J = C \cdot A^2$$

BASIC
EQUATION #1

CONSTANT, AT A GIVEN SHAPE

$$\frac{C' E J}{L^3} = C'' \cdot \frac{E A^2}{L^3} \geq S_{\min}$$

$FS_A =$ INDEPENDENT FROM MATERIAL

$$A \geq \sqrt{\frac{S_{\min} \cdot L^3}{C''}} \cdot \boxed{\frac{1}{\sqrt{E}}}$$

CONTRIBUTION
OF MATERIALS

↓ A

AREA MINIMIZED
BY INCREASING
ELASTIC MODULUS

↑ E

mass

$$m = L \cdot A \cdot \rho$$

← DENSITY

$$m \geq \underbrace{L \cdot FS_{\text{BAR AREA}}}_{\text{INDEPENDENT FROM MATERIAL}} \cdot \left[\frac{1}{\sqrt{E}} \cdot \rho \right] \text{ MATERIAL PROPERTIES}$$

$$m \geq FS_m^{\text{BAR}} \cdot \frac{\rho}{\sqrt{E}}$$

↓ m WITH ↑ $\frac{\sqrt{E}}{\rho} = I$

Mind the
exponent
of E!

PANELS

$$J = \frac{b w^3}{12}$$

W = THICKNESS

$$b \approx L \approx \text{CONSTANT}$$

$$J = C \cdot w^3$$

BASIC EQUATION
#2

$$C' \frac{E J}{L^3} = C'' \cdot \frac{E w^3}{L^3} \geq S_{\min}$$

$$w \geq \underbrace{\left(\frac{S_{\min} \cdot L^3}{C''} \right)^{1/3}}_{F S_w^{\text{PANEL}}} \cdot \frac{1}{E^{1/3}}$$

↓ w ↑ E

mass

$$W \geq F S_w^{\text{PANEL}} \cdot \frac{1}{E^{1/3}}$$

$$m = b \cdot L \cdot W \cdot \rho \geq \underbrace{b \cdot L \cdot F S_w}_{\text{PANEL}} \left[\rho \cdot \frac{1}{E^{1/3}} \right]$$

$$m \geq F S_m^{\text{PANEL}} \cdot \frac{\rho}{E^{1/3}}$$

↓ m WITH

$$\uparrow \frac{E^{1/3}}{\rho} = I$$

Mind the
exponent,
again!

Optimization of other properties

In general:

$$I = \frac{E^n}{\rho}$$

from which:

$$\text{Log} I = \text{Log} \left(\frac{E^n}{\rho} \right) = n \cdot \text{Log} E - \text{Log} \rho$$

$$\text{Log} E = \frac{1}{n} \cdot \text{Log} \rho + \frac{1}{n} \cdot \text{Log} I$$

$$m \geq FS_m \cdot \frac{\rho}{E^n}$$

- In many cases we are not interested at minimizing masses, but **attributes that are linearly dependent on mass**
- The **costs of a component**, C , derives from the *price* (cost for unit mass), C_m , of the adopted material:

$$C = C_m \cdot m \rightarrow C \geq FS_m \cdot \frac{\rho \cdot C_m}{E^n}$$

$$I = \frac{E^n}{\rho \cdot C_m} = \frac{E^n}{C_v}$$

where C_v is now the cost per unit volume; a new **selection graph will have elastic modulus against C_v** .

- Similar expressions rule production energy, carbon footprint...

1st EXAMPLE OF 'ECO-SELECTION' WHAT ABOUT ENVIRONMENT?

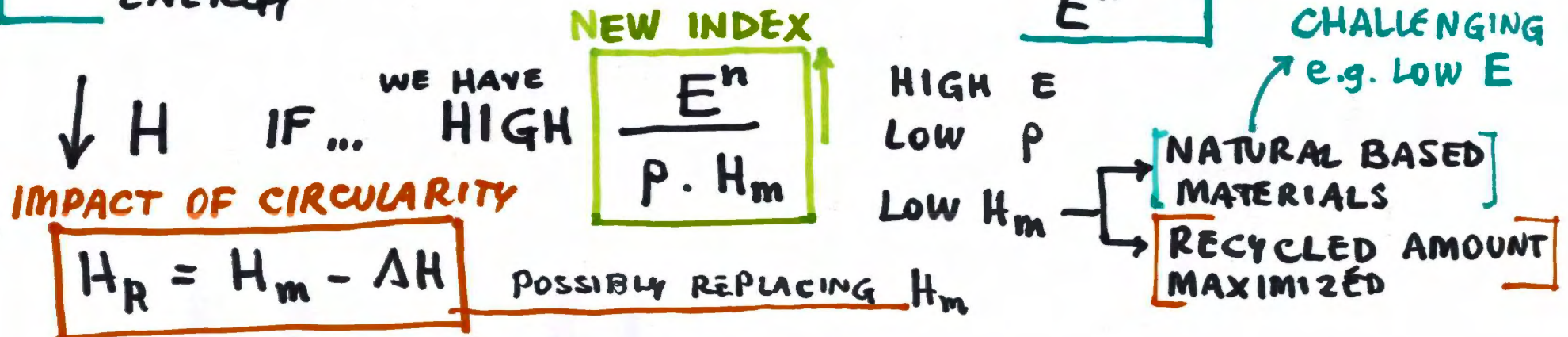
MIND EMBODIED ENERGY H_m

$$[H_m] = \frac{J \leftarrow \text{ENERGY}}{Kg \leftarrow \text{MASS}}$$

H_m IS IMPORTANT, BUT ONLY IF WE KNOW THE MASS

$$[m \geq FS_m \cdot \frac{P}{E^n}] = \text{minimum mass for desired stiffness}$$

$$[H = \text{'REAL' EMBODIED ENERGY} = H_m \cdot m \geq FS_m \cdot \frac{P \cdot H_m}{E^n}] \quad \text{OPTIMIZATION OF ENERGY}$$



