

The Gabriel-Popescu Theorem

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June 2, 2016

Abstract

The purpose of this lecture is to present definitions, lemmas and propositions which will lead us to the Gabriel-Popescu Theorem. We begin with the definition of preradicals, idempotent radicals and their link with the definition of torsion pair, and introduce the definition and the properties of hereditary torsion pair. Then we will introduce the definition of reflexive subcategory, localising functor and Giraud subcategory. The principal part of the lecture is dedicated to the characteristics of Gabriel filter on a ring, which can be viewed as fundamental system of neighborhoods of 0 for a linear topology on a ring, the relationship between a Gabriel filter and a hereditary torsion pair in a category of modules over a ring R , showing also some examples. Finally we will give the statement of the Gabriel-Popescu Theorem.

References

- [1] B. Stenstrom *Ring of Quotients*, Berlin, Heidelberg, New York: Springer-Verlag 1975
- [2] C. Nastasescu *Teorie della Torsione*, Ferrara: Istituto matematico dell'Università di Ferrara 1974