

Introduction to Affine Group Schemes

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Abstract

When talking about affine schemes, one usually thinks about objects like $\mathrm{Spec} A$, for some ring A , equipped with their sheaves of rings $\mathcal{O}_{\mathrm{Spec} A}$. What does it happen when we also consider some group structure on them? Here comes what is usually called an *affine group scheme*. Anyway, our approach to the subject will be slightly different.

It is possible to define affine group schemes as representable functors

$$G: k\text{-Alg} \rightarrow \text{Groups},$$

or equivalently, as Hopf algebras over k , so that we can choose one of these two viewpoints, according to the properties we want to focus on.

We will then introduce basic notions about linear representations and comodules. All this machinery will then show how matrices are at the very heart of the theory.

Some basic motivating examples will be given.

References

- [1] R. Hartshorne: *Algebraic Geometry*, Springer-Verlaag, New York (1977).
- [2] J.S. Milne: *Basic Theory of Affine Group Schemes* (2012).
- [3] W. C. Waterhouse: *Introduction to Affine Group Schemes*, Graduate Texts in Mathematics, vol. 66, Springer-Verlaag, New York (1979).