ENVIRONMENTAL INDICES (OF STIFFNESS)

$$I' = \frac{E^n}{\rho \cdot H_m} = \frac{E^n}{H_{mp}}$$

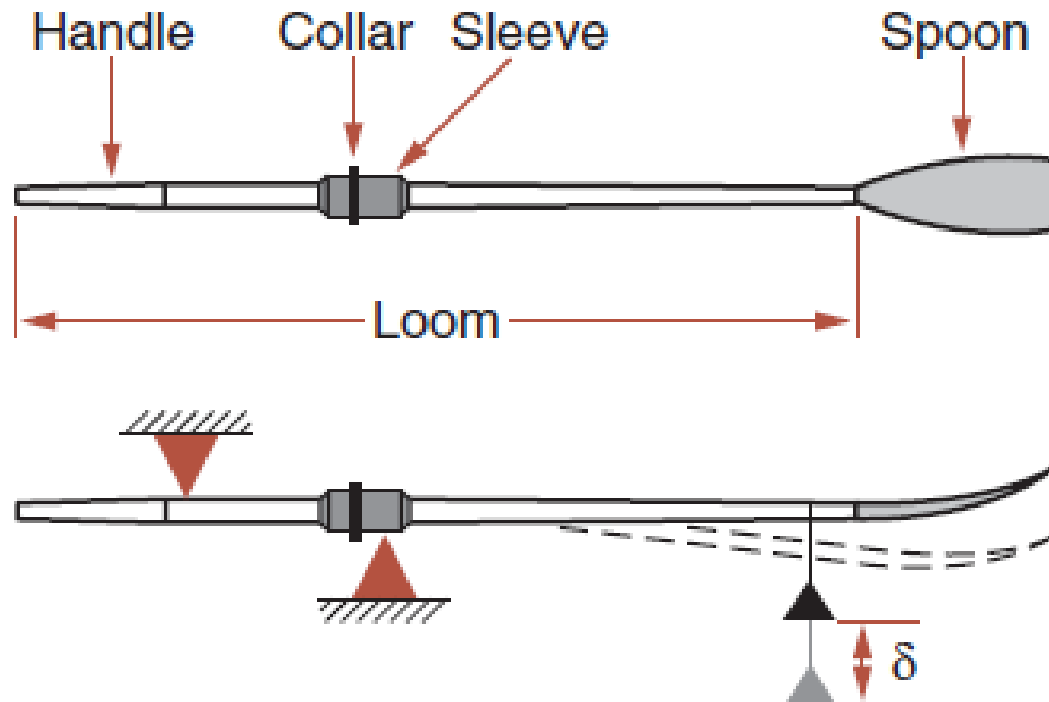
$$I'' = \frac{E^n}{\rho \cdot CO_2}$$

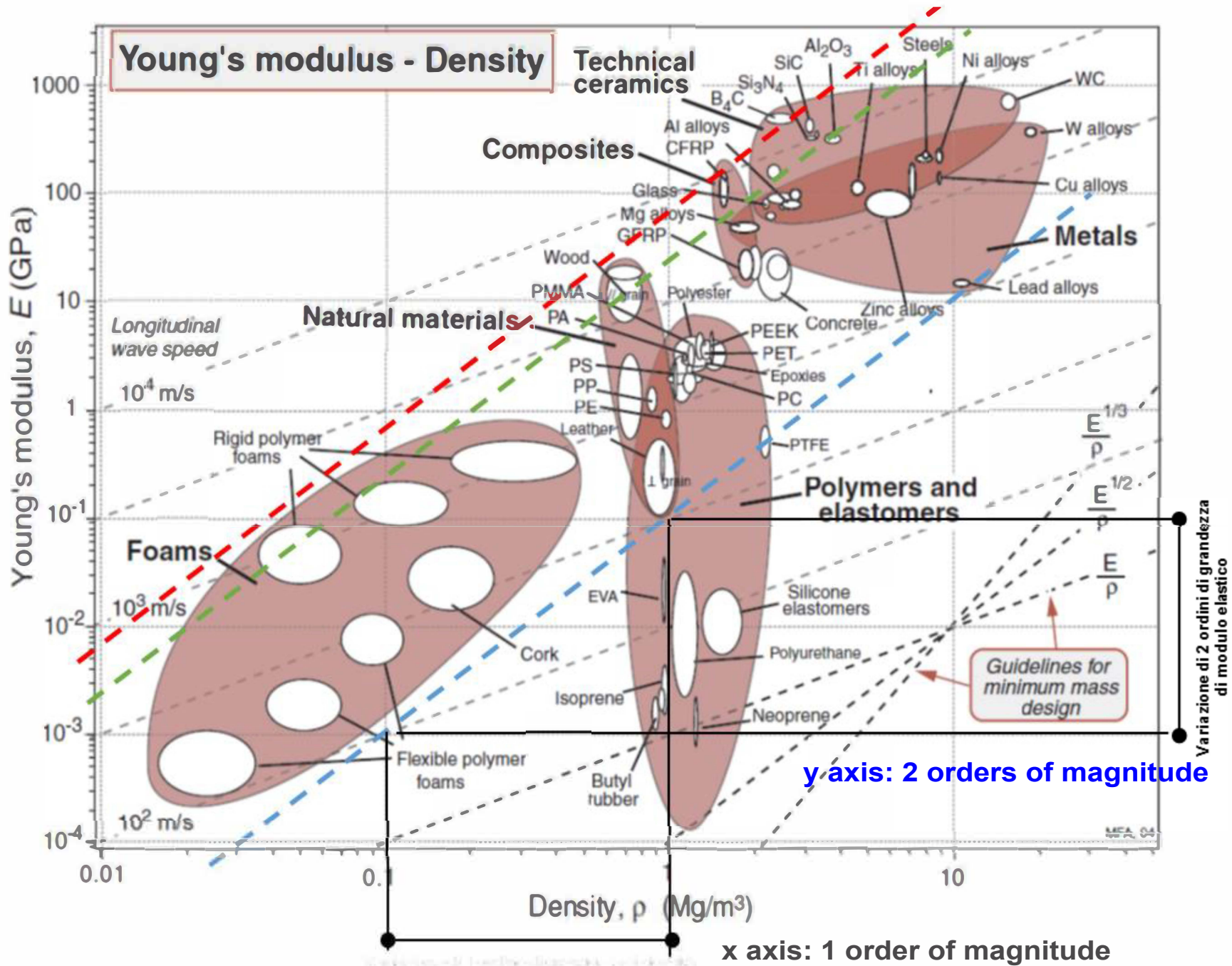
$$[CO_2] = \frac{Kg \text{ } CO_2 \text{ EMITTED}}{Kg \text{ PRODUCT}}$$

