Student Seminar: May 31th, 2013 Danilo Lewanski, University of Padova

On the XIVth Hilbert Problem and its influence on Geometric Invariant Theory

Abstract

Consider an algebraic group G (e.g. $GL(n,k)$) acting on an algebraic variety X. What is the geometric meaning of X/G? In which cases one can give it a structure of algebraic variety? This is strongly related to moduli (intended as *parameters*!) problems which are somehow formalizations of classifications problems in algebraic geometry. In 1900 David Hilbert at the Paris conference of the International Congress of Mathematicians presented a list of unsolved problems that were very influential for 20th century mathematics. The XIVth problem asks whether an invariant algebra is in general finitely generated or not. The answer came in 1959 from M.Nagata. The solution brought to the study of the concept of linear and geometric reductiveness which led, thanks to Mumford's 'Geometric Invariant Theory' published few years later, to the fundamental theorems for the affine and the projective quotients X/G.

MUM D.Mumford, J.Fogarty, F.Kirwan, *Geometric Invariant Theory*, third enlarged edition, Springer-Verlag, 1994.

NEW P.Newstead, *Introduction to moduli problems and orbit spaces*, Tata Institute of fundamental research, Bombay, Springer-Verlag, 1978.

DIE J.Dieudonné and J. Carrell, *Invariant theory old and new*, Academic press,1971.

NAG M.Nagata, *Invariants of a group in an affine ring* J.Math.Kyoto Univ. 3 (1963/64), 369-377