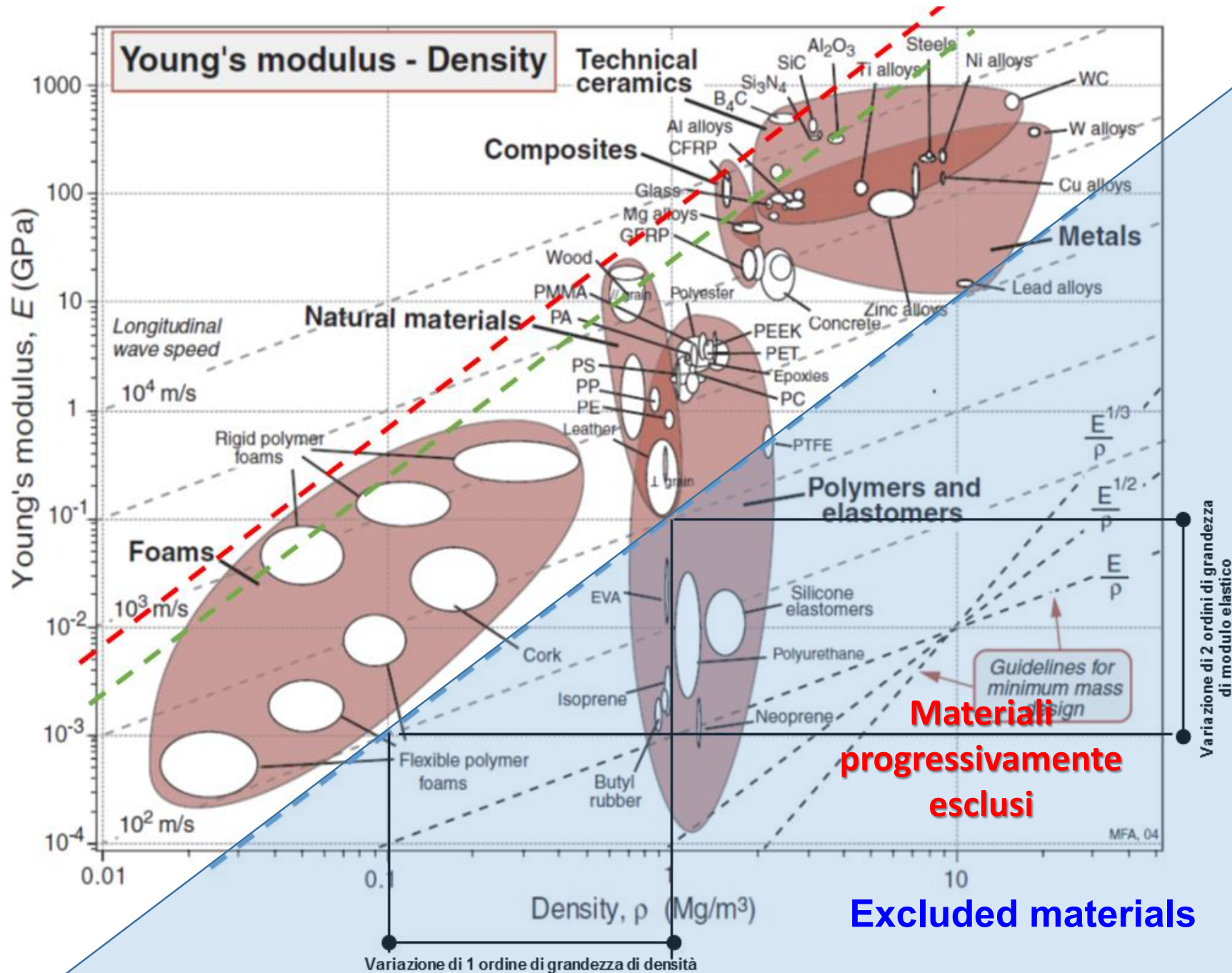
↑ AMOUNT OF CO_2 PER UNIT PRODUCT

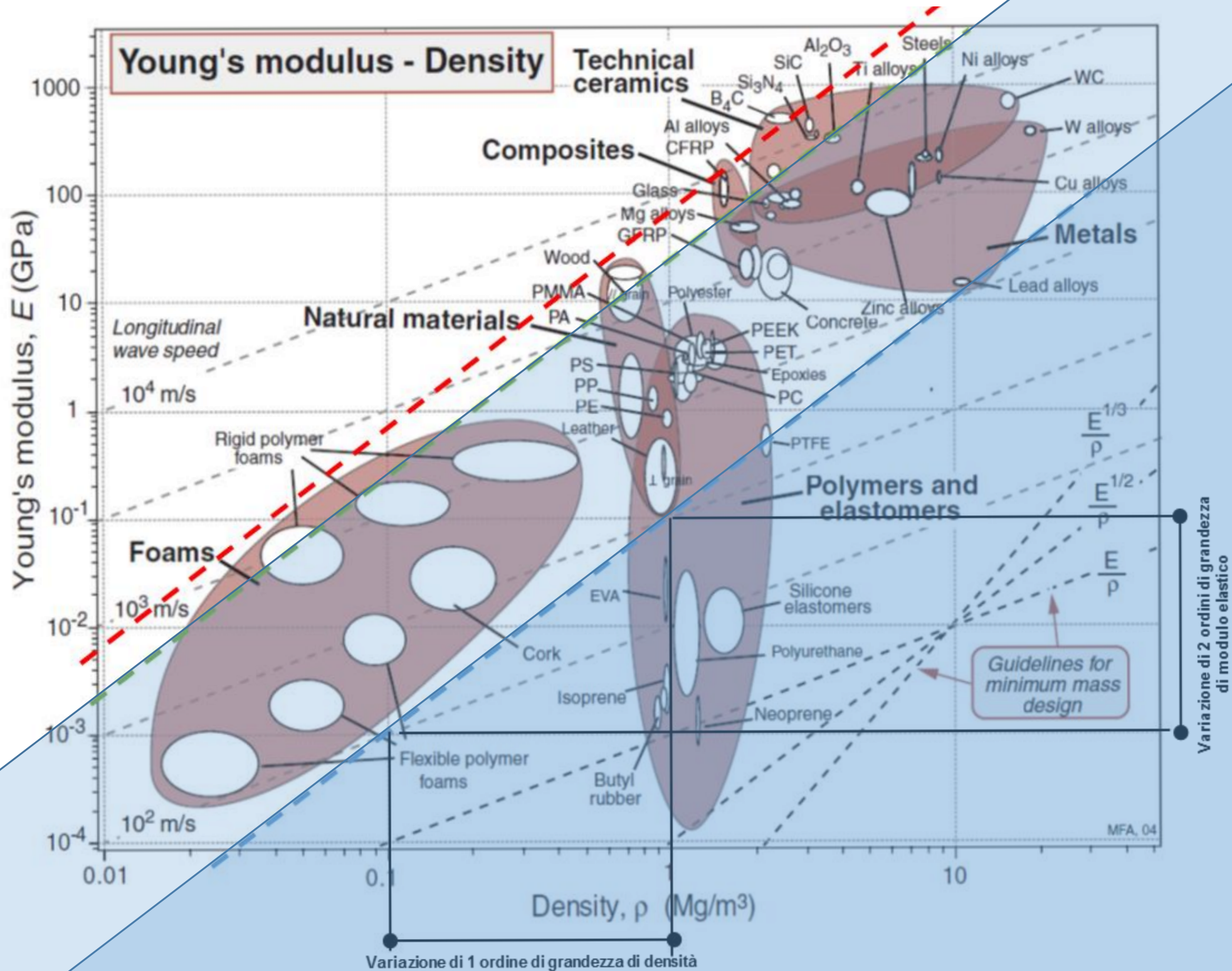
- MIND
1. REDUCING H_m OR CO_2 IS NOT ADVANTAGEOUS BY ITSELF: MATERIALS WITH LOW H_m OR LOW CO_2 ARE CONSIDERED ALSO FOR E AND ρ
 2. H_m AND CO_2 MAY BE REDUCED BY IMPLEMENTING RECYCLING
 $H_R = H_m - \Delta H$
 $CO_{2R} = CO_2 - \Delta CO_2$

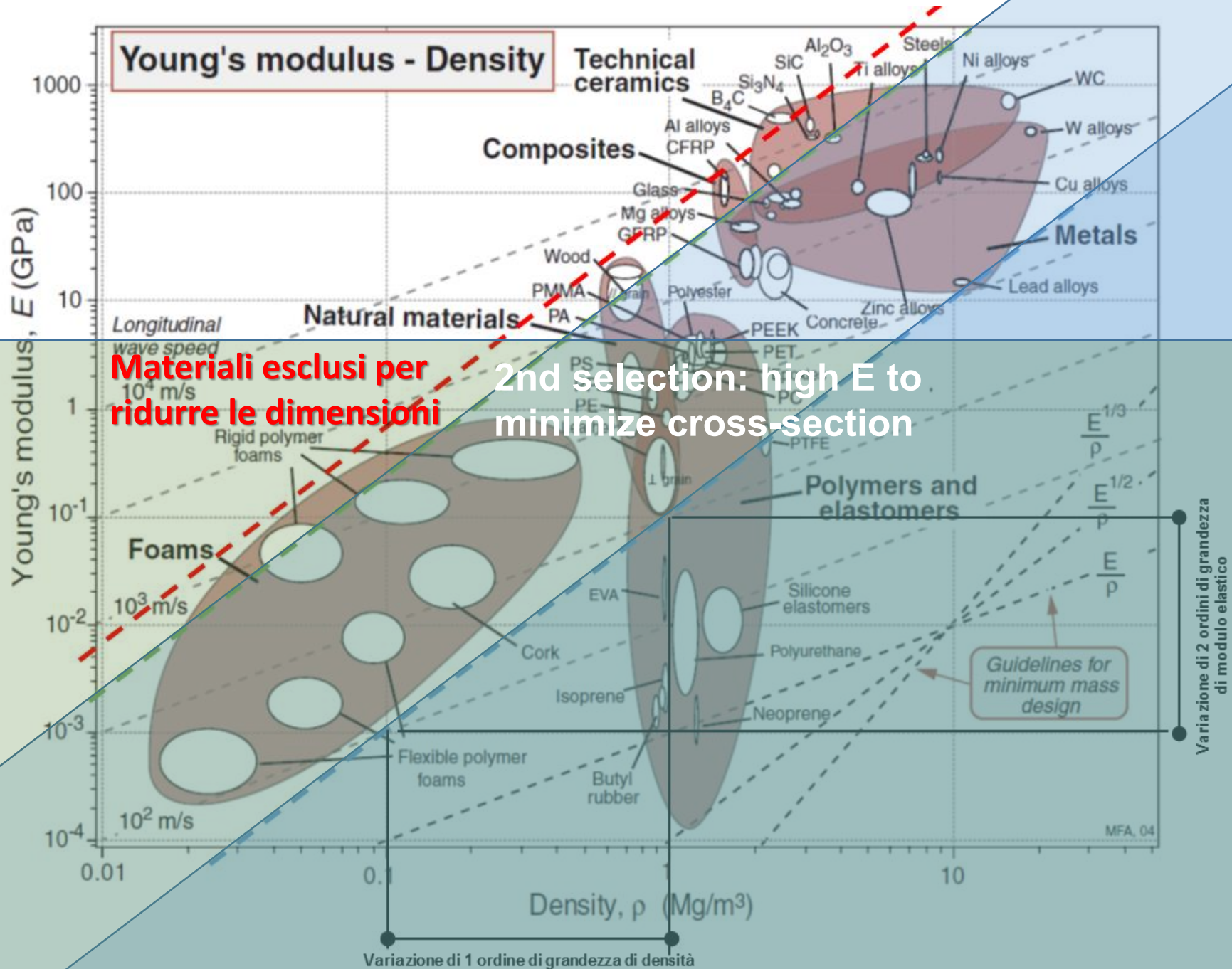
Materials for oars

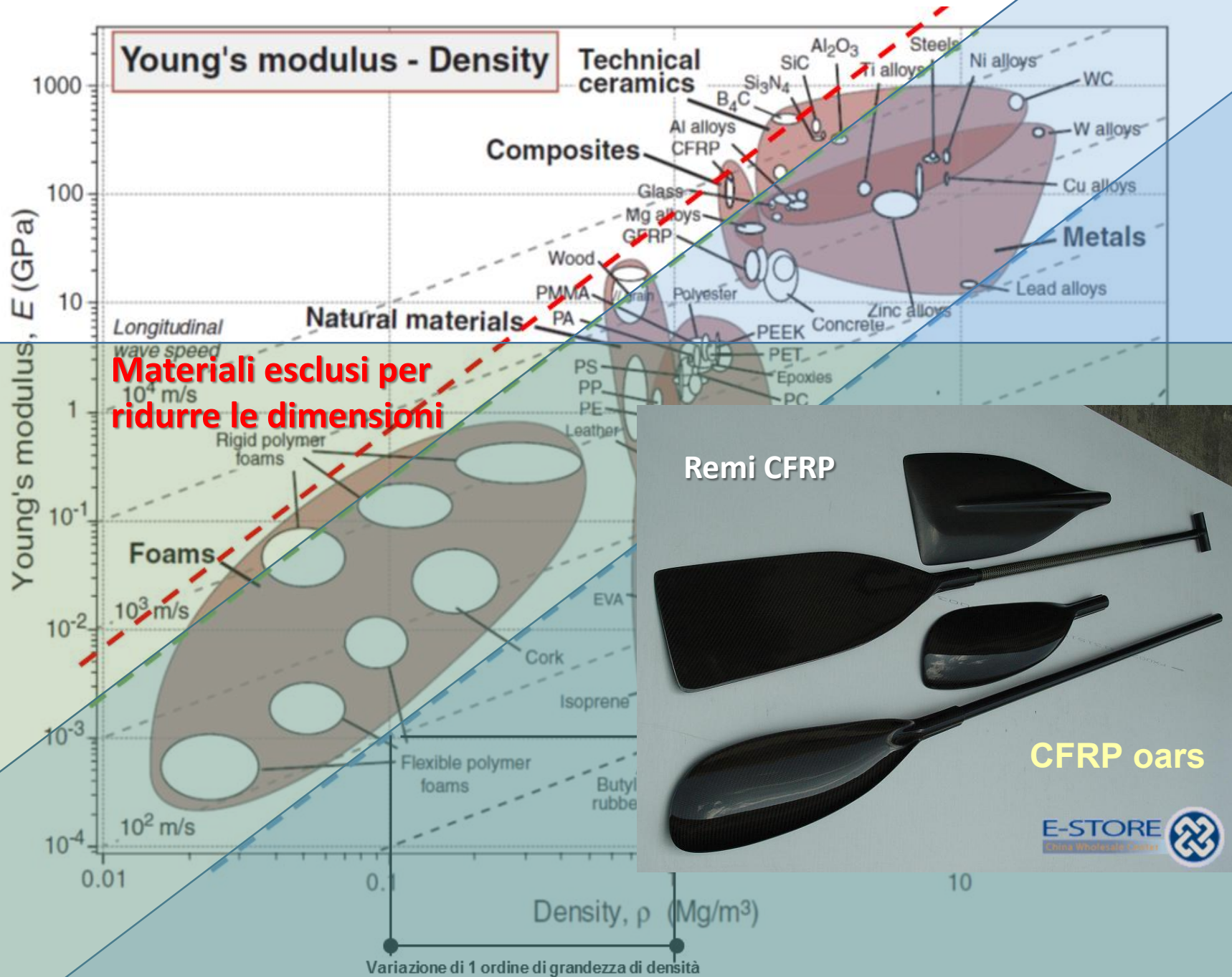






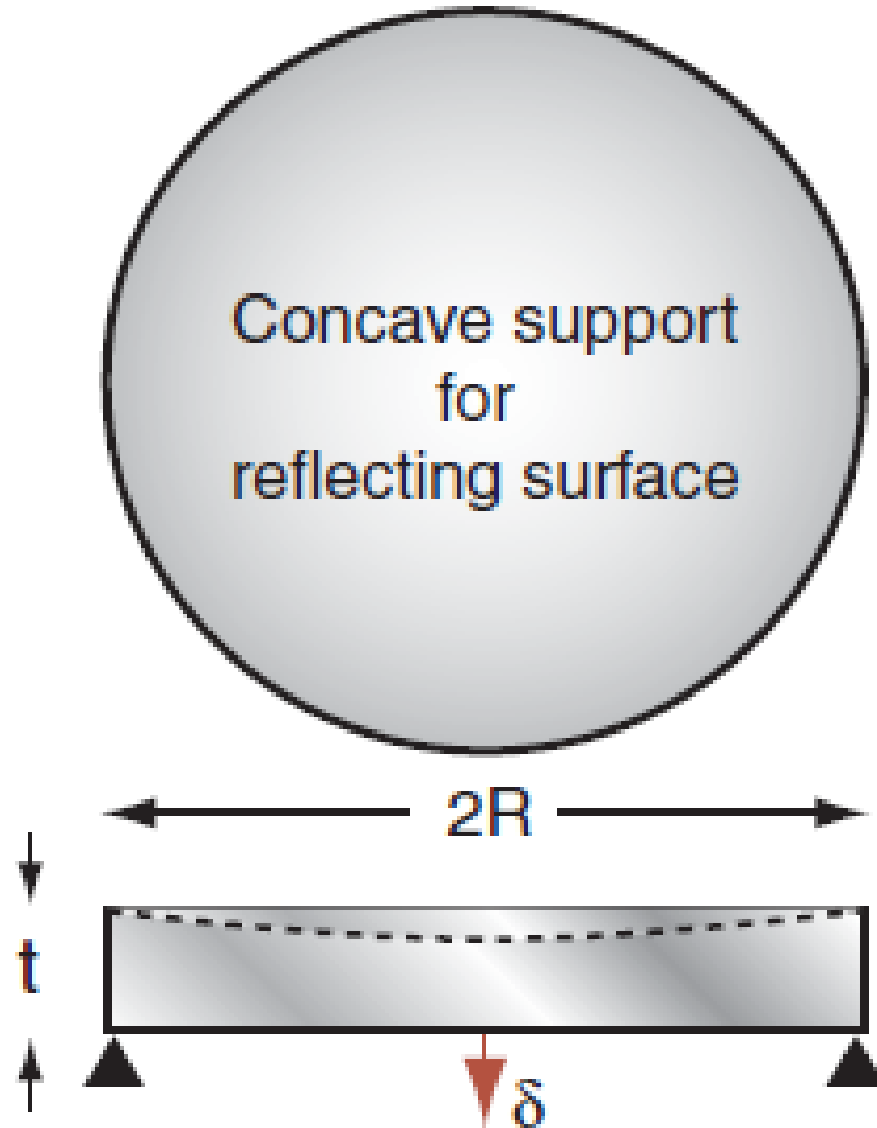


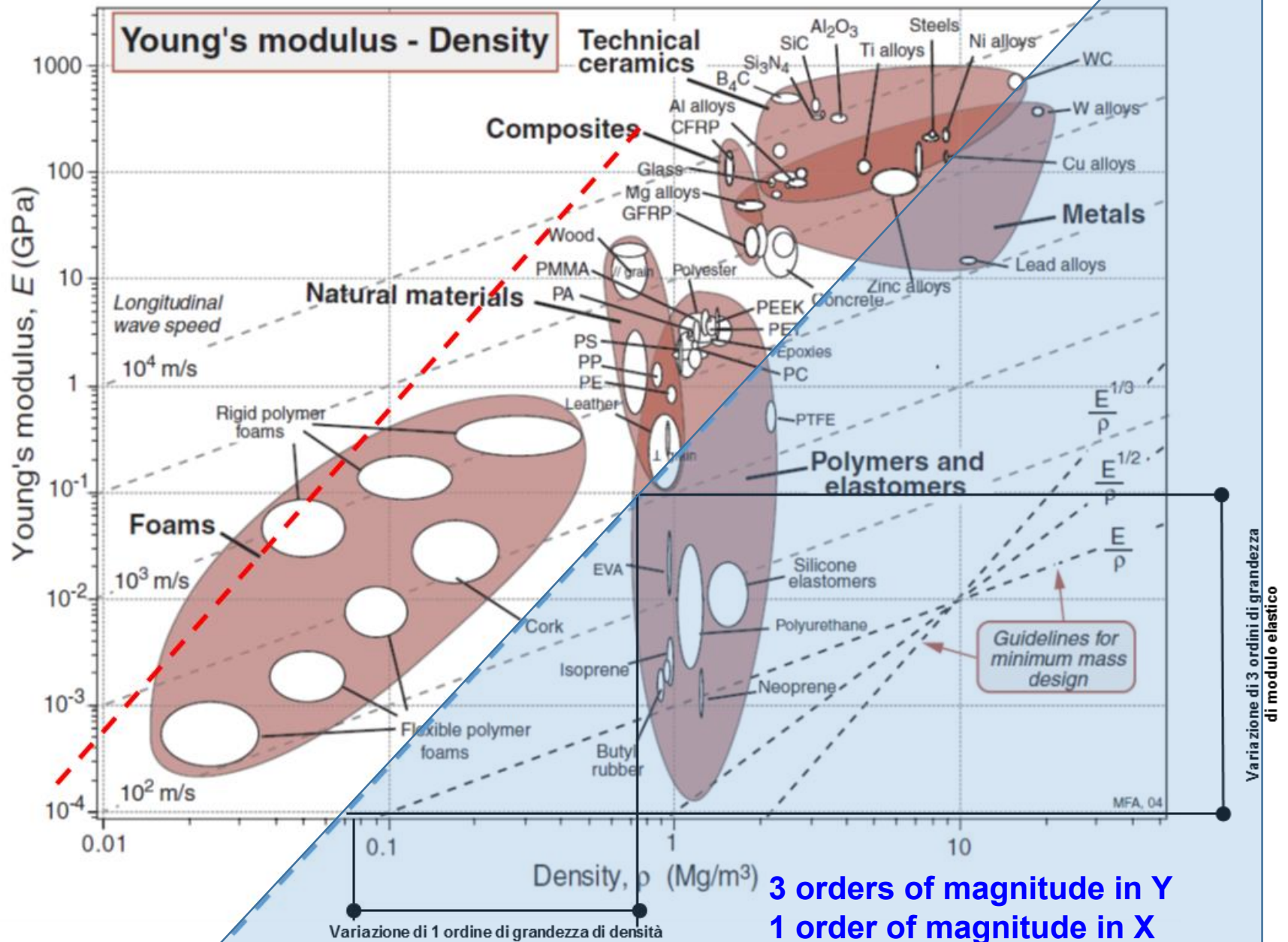




Mirror blanks

Approximate model:
panel





Graphical selection

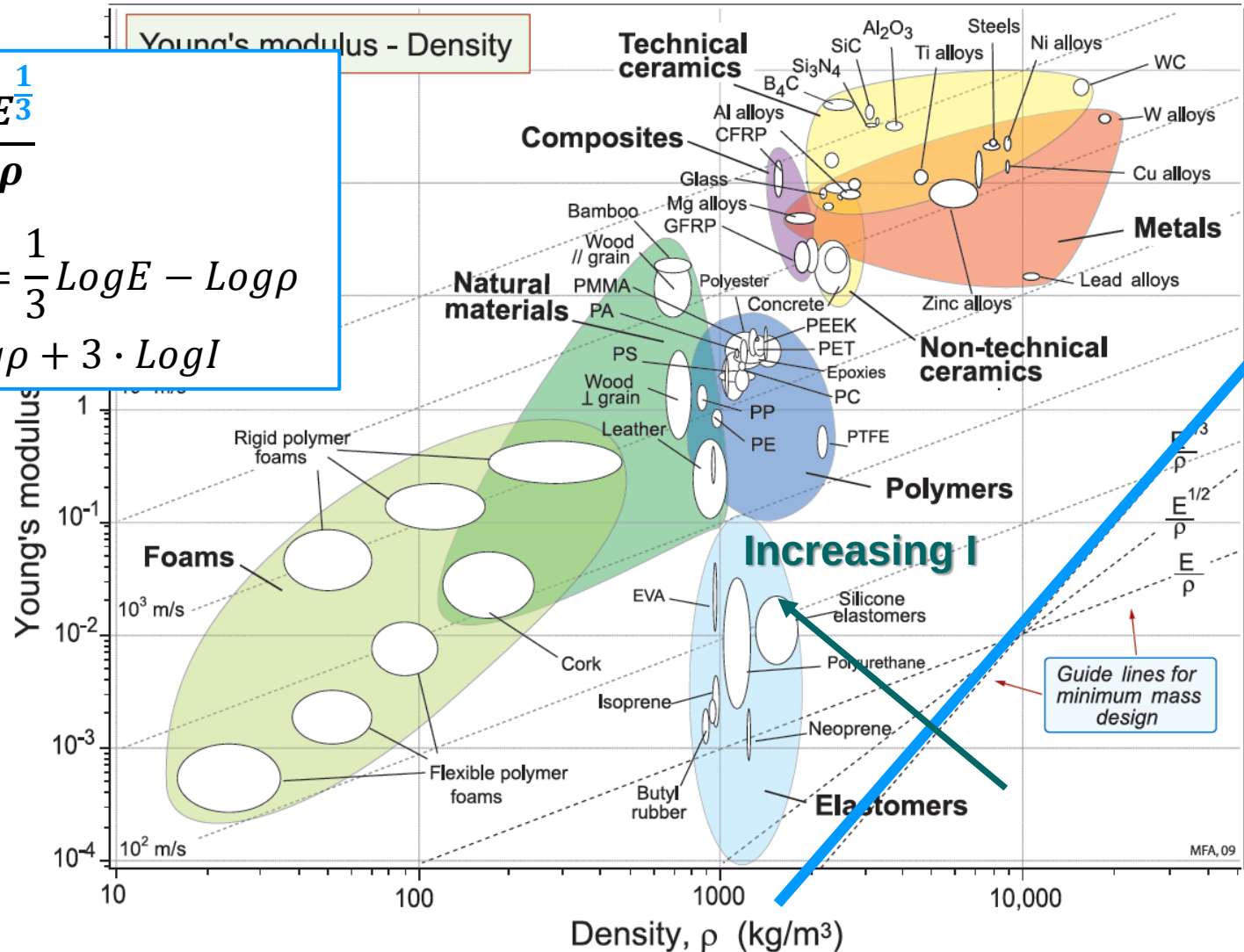
Panels: different 'mechanical exponent'
Dramatic change of selection lines

Light stiff panels

$$I = \frac{E^{\frac{1}{3}}}{\rho}$$

$$\text{Log} I = \text{Log} \left(\frac{E^{\frac{1}{3}}}{\rho} \right) = \frac{1}{3} \text{Log} E - \text{Log} \rho$$

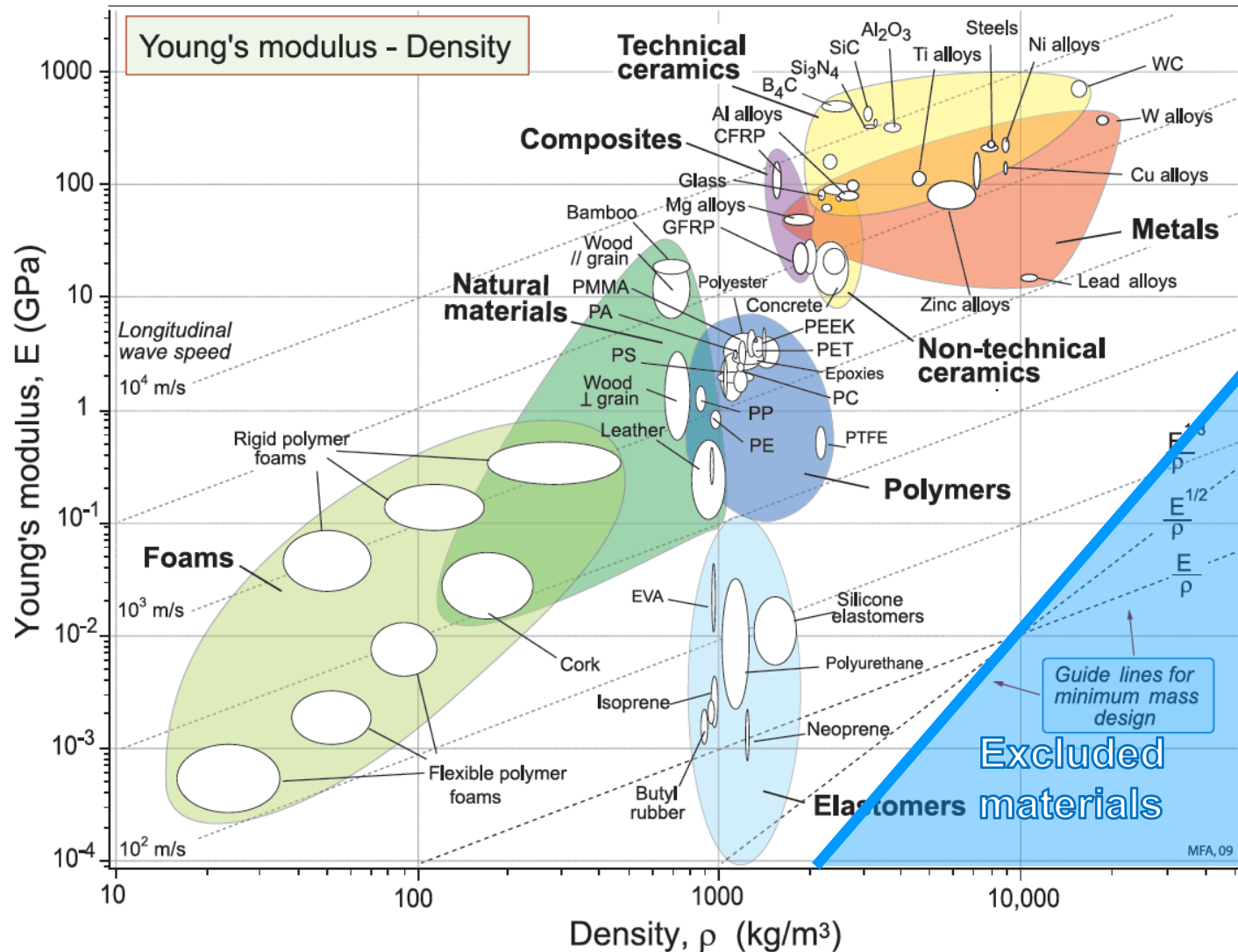
$$\text{Log} E = 3 \cdot \text{Log} \rho + 3 \cdot \text{Log} I$$



Graphical selection

Panels: different 'mechanical exponent'
Dramatic change of selection lines

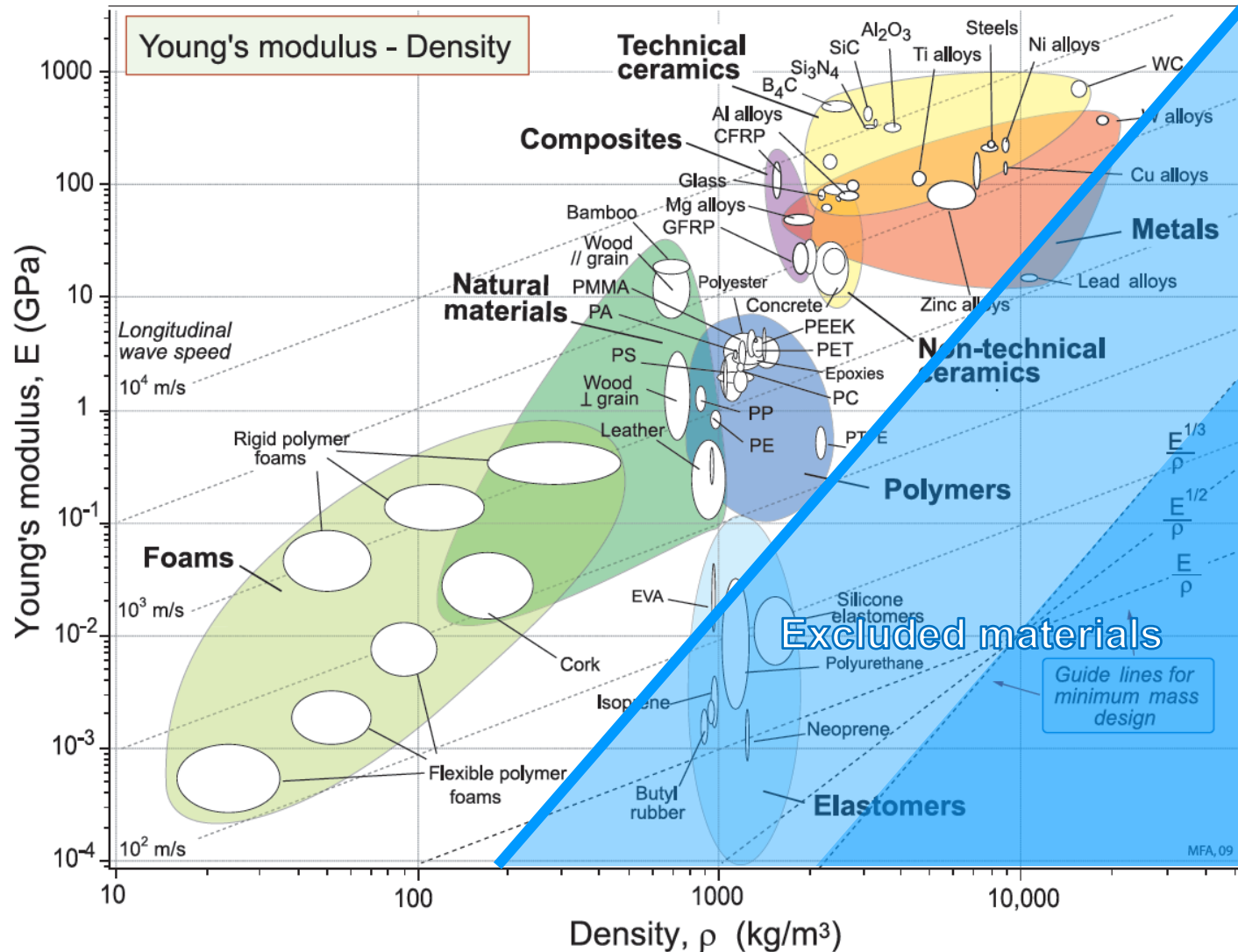
**Materials
selection for
light stiff panels
Reduction of
mass**



Graphical selection

Panels: different 'mechanical exponent'
Dramatic change of selection lines

**Materials
selection for
light stiff panels**
**Reduction of
mass**

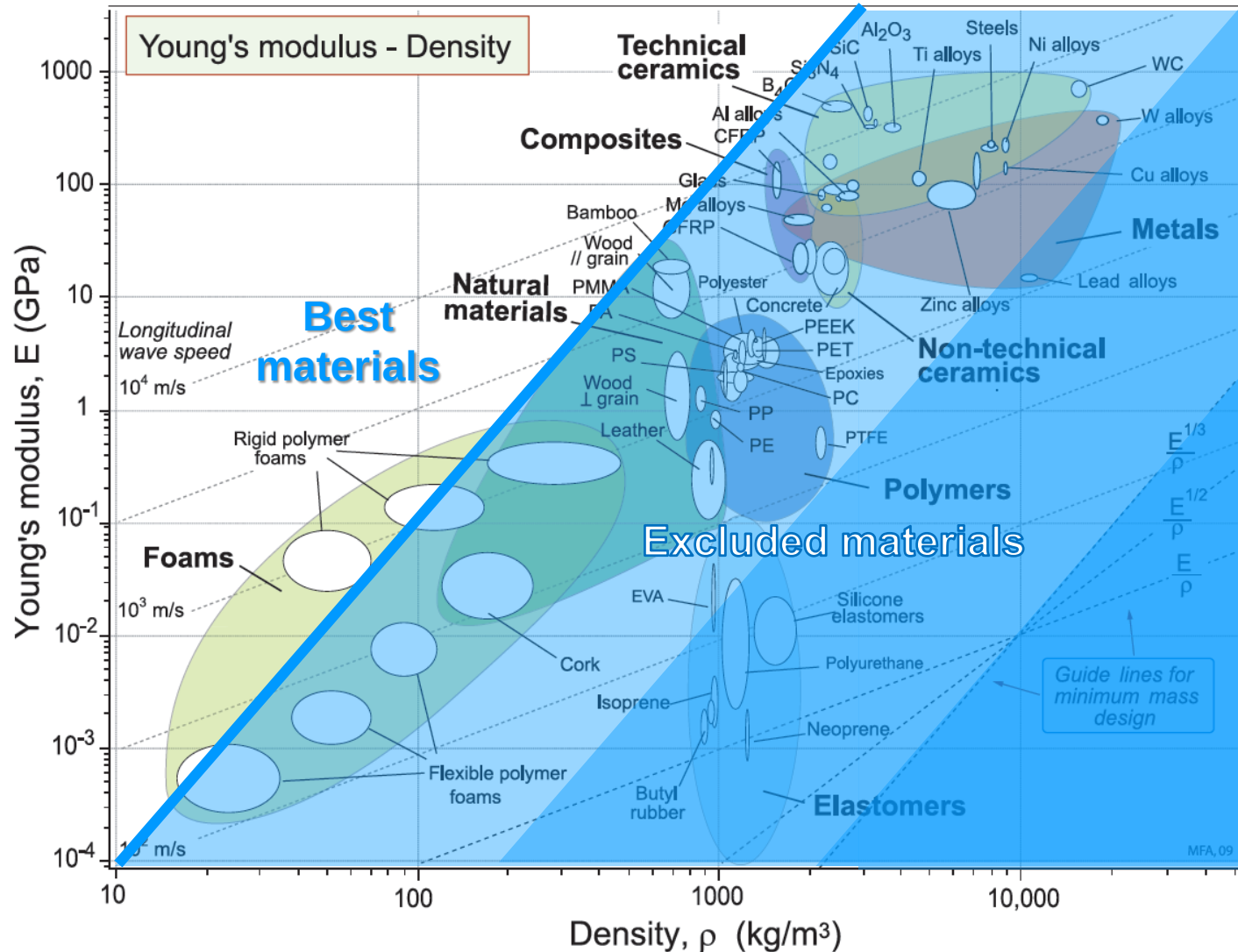




Graphical selection

Panels: different 'mechanical exponent'
Dramatic change of selection lines

Materials selection for light stiff panels
Reduction of mass
Rigid foams become attractive
Among rigid foams, ceramic foams are particularly interesting (lower CTE, higher thermal stability compared to others)





Optimum solution

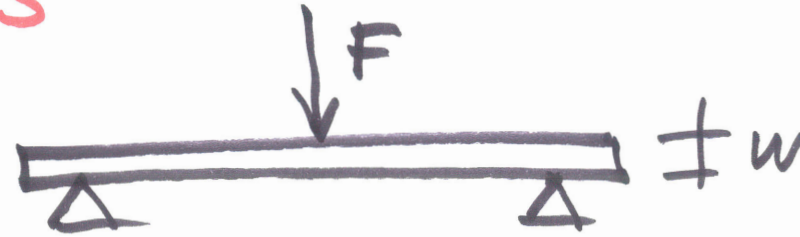
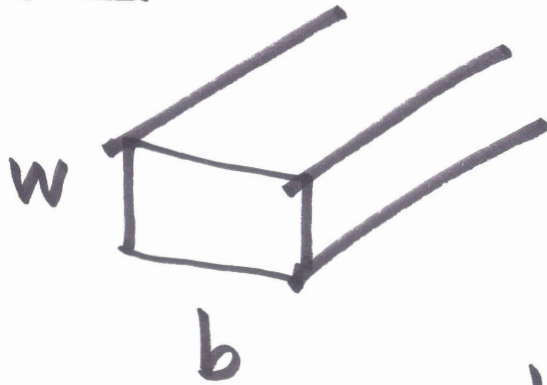
Cellular solid or honeycomb (variant of cellular solid) in glass or glass-ceramic material with extremely low thermal expansion coefficient

E.g. Schott Zerodur® glass-ceramics [www.schott.com]

BARS

CROSS SECTION CASE STUDIES

Detailed demonstrations,
OPTIONAL



$$A = b \cdot w$$

$$b = \alpha \cdot w$$

$$w = b / \alpha$$

α = PROPORTION
b VS. w

$$J = \frac{bw^3}{12} = \frac{bw \cdot w^2}{12} = \frac{bw \cdot \frac{bw}{\alpha}}{12} = \frac{A^2}{12\alpha}$$

RECTANGULAR
CROSS
SECTION

CIRCULAR CROSS SECTION

$$J = \frac{\pi R^4}{4} = \frac{(\pi R^2)^2}{4\pi} = \frac{A^2}{4\pi}$$

$J = C \cdot A^2$

IN ALL CASES